Chapter 1: Fundamental circuit theorems

Introduction

In this chapter, the concept of the most common circuit elements either active or passive is introduced, including basic law. Superposition, Thevenin, Milman and Norton’s theorems will be discussed.

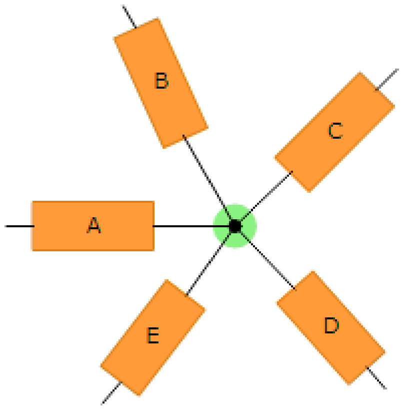
3. Circuit terminology

Some basic terminology in the field of electronic will be given in this section:

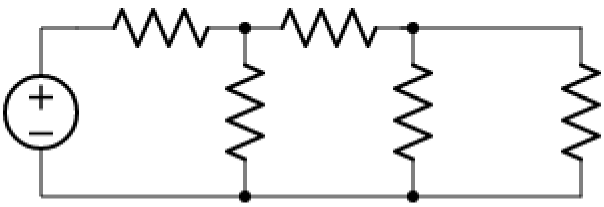
- **Circuit** – A circuit is a closed loop conducting path in which an electrical current flows.
  - **Closed circuit** – A circuit is closed if the circle is complete, if all currents have a path back to where they came from.
  - **Open circuit** – A circuit is open if the circle is not complete, if there is a gap or opening in the path.
  - **Short circuit** – A short happens when a path of low resistance is connected (usually by mistake) to a component. The resistor shown below is the intended path for current, and the curved wire going around it is the short. Current is diverted away from its intended path, sometimes with damaging results. The wire shorts out the resistor by providing a low-resistance path for current (probably not what the designer intended).
- **Elements** are either sources or components.
  - **Sources** provide energy to a circuit. There are two basic types.
    - Voltage source
    - Current source
  - **Components** come in three basic types, each characterized by a different voltage-current relationship.
    - Resistor

- Capacitor
- Inductor

- **Path** – a single line of connecting elements or sources.
- **Node** – a node is a junction, connection or terminal within a circuit were two or more circuit elements are connected or joined together giving a connection point between two or more branches. A node is indicated by a dot.

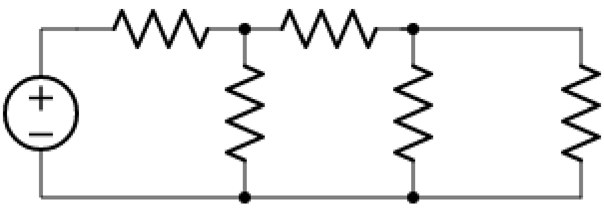


Question : How many nodes are in this circuit? (4)



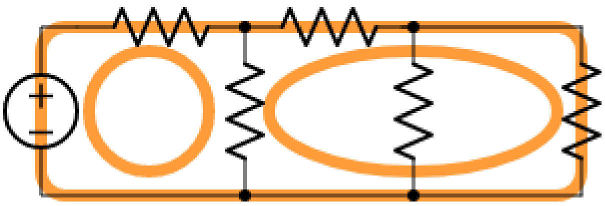
- **Branch** – a branch is a single or group of components such as resistors or a source which are connected between two nodes. The number of branches in a circuit is equal to the number of elements.

Question: How many branches are in this circuit? (6)



- **Loop** – a loop is a simple closed path in a circuit in which no circuit element or node is encountered more than once. To draw a loop, select any node as a starting point and draw a path through elements and nodes until the path comes back to the node where you started.

Question: How many loop are in this schematic? (6)



- **Mesh** – a mesh is a single closed loop series path that does not contain any other paths. There are no loops inside a mesh. Whereas, mesh analysis is governed by Kirchhoff's voltage law.
- **Reference Node** – During circuit analysis we usually pick one of the nodes in the circuit to be the *reference node*. Voltages at other nodes are measured relative to the reference node. Any node can be the reference, but two common choices that simplify circuit analysis are,

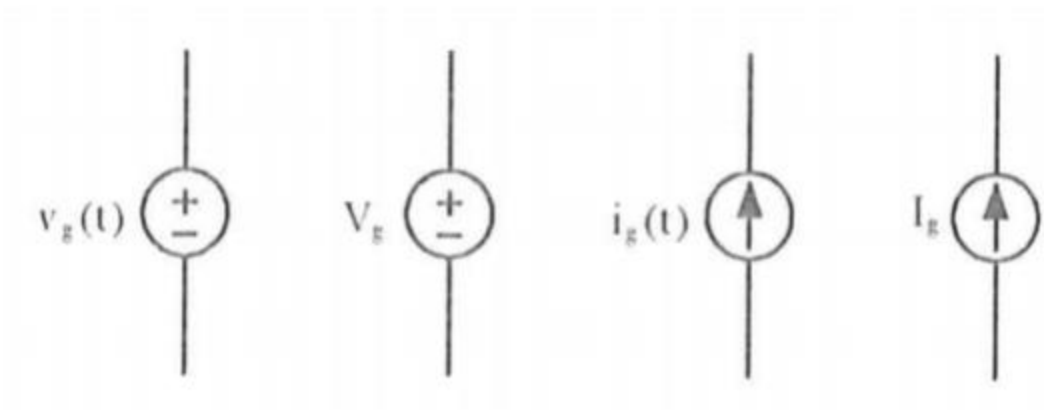
✓ the negative terminal of the voltage or current source powering the circuit, or

✓ the node connected to the greatest number of branches.

- **Ground** – The reference node is often referred to as *ground*. The concept of *ground* has three important meanings.

  you will come across various symbols for ground:

- **DC and AC sources:** In AC sources, the voltage or current supplied varies with time, they are symbolized with non-capital letters and normally time dependence is indicated. In DC sources the voltage or current supplied is constant and will be symbolized by omitting the explicit dependence with time and using capital letters.



4. Basic circuits elements

There are two types of elements found in electric circuits: passive elements and active elements. An active element is capable of generating energy while a passive element is not.

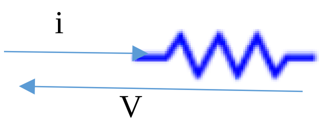
4.1. Passive elements

A passive element dissipates energy only. Resistors, capacitors, and inductors are the three fundamental passive circuit elements used in electric circuits.

4.1.1. Resistor

A resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element. Resistors reduce the current flow and lower voltage levels within circuits. Most circuits often have more than one resistor to limit the flow of charges in a circuit.

The symbol of this element is depicted in the Figure bellow:

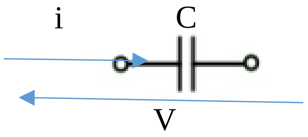


The inverse of the resistance is called conductance, it is symbolized by the letter G and it is measured in Siemens [S]. Conductance symbolizes the ease with which the current flows through a certain element.

$$G = 1/R \text{ [S]}$$

4.1.2. Capacitor

An electric circuit element that has an ability of storing electrical energy in the form of electric field is called a **capacitor**. The property of the capacitor by virtue of which it stores electrical energy is known as **capacitance**. The symbol of this element is depicted in the Figure bellow:



The expression of the current of a capacitor is given by,

$$i = C\frac{dv}{dt}$$

4.1.3. Inductor

Inductor is basically a wire of finite length twisted into a coil. An inductor is also a basic circuit element that used to introduce inductance in an electrical or electronic circuit. The inductor has a property, known as **inductance**. That is used in most power electronic circuits to store energy in the form of magnetic energy when electricity is applied to it.

The circuit symbol of a typical inductor is shown in the following figure.

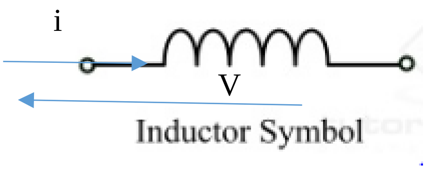


Figure 1. 1: Inductor symbol

The voltage across an inductor is given by,

$$v = L\frac{di}{dt}$$

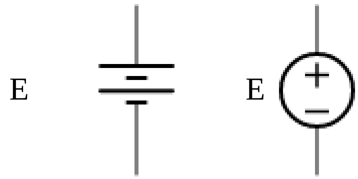
4.2. Active elements

Such as voltage source and current source.

4.2.1. Voltage sources

A voltage source is an active element that provides a specified and constant voltage which is completely independent of any other circuit elements. However, the rated voltage across the terminals of real or practical voltage sources drops off as the load current it supplies increases.

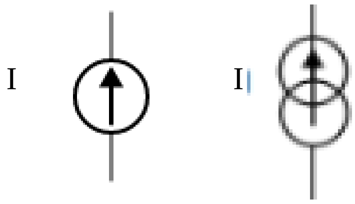
The two common symbols for constant voltage sources:



4.2.2. Current sources

A current source is an active circuit element that is capable of supplying a constant current flow to a circuit regardless of the voltage developed across its terminals.

The two common symbols for a constant current source:



The arrow indicates the direction of positive current flow.

5. Circuit elements association

Circuit elements of the same type can be associated to ease and simplify circuit solving. Element associations can be in series or in parallel. We can group the elements to simplify the analyze of circuits:

5.1. Serial Association

Two circuit elements are connected in series when they are placed one after the other, that is, when they have a common terminal that they do not share with any other element of the circuit.

When two elements are connected in series, the same current circulates through both (the voltage in each of them will be, in general, different).

In the series circuits

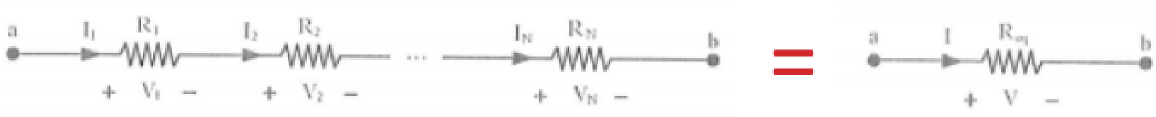
1) The same current flows through each resistance. $I_1 = I_2 = \ldots = I_N = I$

2) The supply voltage **V** is the sum of the individual voltage drops across the resistances.

$$\mathbf{V = V_1 + V_2 + V_3 + \ldots + V_N}$$

5.1.1. Resistors in series

Consider the resistances shown in the bellow figure.



The total or equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

For **N** resistances in series,

$$R_{eq} = R_1 + R_2 + \ldots + R_3 = \sum_{n=1}^{N} R_n$$

5.1.2. Capacitor in series

Capacitances in series combine like resistors in parallel.

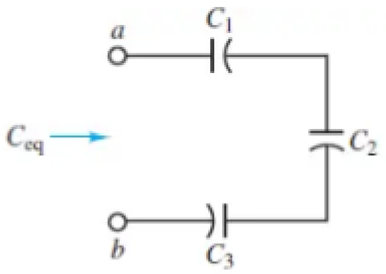


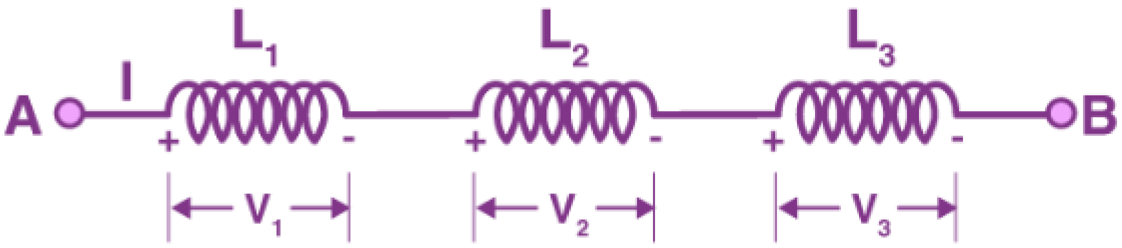
Figure 1. 2: Capacitance in circuit with series capacitors

The equivalent capacitance of capacitors connected in series is given by,

$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

5.1.3. Inductor in series

Consider the connection below:

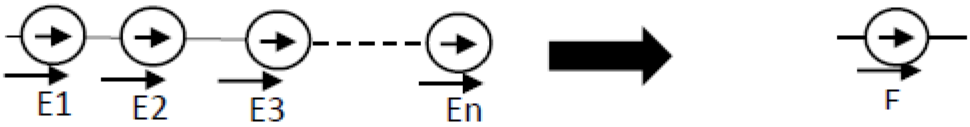


The equivalent inductance of inductors connected in series is given by,

$$L_s = L_1 + L_2 + L_3 + \ldots + L_n$$

5.1.4. Voltage sources in series

Two voltage sources can always be connected in series regardless of their value. Their connection is equivalent to a single source whose voltage is the algebraic sum of the other two.



$$E = E_1 + E_2 + E_3 + \cdots + E_n$$

5.2. Parallel Association

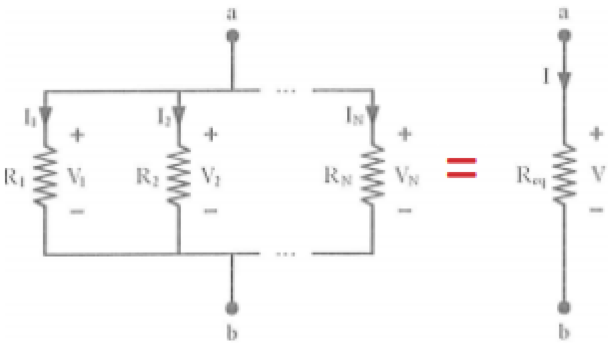
Two circuit elements are connected in parallel when they share their two terminals.

When two elements are connected in parallel, the voltage in both is identical (the current flowing through each of them will be, in general, different).

5.2.1. Resistor in parallel

Consider a parallel circuit shown in the **Fig. 2.6**.

$$\frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_N} = \sum_{n=1}^{N}\frac{1}{R_n}$$



$$\frac{\mathbf{1}}{\boldsymbol{R}} = \frac{\mathbf{1}}{\boldsymbol{R_1}} + \frac{\mathbf{1}}{\boldsymbol{R_2}} + \frac{\mathbf{1}}{\boldsymbol{R_3}} + \cdots + \frac{\mathbf{1}}{\boldsymbol{R_N}} \qquad (2.10)$$

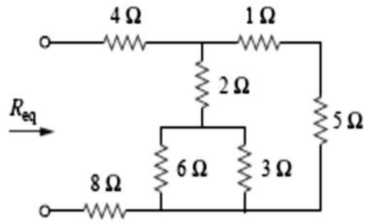
Note that **Req** is always smaller than the resistance of the smallest resistor in the parallel combination.

$$V_1 = V_2 = \ldots = V_N = V$$

$$I_1 + I_2 + \ldots + I_N = I$$

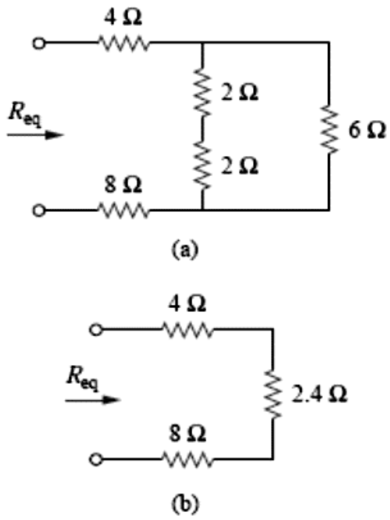
Example

Find **Req** for the circuit shown in **Fig. 1**.



Solution

R_{eq} = 4 Ω + 2.4 Ω + 8 Ω = 14.4 Ω



5.2.2. Capacitor in parallel

Capacitances in parallel combine like resistors in series.

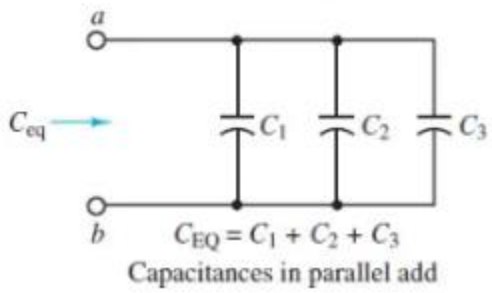


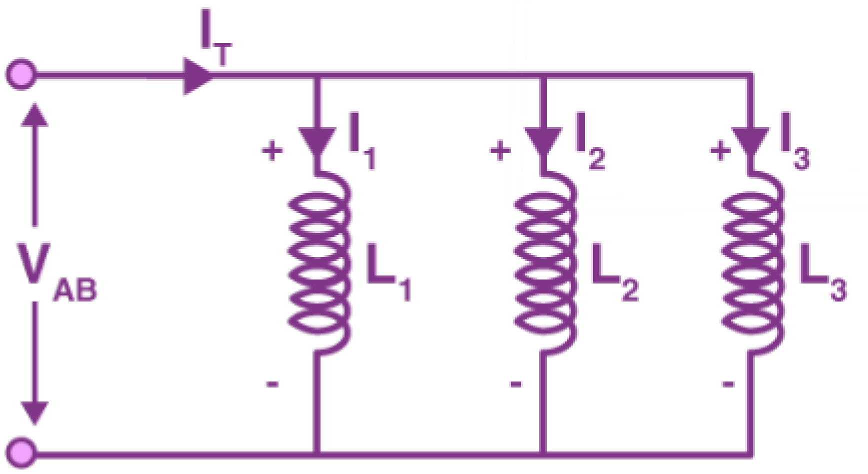
Figure 1. 3: Equivalent capacitance in circuit with parallel capacitors

The equivalent capacitance of capacitors connected in parallel is given by,

$$C_p = C_1 + C_2 + C_3 + \ldots + C_n$$

5.2.3. Inductor in parallel

Consider the example below:



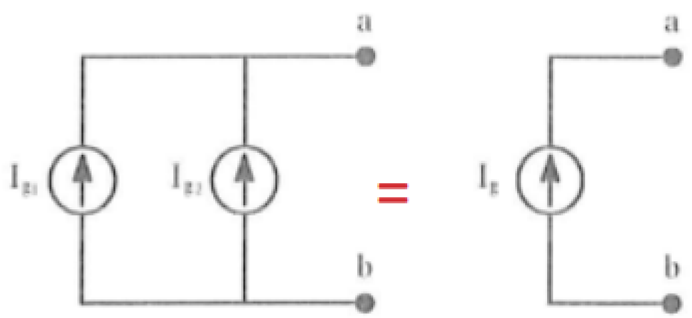
The equivalent inductance of inductors connected in parallel is given by,

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}$$

when inductors are connected in parallel form, their effective inductance decreases. Inductors in parallel are somewhat similar to the capacitors in series.

5.2.4. Current sources in parallel

Two current sources can always be connected in parallel, regardless of their current value. The parallel of two current sources corresponds to a single current source with a current value equal to the algebraic sum of the other two currents.



$$I_g = I_{g1} + I_{g2}$$

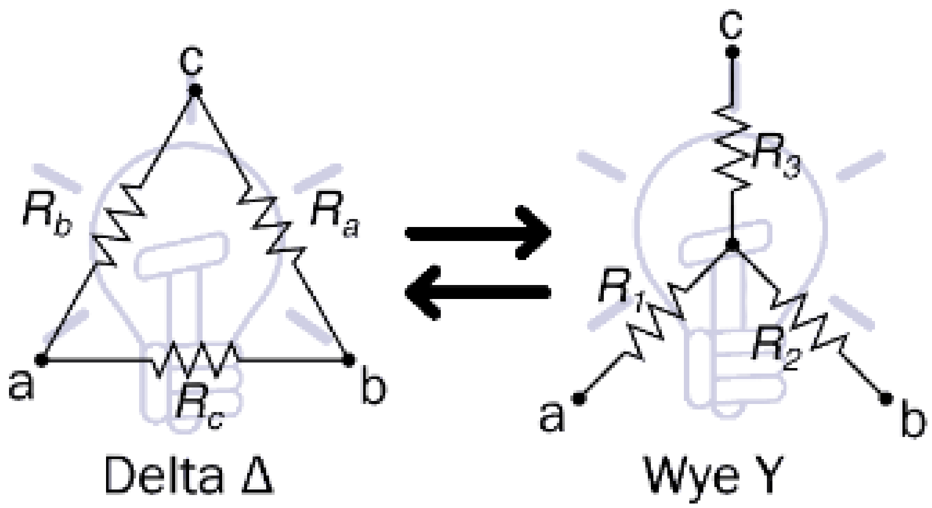
5.3. Series-parallel circuits

A series-parallel configuration is one that is formed by a combination of series and parallel elements. A complex configuration is one in which none of the elements are in series or parallel.

5.4. Kennelly theorem

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series.

The delta (Δ) interconnection is also referred to as Pi interconnection & the wye (Y) interconnection is also referred to as Tee (T) interconnection.



From Delta (Δ) to Wye (Y) Interconnection:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

From Wye (Y) to Delta (Δ) Interconnection

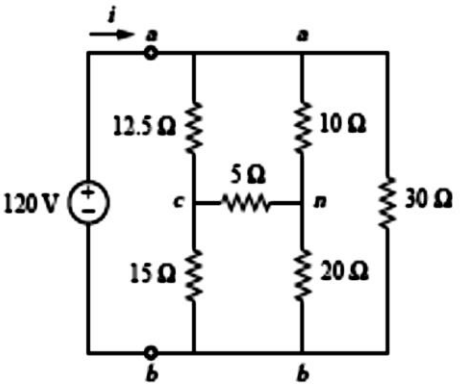
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

5.5. Example

Obtain the equivalent resistance $\mathbf{R_{ab}}$ for the circuit in **Fig. 1** and use it to find current **i**.



5.6. Solution

If we convert the **Y**-network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$\mathbf{R_1 = 10\ \Omega,\quad R_2 = 20\ \Omega,\quad R_3 = 5\ \Omega}$

Thus, from Eqs. (2.24) to (2.26) we have

$$\mathbf{Ra} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R1}} = \frac{\mathbf{10\times20+20\times5+5\times10}}{\mathbf{10}} = \frac{350}{10} = 35\ \Omega$$

$$\mathbf{Rb} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R2}} = \frac{350}{20} = 17.5\Omega$$

$$\mathbf{Rc} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R3}} = \frac{350}{5} = 70\Omega$$

With the **Y** converted to **Δ**, the equivalent circuit (with the voltage source removed for now) is shown in **Fig. 2 (a)**. Combining the three pairs of resistors in parallel, we obtain

$\mathbf{70 \parallel 30 = 70 \times 30 / (70 + 30) = 21\ \Omega}$

$$12.5 \parallel 17.5 = 12.5 \times 17.5 / (12.5 + 17.5) = 7.2917\ \Omega$$

$$15 \parallel 35 = 15 \times 35 / (15 + 35) = 10.5\ \Omega$$

so that the equivalent circuit is shown in **Fig. 2 (b)**. Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = 17.792 \times 21 / (17.792 + 21) = 9.632\ \Omega$$

Then

$$i = v_s / R_{ab} = 120 / 9.632 = 12.458\ A$$

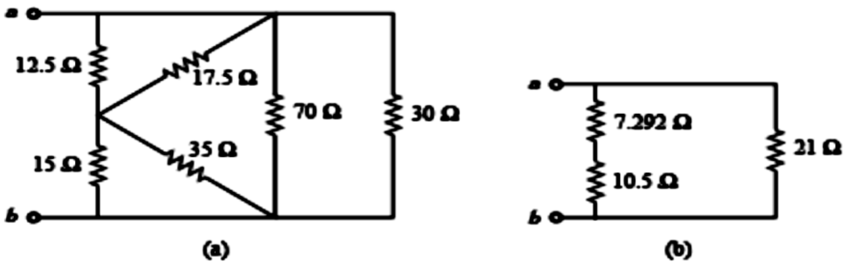


Figure 1. 4: ***Equivalent circuits to Fig. 1, with the voltage removed.***

6. Ideal and real sources

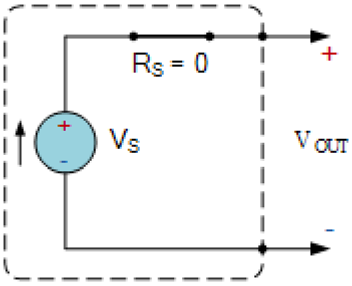
An ideal source is a theoretical concept that represents an idealized behavior of a source. An ideal source has no internal resistance or impedance and can provide infinite power to the circuit.

6.1. Ideal voltage and current sources

Ideal sources have no internal resistance or impedance and can provide infinite power to the circuit.

6.1.1. Ideal voltage sources

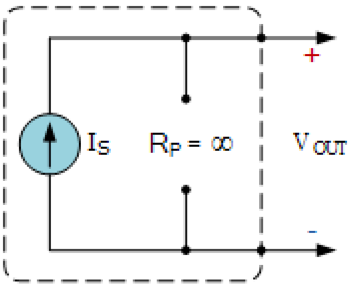
An ideal voltage source maintains a constant voltage across its terminals regardless of the load impedance or current.



Ideal Voltage
Source

3.1.1. Ideal current sources

An ideal current source maintains a constant current through its terminals regardless of the load impedance or voltage.



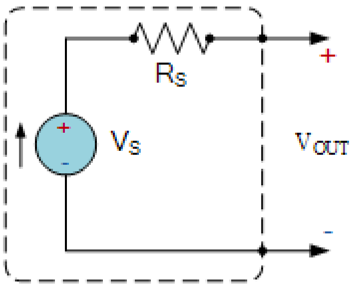
Ideal Current
Source

6.2. Real voltage and current sources

Real sources have some internal resistance or impedance and can provide limited power to the circuit.

6.2.1. Real voltage sources

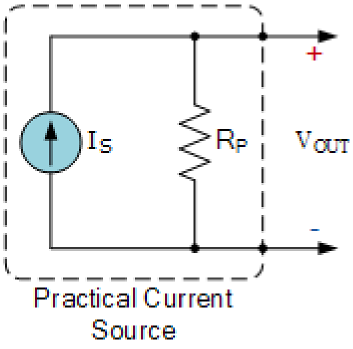
It is composed of a current generator in series with an internal resistance r.



Practical Voltage
Source

6.2.2. Real current sources

It is composed of a current generator in parallel with an internal resistance r.

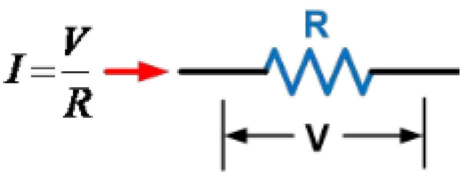


7. Basic laws

The theory is comprised of a number of different laws, ideas, and definitions. These include Ohm's law and Kirchhoff's law, which describe the relationship between current, voltage, and resistance.

7.1. Ohm's law

Ohm's law states that the electrical current flowing through any conductor is directly proportional to the potential difference (voltage) between its ends, assuming the physical conditions of the conductor do not change.



$$\text{Current}, (I) = \frac{\text{Voltage}, (V)}{\text{Resistance}, (R)} \text{ in Amperes}, (A)$$

where

I = Electrical Current Flowing through the Resistor

V = Voltage Drop of the Resistor

R = R is the resistance of the resistor, measured in Ohms (Ω)

7.2. Kirchhoff's laws

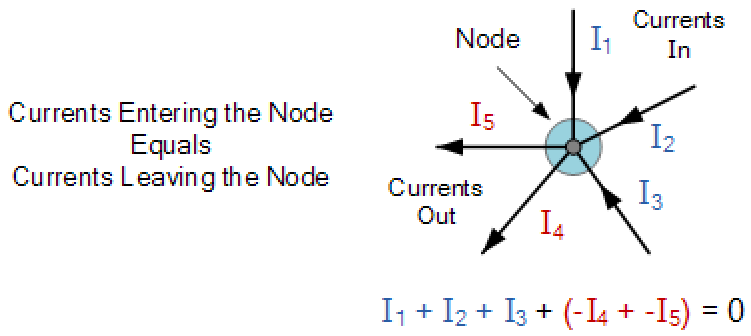
One of the simplest methods to analyze a r circuit is to apply Kirchhoff's laws to the circuit. Kirchhoff's laws are two rules:

7.2.1. Kirchhoff's current law (KCL)

KCL Rule: Algebraic sum of electrical current that merge in a common node of a circuit is zero.

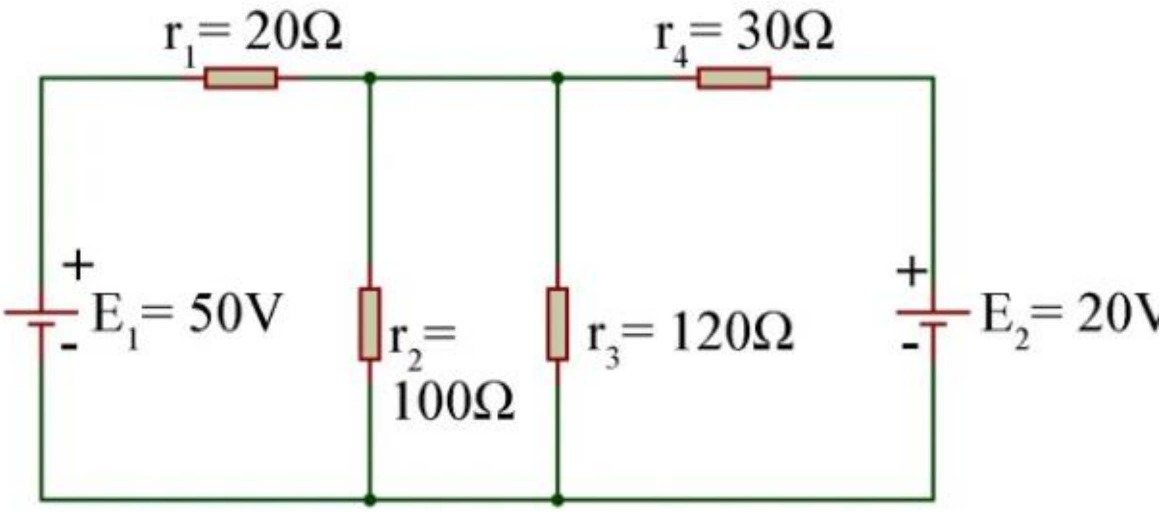
$$\Sigma I_{in} = \Sigma I_{out}$$

Assigning a positive sign to a current that enters a node requires that we assign a negative sign to one that leaves and vice versa.

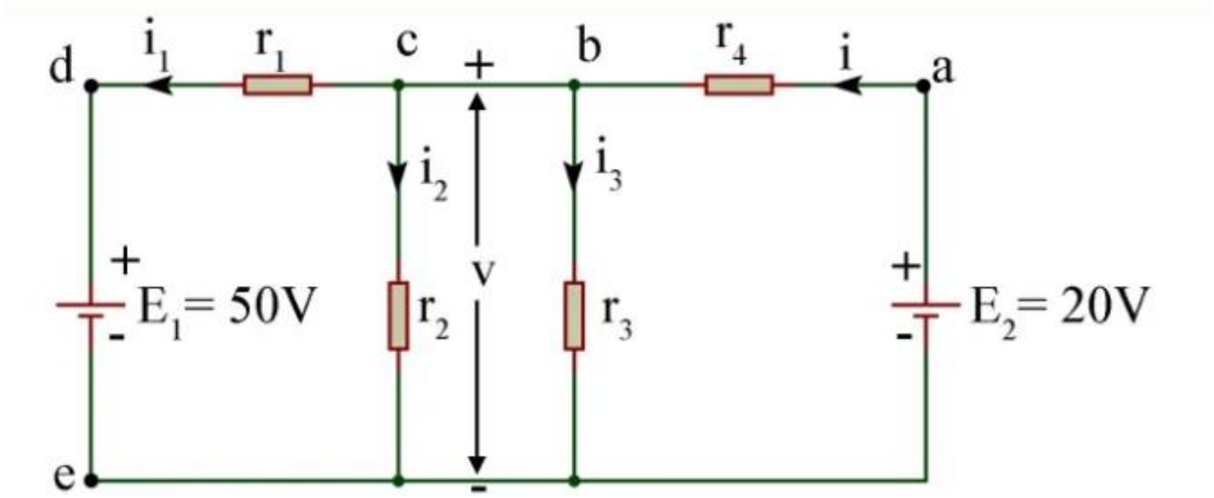


Example 1

Using Nodal KCL method, find the current through resistor R_2.



Solution: Let us redraw the circuit with naming of the nodes and branch current as shown in figure



At node "b", $i = i_1 + i_2 + i_3$

Assuming the polarity of the voltage v at node c or b, we thus get

$$\frac{20 - v}{r_4} = \frac{v - 50}{r_1} + \frac{v}{r_2} + \frac{v}{r_3}$$

or, $$\frac{v - 20}{30} + \frac{v - 50}{20} + \frac{v}{100} + \frac{v}{120} = 0$$

$$\therefore v = 31.18V$$

$$\therefore i_2 = \frac{v}{r_2} = \frac{31.18}{100}A = 0.3118A$$

i.e. current through r_2 = 311.8mA.

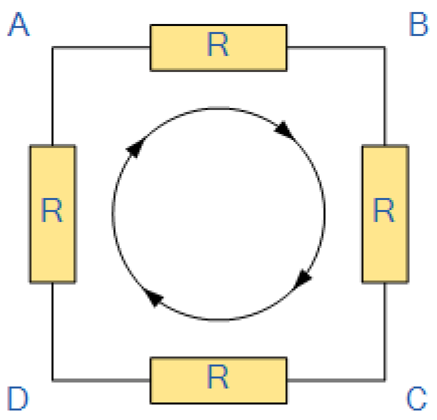
7.2.2. Kirchhoff’s voltage law (KVL)

KVL Rule: The sum of voltages around a closed loop circuit is equal to zero.

$$\sum_{k=1}^{n} V_k = 0$$

To use Kirchhoff's law of tensions, we must assign an algebraic sign (reference direction) to each loop. As we trace a closed path, each tension will appear in the loop as an increase or a drop in the direction in which we draw the loop.

In a circuit you can write as many different equations as there are loops in the circuit. However only some of then will be independent (defined by the number of loops).

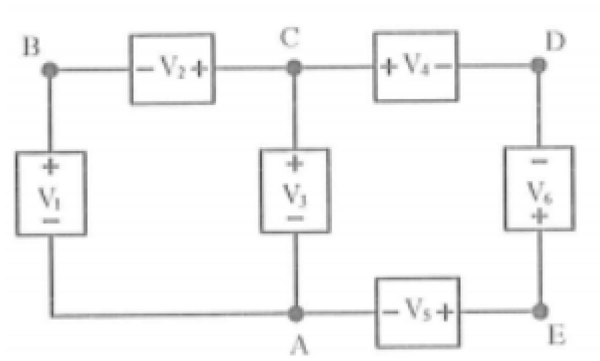


$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$

• Loop ABCA: V1+V2-V3 = 0

• Loop ACDEA: V3-V4+V6-V5 = 0

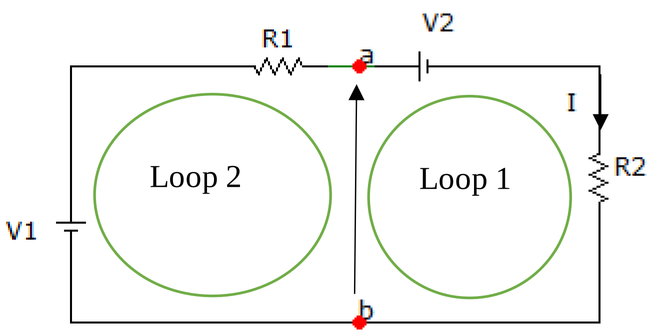
• Loop ABCDEA: V1+V2-V4+V6-V5= 0



Example 1

Using KVL, find the current I and the voltage Vab in the following circuit? my

Given: R1=80KΩ; R2=40KΩ; V1=6V; V2=12V.



1)The current I=?

Using KVL, we find:

$$V1 - V2 = (R1 + R2)I \qquad \Rightarrow I = \frac{V1-V2}{(R1+R2)}$$

NA: $I = \frac{6-12}{40+80}$**=--0.05mA**

2) the voltage Vab=?

Using KVL, we find:

Loop 1: $Vab = V2 + R2.I$

Loop 2: $Vab = V1 - R2.I$

NA:

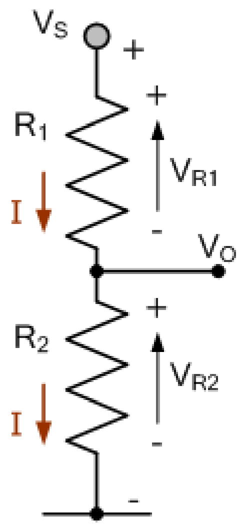
Vab=12+40×(-0.05)=10V

Vab=6-80×(-0.05)=10v

8. Voltage and current circuits dividers

A series circuit acts as a **voltage divider** as it divides the total supply voltage into different voltages across the circuit elements. A parallel circuit acts as a **current divider** as it divides the total circuit current in its all branches.

8.1. Voltage Dividers



Voltage Divider circuits are used to produce different voltage levels from a common voltage source but the current is the same for all components in a series circuit.

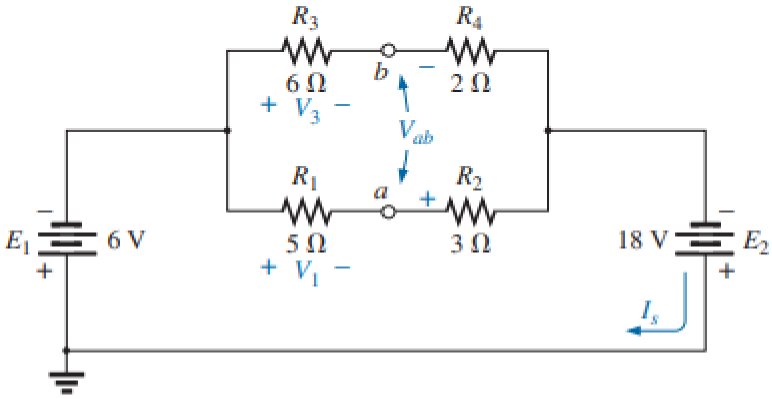
$$V_{R1} = V_S\left(\frac{R_1}{R_1 + R_2}\right)$$

8.1.1. Example

Consider the network presented in the figure below.

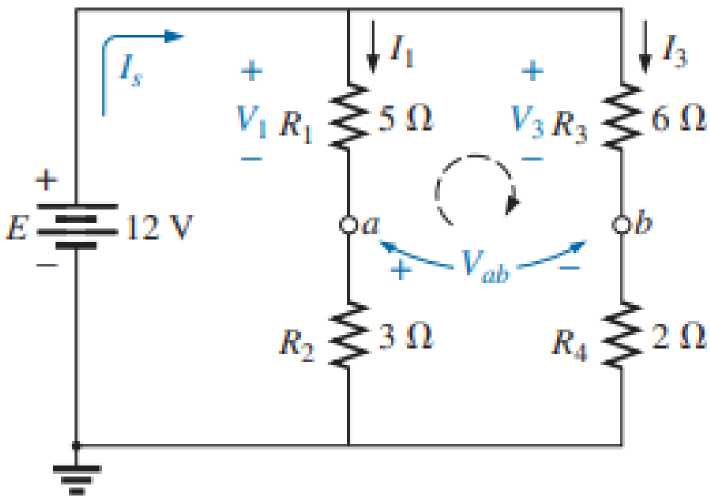
a. Find the voltages using KVL V1, V3, and Vab.

b. Calculate the current Is using KCL.



8.1.2. Solution

1) The voltage V1, V3, and Vab.



a. Note the similarities with Fig. 7.16, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop in Fig. 7.21 in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$$

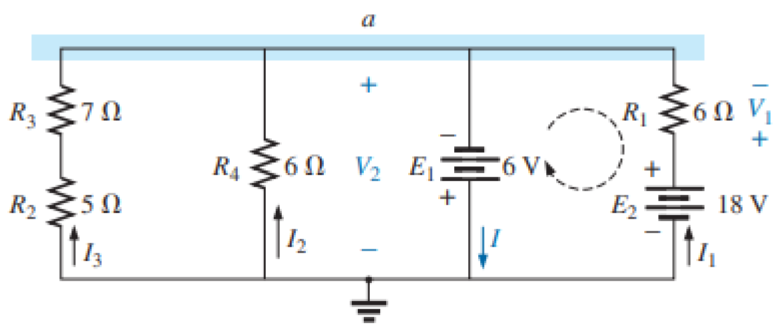
b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

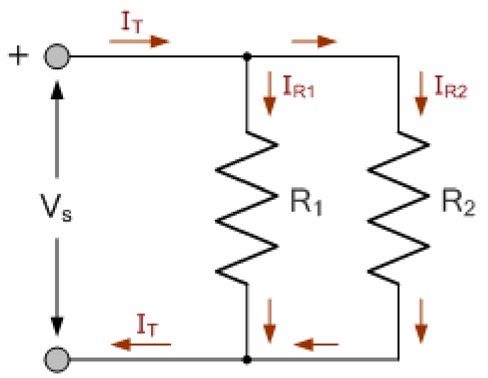
Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$



8.2. Current Dividers

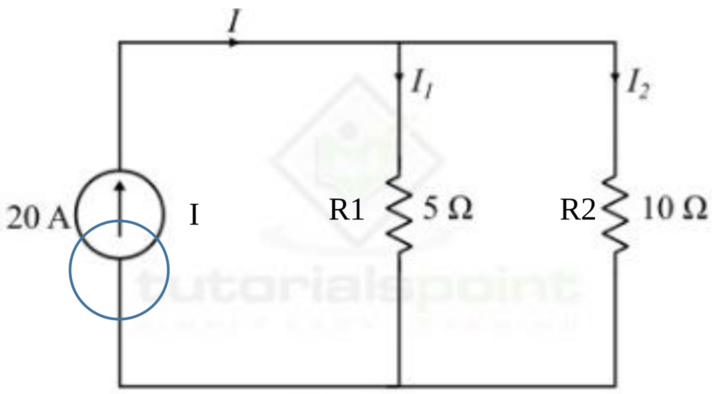
Current Divider circuits have two or more parallel branches for currents to flow through but the voltage is the same for all components in the parallel circuit.



$$I_{R2} = I_T\left(\frac{R_1}{R_1 + R_2}\right)$$

8.2.1. Example

Find the currents I_1 and I_2 in the parallel circuit shown in Figure 3.



8.2.2. Solution

Using the current division rule, the current through resistor R_1 is,

$$I_1 = I \times \frac{R_2}{R_1 + R_2} = 20 \times \frac{10}{5 + 10}$$

$$\therefore I_1 = 13.33 \text{ A}$$

The current through resistor R_2 will be,

$$I_2 = I \times \frac{R_1}{R_1 + R_2} = 20 \times \frac{5}{5 + 10}$$

$$\therefore I_2 = 6.67 \text{ A}$$

9. Fundamental theorem's

Electric circuit theorems are always beneficial to help find voltage and currents in multi-loop circuits. These theorems use fundamental rules or formulas and basic equations of mathematics to analyze basic components of electrical or electronics parameters such as voltages, currents, resistance, and so on. These fundamental theorems include the basic theorems like Superposition theorem, Thevenin's theorem, Norton's theorem, and Milman theorems.

9.1. Superposition Theorem

Superposition states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to each independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

Steps to apply superposition principle:

1) Turn off all independent sources except one source. Find the output (v or i) due to that active source.

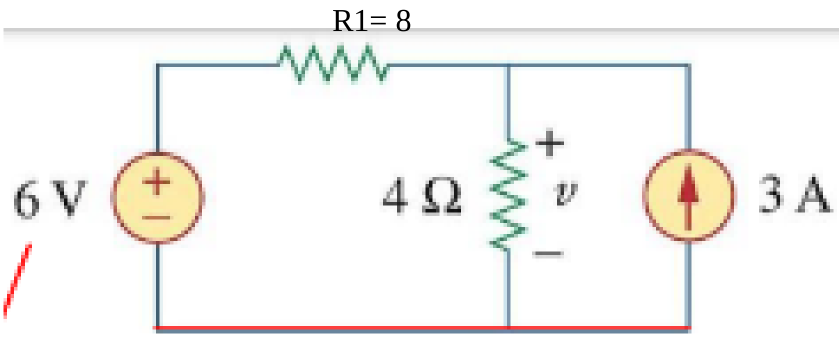
2) Repeat Step 1 for each of the other independent sources.
3) Find total contribution by adding all contributions from independent sources.

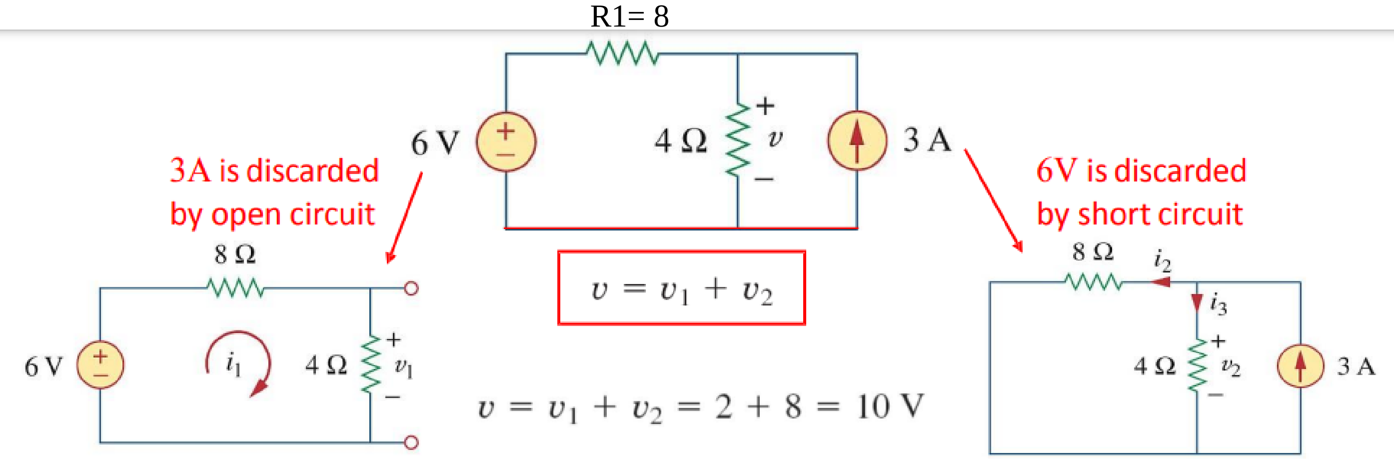
Note: In Step 1, this implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).

Dependent sources are left intact because they are controlled by others.

9.1.1. Example

Use the superposition theorem to find **v** in the circuit.





$$v = v_1 + v_2$$

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$

$$v_1 = 4i_1 = 2 \text{ V}$$

$$i_3 = \frac{8}{4 + 8}(3) = 2 \text{ A}$$

$$v_2 = 4i_3 = 8 \text{ V}$$

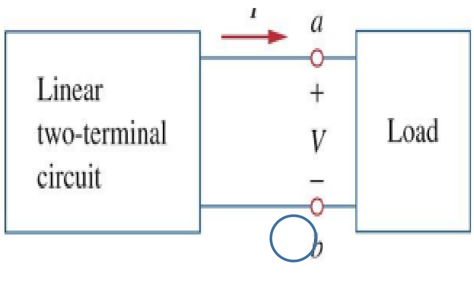
9.1.2. Solution

The voltage v=? by using the superposition theorem

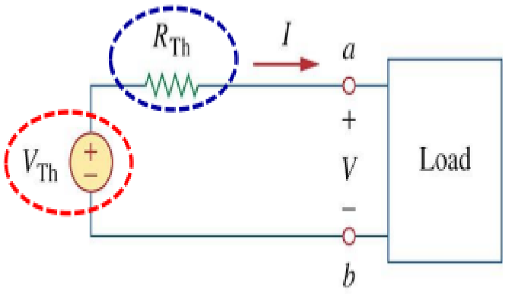
9.2. Thevenin's Theorem

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source VTh in series with a resistor RTh, where :

- VTh is the open-circuit voltage at the terminals.
- RTh is the input or equivalent resistance at the terminals when the independent sources are turned off.



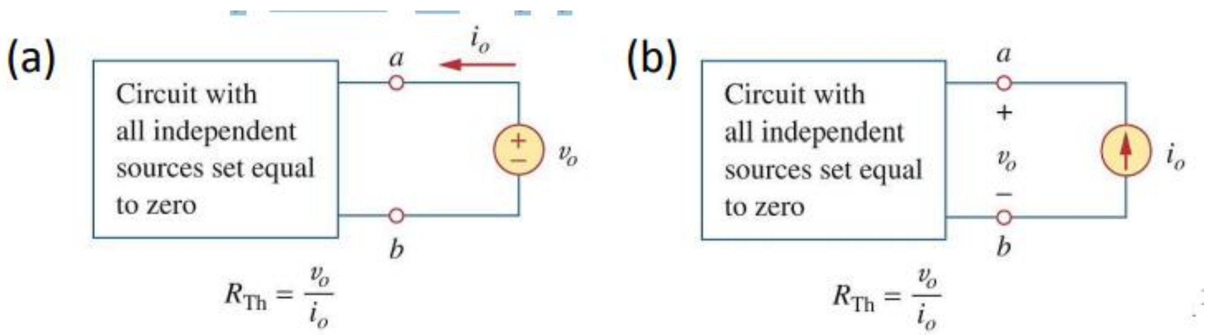
(a)



(b)

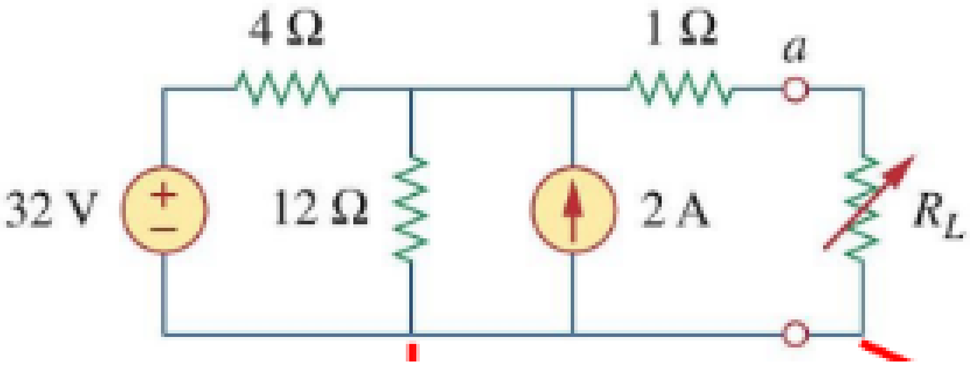
To find RTh:

- **Case 1:** If the network has no dependent sources, we turn off all independent. Source. RTh is the input resistance of the network looking between terminals a &b.
- **Case 2:** If the network has depend. Sources. Depend. sources are not to be turned off because they are controlled by circuit variables.
  - (a) Apply vo at a & b and determine the resulting io . Then RTh = vo /io . Alternatively,
  - (b) insert io at a & b and determine vo . Again RTh = vo /io .



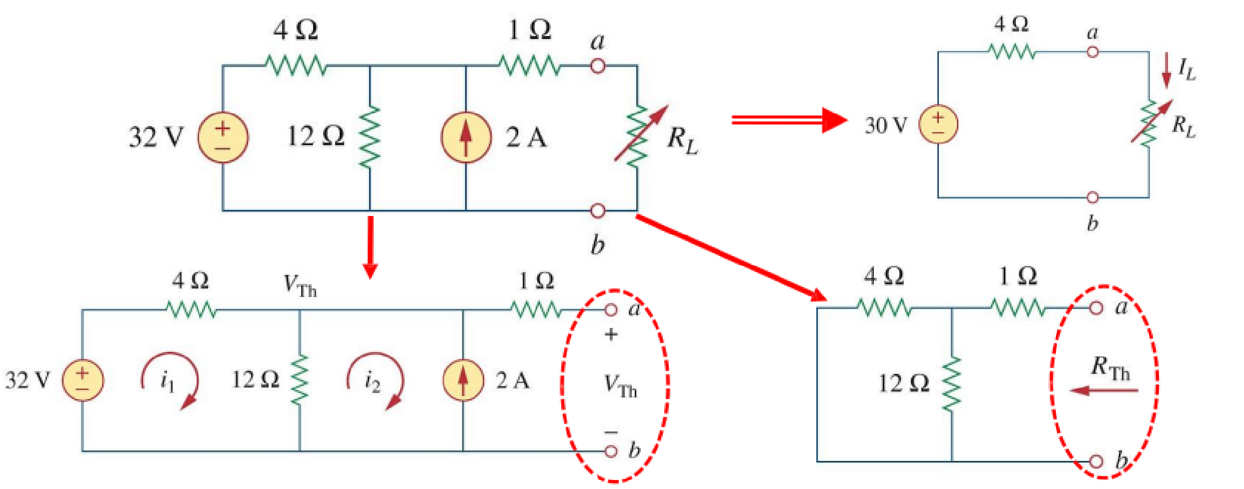
9.2.1. Example

Find the Thevenin equivalent circuit at the terminals a & b.



9.2.2. Solution

The Thevenin equivalent circuit:



Indep. voltage source as a short circuit & the current source as an open circuit.

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

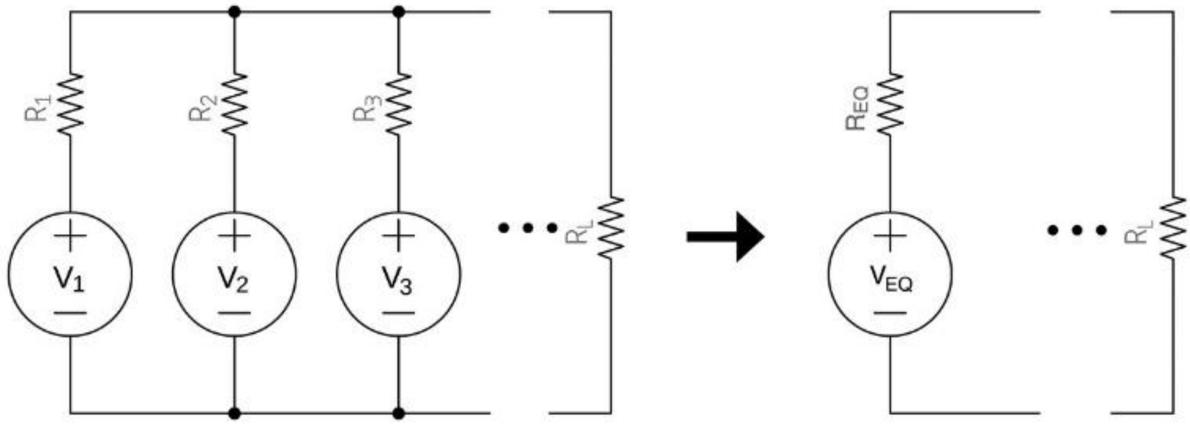
for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

9.3. Milman's Theorem

Millman's theorem states that any circuit containing multiple voltage sources, each one in series with its own resistance, can be replaced by one voltage source (V_{EQ}) in series with a resistance (R_{EQ}).



As per Millman's Theorem

If the N number of sourcesis given, the.

$$Veq=\frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}+\frac{E_3}{R_3}+\cdots+\frac{E_N}{R_N}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_N}}$$

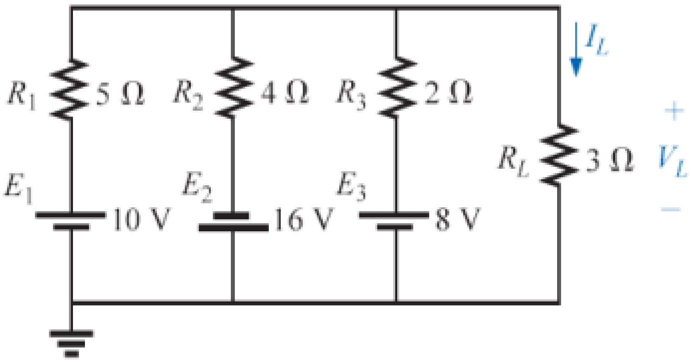
And Req will be

$$Req=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_N}}$$

9.3.1. Example

Using Millman's theorem, find

1) The current through
2) Voltage across the resistor RL of the circuit bellow.



9.3.2. Solution

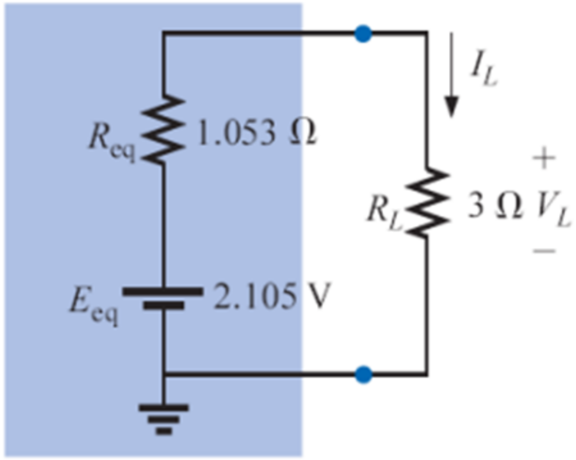
$$E_{eq} = \frac{+\dfrac{E_1}{R_1} - \dfrac{E_2}{R_2} + \dfrac{E_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$E_{eq} = \frac{+\dfrac{10\text{ V}}{5\,\Omega} - \dfrac{16\text{ V}}{4\,\Omega} + \dfrac{8\text{ V}}{2\,\Omega}}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{2\text{ A} - 4\text{ A} + 4\text{ A}}{0.2\text{ S} + 0.25\text{ S} + 0.5\text{ S}}$$

$$= \frac{2\text{ A}}{0.95\text{ S}} = \mathbf{2.105\ V}$$

with $$R_{eq} = \frac{1}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{1}{0.95\text{ S}} = \mathbf{1.053\ \Omega}$$

The resultant source is shown in Fig. (13-1-a).



Fig(1-13-a)

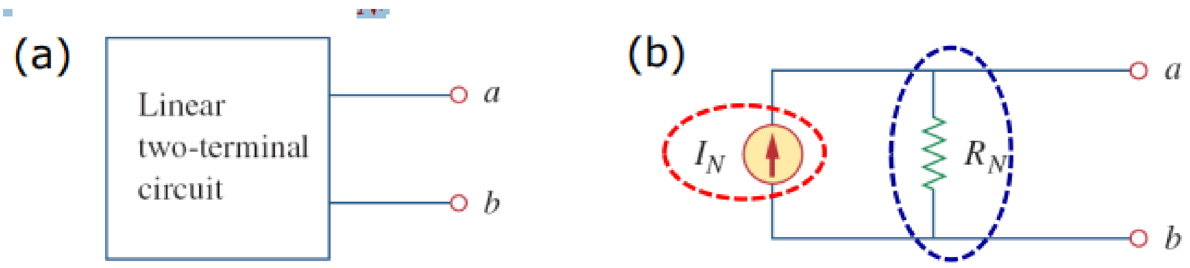
$$I_L = \frac{2.105\ \text{V}}{1.053\ \Omega + 3\ \Omega} = \frac{2.105\ \text{V}}{4.053\ \Omega} = \mathbf{0.519\ A}$$

with $$V_L = I_L R_L = (0.519\ \text{A})(3\ \Omega) = \mathbf{1.557\ V}$$

9.4. Norton's Theorem

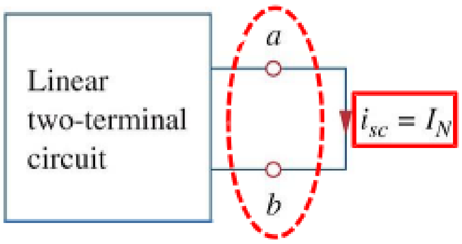
Norton's theorem reduces an electric circuit down to a single resistance in parallel with a constant current source

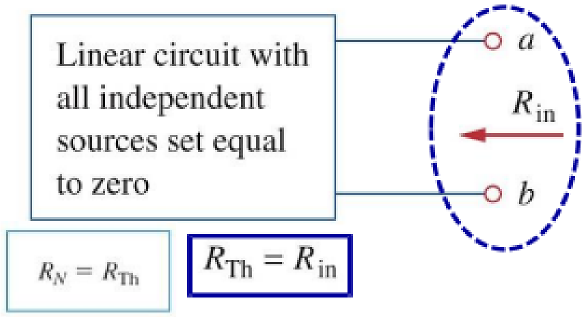
It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a current source IN in parallel with a resistor RN,



where

- IN is the short-circuit current through the terminals.
- RN is the input or equivalent resistance at the terminals when the indepen. sources are turned off.





Note: The Thevenin and Norton equivalent circuits are related by a source transformation.

$$I_N = \frac{V_{Th}}{R_{Th}}$$

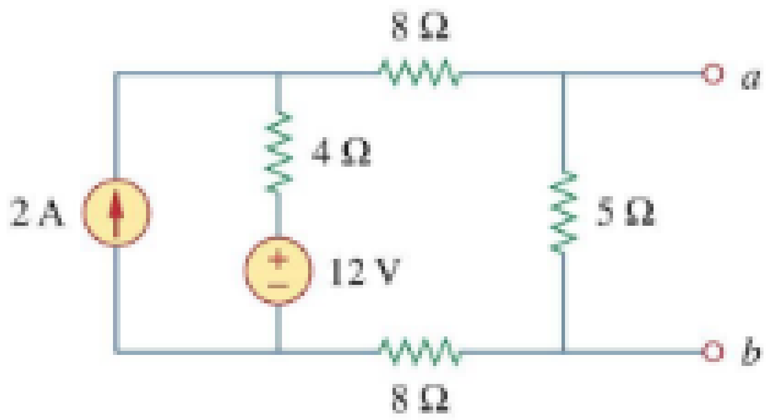
$$V_{TH} = v_{oc}$$

$$I_N = i_{sc}$$

$$R_{TH} = v_{oc} / i_{sc} = R_N$$

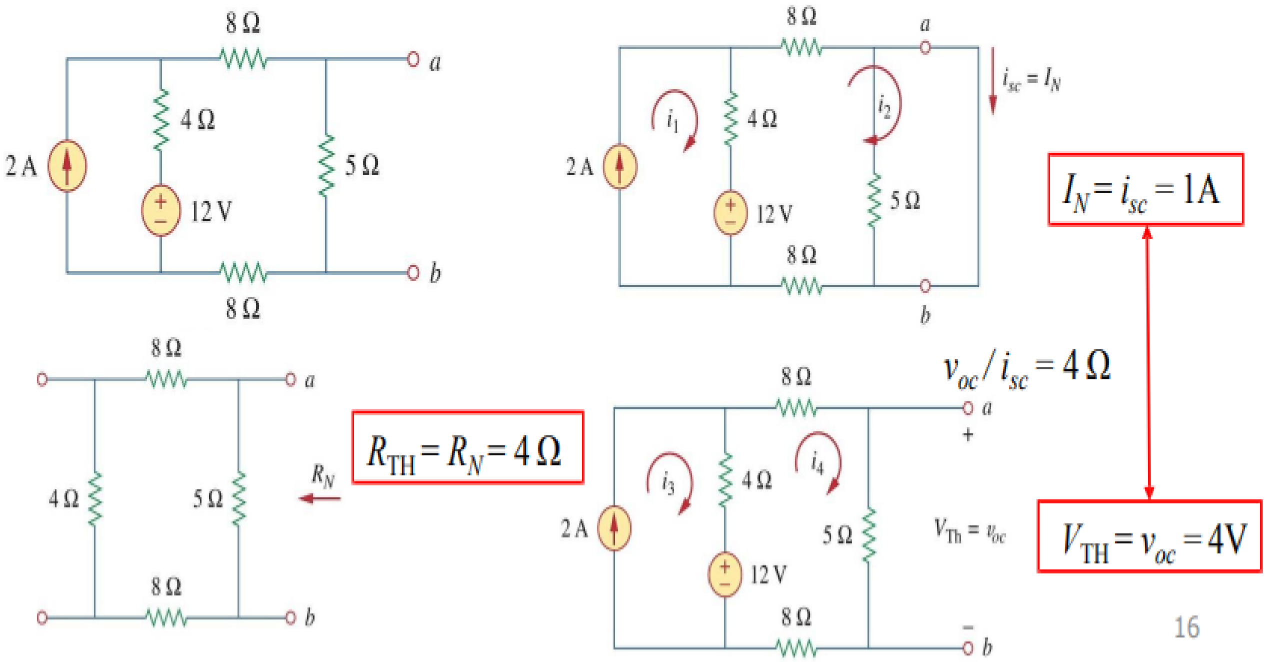
9.4.1. Example

Find the Norton equivalent circuit at the terminals a & b.



9.4.2. Solution

The Norton equivalent circuit



10. Northon and Thevenin equivalent circuits