7.1 Inverter:

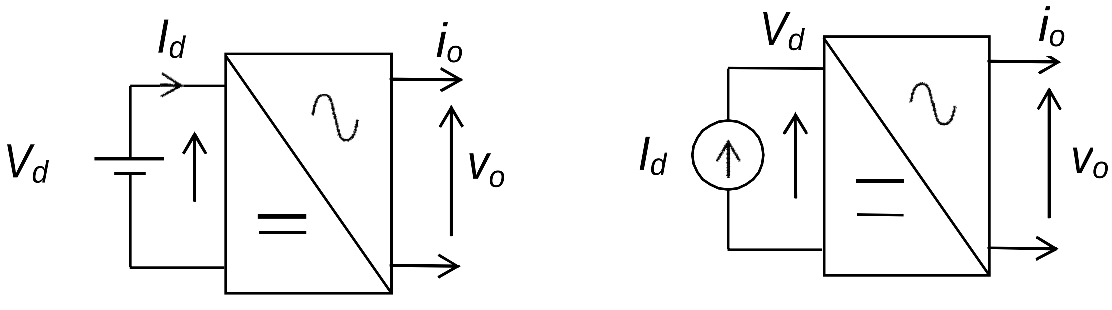
To produce an ac output waveform from a dc power supply is the main objective of inverters. These types of converters are required in many applications, like adjustable speed drives (ASDs), uninterruptible power supplies, static var compensators, active power filters, flexible ac transmission systems, and grid-connected photovoltaic systems, which are only a few applications. For sinusoidal ac outputs, the magnitude, frequency, and phase of the waveform should be controllable.

7.2 Inverters Classification:

The inverters can be classified based on a number of factors like, the nature of output waveform (sine, square, quasi square, PWM etc), the power devices being used (thyristor transistor, MOSFETs IGBTs), the configuration being used, (series. parallel, half bridge, Full bridge), the type of commutation circuit that is being employed and voltage source and current source inverters.

Voltage source inverters are supplied from ideal (or low impedance) DC voltage sources. The DC source can be fixed or variable, with very low source impedance. With such a source, the output AC voltage is determined completely by the DC source voltage and the states of the switching devices in the inverter. This of course assumes that the load current does not become discontinuous. The DC source may be in the form of batteries, generators, solar cells or for large power applications, controlled or uncontrolled AC-DC rectifier circuits terminated with a C- or LC-filter which offers low impedance path to the lagging components of the load current which must sink into the DC source.

Current source inverters have a stiff DC current supply, usually through a large inductor or a closed-loop controlled current source. The DC source impedance of such inverters is very high. The output AC current is determined completely from the magnitude of the source current and the states of the switching devices used in the inverter. The output AC voltage form such an inverter is determined by the load impedance and the inverter output current.



VSI, with fixed or variable V_d CSI, with fixed or variable I_d

Fig7.1 Inverter diagram

For inverter circuits, the output voltage and current waveforms, v_o and i_o respectively, are assumed to be AC quantities. These are normally stated in terms of their RMS values. Deviations of these waveforms from their fundamental and sinusoidal component are normally expressed in terms of the THD (Total Harmonic Distortion) Factors.

7.4 Single phase Half-Bridge Square-Wave Inverter:

A single phase inverter is shown in **Figure7. 2**.The switches direction S_1 and S_2 are unidirectional, i.e. they conduct current in one The current through S_1 is denoted as i_1 and the current through S_2 is i_2 . The switching sequence is so design (**Figure 7.3**) that switch S_1 is **on** for the time duration $0 \le t \le T_1$ and the switch S_2 is **on** for the time duration $T_1 < t \le T_2$. When switch S_1 is turned **on**, the instantaneous voltage across the load is

$$v_o = \frac{V_{in}}{2} \qquad (7.1)$$

When the switch S_2 is only turned ***on***, the voltage across the load is

$$v_o = -\frac{V_{in}}{2} \qquad (7.2)$$

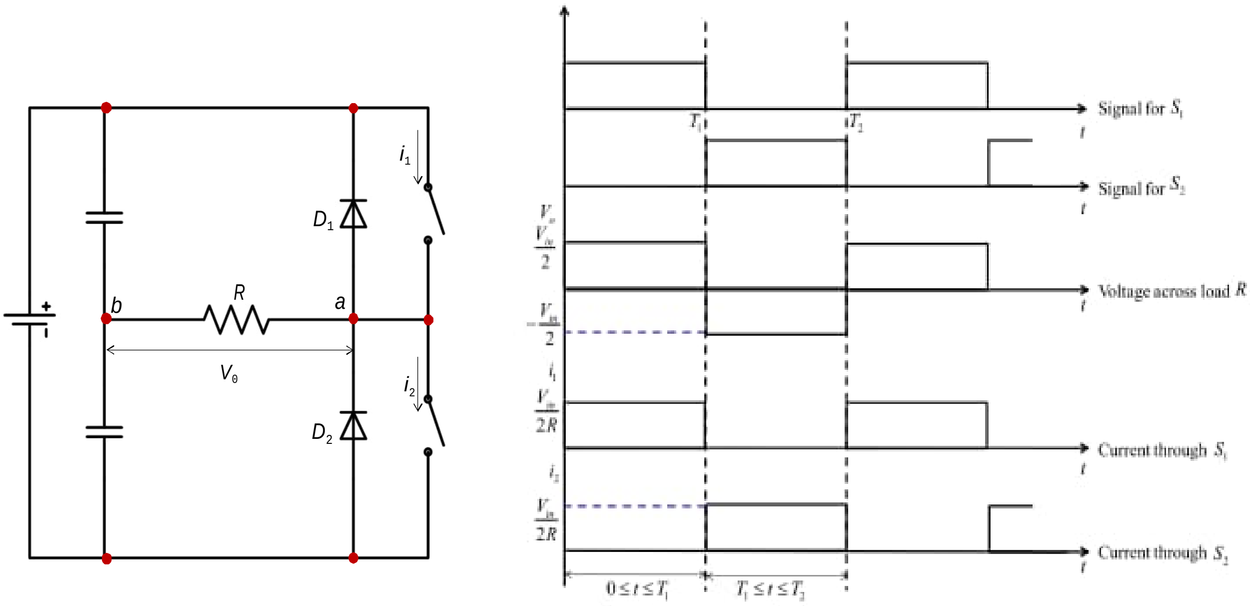


Fig7.2 (a) Half-bridge inverter under resistive, (b) Switching and output waveforms

The r.m.s value of output voltage v_O is given by

$$V_{o,rms} = \left(\frac{1}{T_1}\int_0^{T_1} \frac{V_{in}{}^2}{4} dt\right)^{1/2} = \frac{V_{in}}{2} \qquad (7.3)$$

The instantaneous output voltage v_O is rectangular in shape **Figure7.2 (b)**. The instantaneous value of v_O can be expressed in Fourier series as:

$$v_o = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \qquad (7.4)$$

Due to the quarter wave symmetry along the time axis (**Figure 7.3**), the values of a_0 and a_n are zero. The value of b_n is given by

$$b_n = \frac{1}{\pi}\left[\int_{-\frac{\pi}{2}}^{0} \frac{-V_{in}}{2} d(\omega t) + \int_{0}^{\frac{\pi}{2}} \frac{V_{in}}{2} d(\omega t)\right] = \frac{2V_{in}}{n\pi} \qquad (7.5)$$

Substituting the value of b_n from equation **(7.5)** into equation**(7. 4)** gives

$$v_o = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V_{in}}{n\pi} \sin(n\omega t) \qquad (7.6)$$

The current through the resistor (i_L) is given by

$$i_L = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V_{in}}{R\, n\pi} \sin(n\omega t) \qquad (7.7)$$

For n = 1, equation**(7. 6)** gives the rms value of the fundamental component as

$$V_{o1} = \frac{2V_{in}}{\sqrt{2}\pi} \approx 0.45V_{in} \qquad (7.8)$$

the load current cannot change immediately with the output voltage. The working of the DC-AC inverter with inductive load is as follow is:

Case1: In the time interval $0 \leq t \leq T_1$the switch S_1 is **on** and the current flows through the inductor from points **a** to **b.** When the switch S_1 is **off** (case 1) at $t=T_1$, the load current would continue to flow through the capacitor C_2 and diode D_2 unitl the current falls to zero.

Case 2: Similarly, when S_2 is turned **off** at $t=T_2$, the load current flows through the diode D_1 and the capacitor C_1 until the current falls to zero.

When diodes D_1 and D_2 conduct, energy is fed back to the dc source and these diodes are known as feedback diodes. These diodes are also known as freewheeling diodes. The instantaneous load current is obtained as:

$$i_L = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V_{in}}{n\pi\sqrt{R^2+(n\omega L)^2}} \sin(n\omega t - \theta_n) \qquad (7.9)$$

Where; $\theta = tan^{-1}\left(\frac{n\omega L}{R}\right)$

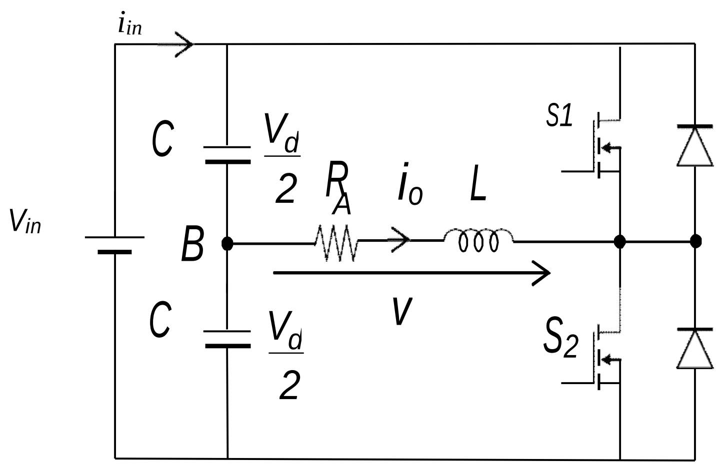


Fig7.3 (a) Half-bridge inverter with RL load,

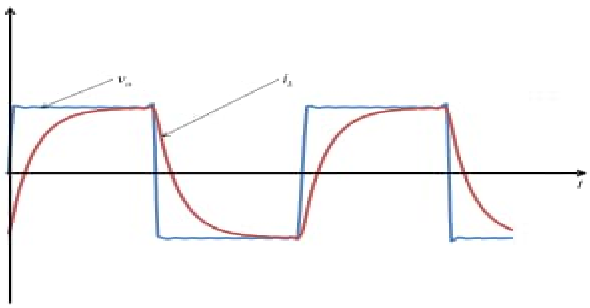


Fig.7.3 (b): Instantaneous output voltage and load current through RL load

7.6 Single Phase Bridge DC-AC Square-Wave Inverter with R Load

A single phase bridge DC-AC inverter is shown in Figure (7.8). In this circuit four switches are used. Switches S1 and S2 are switched together while switches S3 and S4 are switched together alternately to S1 and S2 in a complementary manner. The four feedback diodes D1-D4 conduct currents as indicated in the figure below.

when the switches S1 and S2 are turned on simultaneously for a duration $0 \leq t \leq T_1$, the input voltage Vin appears across the load and the current lows from point a to b. If the switches S3 and S4 are turned on for a duration $T_1 \leq t \leq T_2$, the voltage across the load is reversed and the current through the load flows from point b to a. the voltage and current waveforms across the resistive load are shown in figure (7.9). the instantaneous output voltage can expressed in Fourier series as;

$$v_o = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \qquad (7.10)$$

Due to the square wave symmetry along the x-axis (as seen in Figure 9), both a_o and a_n are zero, and b_n is obtained as

$$b_n = \frac{1}{\pi}\left[\int_{-\frac{\pi}{2}}^{0} -V_{in}\, d(\omega t) + \int_{0}^{\frac{\pi}{2}} V_{in}\, d(\omega t)\right] = \frac{4V_{in}}{n\pi} \qquad (7.11)$$

Substituting the value of b_n from equation 12 into equation 11 gives

$$v_o = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{in}}{n\pi} \sin(n\omega t) \qquad (7.12)$$

The instantaneous current through the resistive load is given by

$$i_L = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{in}}{Rn\pi} \sin(n\omega t) \qquad (7.13)$$

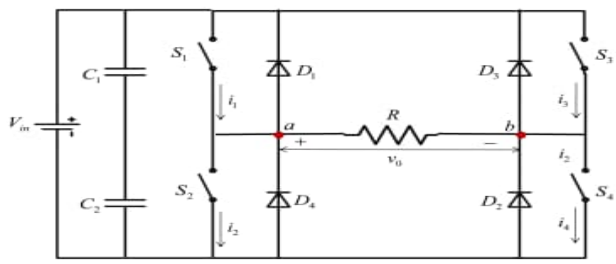


Fig.7.8: Full bridge DC-AC inverter with resistive load

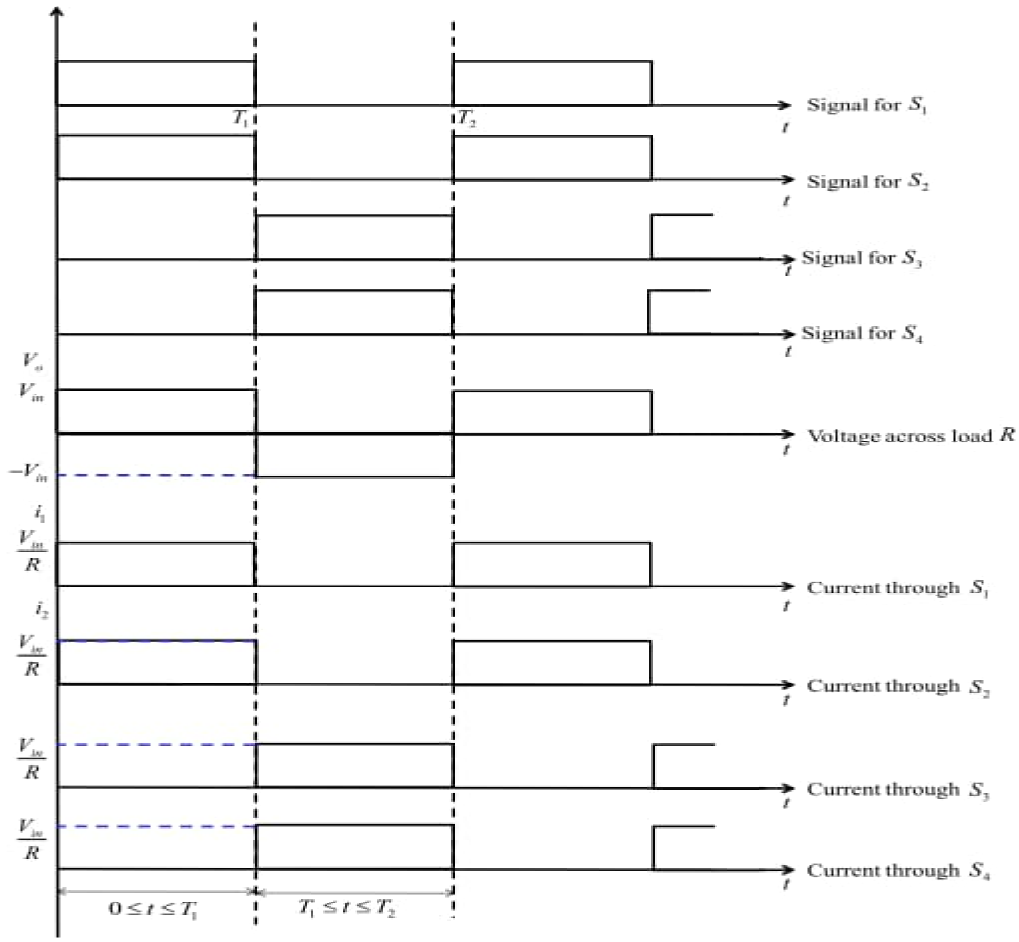


Fig.7. 9 voltage and current waveforms for full bridge DC-AC inverter with R load

7.7 Single Phase Bridge DC-AC Square-Wave Inverter with RL Load

The function of the inverter in case of RL load can be explained as follows:

Case1: In the time interval $t = T_1$ the switches S_1 and S_2 are turned off and the pairs of switches S_3 and S_4 are turned on**.** Due to the inductive load, the current through the load (i_L) will not change its direction at $t = T_1$ and will continue to flow through the load from point a to b, through the diodes D_3 and D4 , till it becomes zero as shown in Figure 7.10a. Once, i_L=0, S_3 and S_4 and start conducting and the load current i_L builds up in opposite direction (point b to a).

Case 2: Similarly, at t=T_2, the switches S_1 and S_2 are turned on and the pairs of switches S_3 and S_4 are turned off, the current takes time to become zero and diodes D_1 and D_2 conduct as long as its non-zero as shown in Figure 7. 10b.The instantaneous load current is obtained as:

$$i_L = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{in}}{n\pi\sqrt{R^2+(n\omega L)^2}} \sin(n\omega t - \theta_n) \qquad (7.9)$$

Where; $\theta = tan^{-1}\left(\frac{n\omega L}{R}\right)$

The current and voltage waveforms for RL load are shown in Figure (7.11).In this figure the conduction is divided into 4 distinct zones. In Zone I the diode D_1 and D_2 conduct until i_L becomes zero. Once, i_L equals zero, the switches S_1 and S_2 conduct and it is marked as Zone II. At time t = T_2 , the diodes D_3 and D_4 conduct and this is marked as Zone III in Figure(7. 10). Finally, in Zone IV the switches S_3 and S_4 conduct.

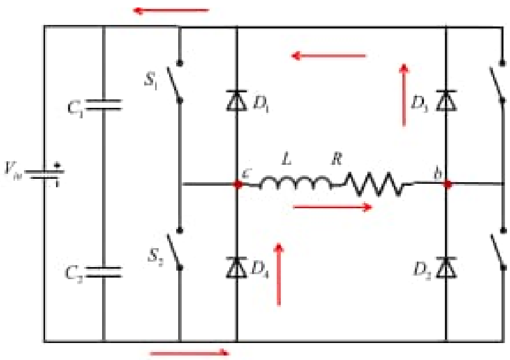


Fig.7.10a: Current flow in case 1

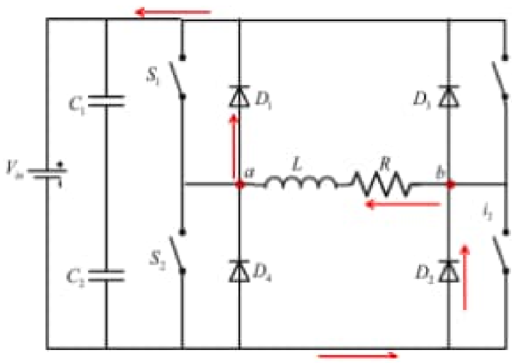


Fig.7. 10b: Current flow in case 2

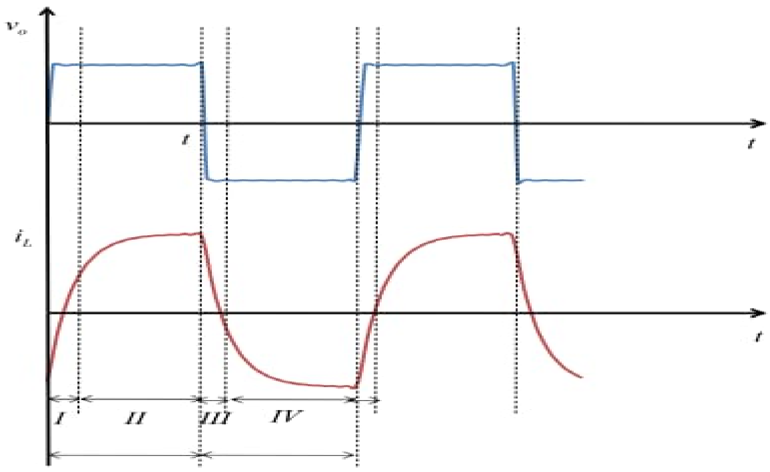


Fig. (7.11): Voltage and current waveforms in case of RL loads

7.7 Performance Parameters

- Harmonic factor of the nth harmonic (HFn)

$$HF_n = \frac{V_{on}}{V_{o1}} \quad \text{for } n > 1 \qquad (7.10)$$

V_{on} rms value of the nth harmonic component

V_{o1} rms value of the fundamental component

- Total Harmonic Distortion (THD): Measures the “closeness” in shape between a waveform and its fundamental component.

$$THD = \frac{1}{V_{01}} \left(\sum_{n=2,3,\dots\dots}^{\infty} {V_{on}}^2 \right)^{\frac{1}{2}} \qquad (7.11)$$

7.8 Single-phase SPWM Inverters

In sinusoidal PWM, also called ***sine-PWM***, a sinusoidal modulating voltage e_c of the desired output frequency f_o is compared with a higher frequency triangular or saw tooth carrier waveform to generate the switching signals for the inverter. The amplitude of e_c also determines the amplitude (or RMS value) of the fundamental output voltage. The inverter can be half- or full-bridge circuits of figure 19.3 or 19.4. The resulting switching pulses have widths approximately proportional to the sine of the angular position at the center of the pulses. The widths of these pulses are also proportional to the amplitude of the modulating signal e_c , relative to the amplitude of the carrier.

7.9 Single-phase Full-bridge SPWM Inverter

The full bridge inverter can be either bipolar or unipolar switched. In the bipolar switching scheme, transistors T_1 and T_2 are switched ON together, when e_c > vtri , as are T_3 and T_4, when $e_c < v_{tri}$. The control voltage e_c is sinusoidal, and of frequency equal to the desired output frequency. Its amplitude is determined by the required RMS output voltage. The carrier frequency is generally much higher than the frequency of the modulations waveform (e_c). Regardless of the direction of current flow in the load, the load voltage waveform is determined by the state of the switches. The amplitude of each SPWM voltage pulse across the load is now $\pm V_d$. Two switches in the same leg of the inverter are never turned on together because that would constitute a short circuit across the DC source.

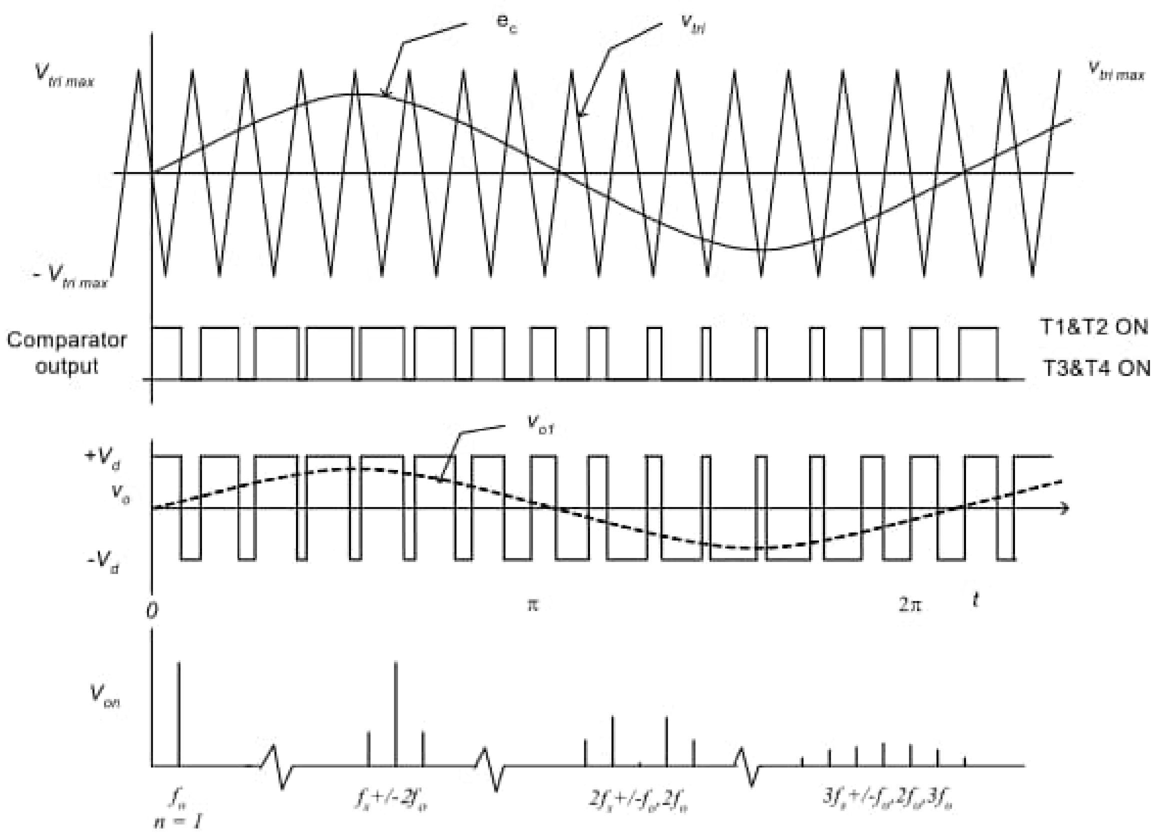


Fig.7.12 Single-phase full bridge inverter output voltage and its spectrum

The comparator out which produces the switching signals of the diagonal transistor pairs are based on the following rule :

when ec > vtri , T1 & T2 are ON and T3 & T4 are OFF
when ec < vtri , T3 & T4 are ON and T1 & T2 are OFF

Let us first define two important terms associated with SPWM. Depth of Modulation,
m = ec max /^Vtri
Frequency Modulation Ratio, mf = fs/f1
where f1 is the frequency of the sinusoidal modulating waveform which is also the fundamental output frequency, and fs is the frequency of the carrier (the inverter switching frequency).
Fourier analysis of the output voltage waveform can be carried out taking a pair of PWM pulse-pairs which repeat in each cycle of the AC output. Note that the output AC cycle consists of many such symmetrical PWM output pulse-pairs for each AC cycle of output voltage. Note also that each pulse pair has a different (sine weighted) width and is shifted from its neighbouring pulse pair by 1/fs .

If the PWM switching or carrier frequency is far higher the frequency of the modulating waveform, it can be assumed that the modulating wave changes little over a switching period. The average output voltage over each switching period is then equal to the depth of modulation (or the effective duty cycle over the switching period) times the supply voltage Vd . It should be expected that the fundamental output voltage waveform should be given by the average voltage voltages during each switching period. This is given the dotted sinusoid of figure above. Thus,

$$V_{o1max} = m \cdot V_d \quad \text{for} \quad 0 \le m \le 1.$$

In other words, the amplitude of the fundamental output voltage is proportional to the depth of modulation.
The output voltage in case of sinusoidal PWM can be expressed as

$$v_o = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{in}}{n\pi} \sum_{k=1,2,\ldots}^{n_p} \sin\frac{n\delta_k}{2}\left[\cos\left(n\omega t - \frac{n\delta_k}{2} - n\alpha_k\right)\right]$$

where;

n_p is the number of pulses in the half cycle
δ_k is the width of the kth pulse
α_k is the starting angle of the kth pulse

The width of the kth pulse (δk) is approximately given by

$$\delta_k = \left(\frac{\pi}{n_p}\right) m_a \sin(\alpha_k)$$

where

$$m_a = \frac{A_r}{A_c}$$

is the modulation index
The value of the starting angle of the kth pulse (k□) is given by numerically solving the following equation

$$m_a \sin(\alpha_k) = -\frac{m_f}{\pi}\alpha_k + (2k-1)$$

where;

$m_f = 2n_p$

The angles θ and α for a sine PWM with 6 pulses per half cycle are calculated using equations 13 and 14 and listed in Table 1. The waveforms of the voltage and current are shown in Figure 7. The rms value of the output voltage is

$$v_o = V_{in}\sqrt{\sum_{m=1}^{2n_p}\frac{\delta_m}{\pi}}$$

The current for an RL load is given by

$$i_L = \sum_{n=1,3,5\ldots}^{\infty}\frac{4V_{in}}{n\pi Z_n}\sum_{k=1,2,\ldots}^{n_p}\sin\frac{n\delta_k}{2}\left[\cos\left(n\omega t - \frac{n\delta_k}{2} - n\alpha_k - \theta_n\right)\right]$$

where

$$Z_n = \sqrt{R^2 + (n\omega L)^2}$$

$$\theta_n = \tan^{-1}\left(\frac{\omega L}{R}\right)$$