DC to DC Converters

5.1 DC Choppers

A chopper is a device which is used to obtain a variable dc voltage from a constant dc voltage source. A chopper is also known as dc-to-dc converter. Uses Power electronics switches like BJT,MOSFET, IGBT…Choppers are widely used in trolley cars, battery operated vehicles, traction motor control, control of large number of dc motors, etc… They are also used in regenerative braking of dc motors to return energy back to supply and also as dc voltage regulators.

Choppers are of two types:

- Step-down choppers (buck converter)
- Step-up choppers.(boost converter)

In step-down choppers, the output voltage will be less than the input voltage whereas in step-up choppers output voltage will be more than the input voltage.

5.2 Principle of Step-Down Chopper with R load

Figure 5.1 shows a step-down chopper with resistive load. The switch in the circuit can be an IGBT switch.

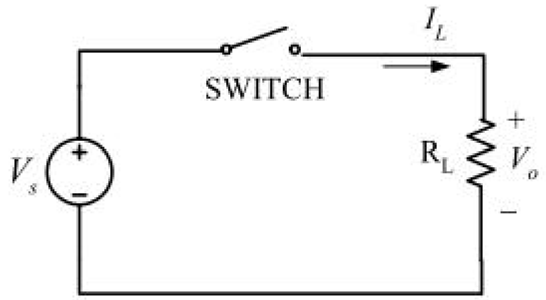


Fig. 5.1: Step-down Chopper with Resistive Load

When the switch (exp;IGBT) is ON, supply voltage appears across the load and when switch is OFF, the voltage across the load will be zero. The output voltage and current waveforms are as shown in figure 5.2.

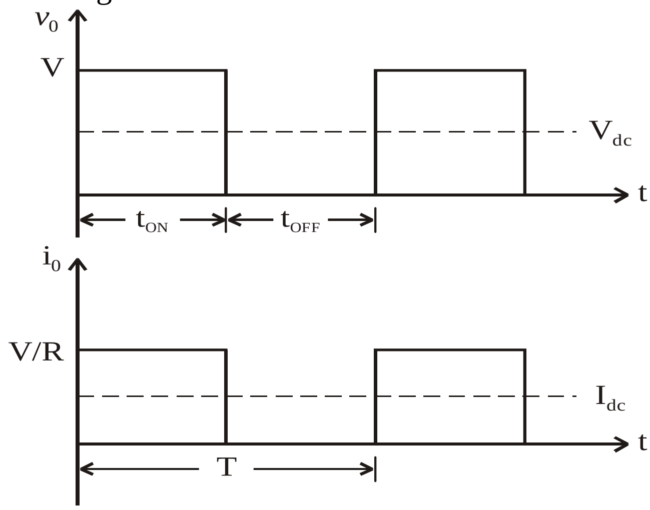


Fig. 5.2: Step-down choppers — output voltage and current waveforms

V_{dc} = average value of output or load voltage

I_{dc} = average value of output or load current

t_{ON} = time interval for which the switch conducts

t_{OFF} = time interval for which the switch is OFF.

$T = t_{ON} + t_{OFF}$ = period of switching or chopping period

$f = \frac{1}{T} =$ frequency of chopper switching or chopping frequency.

Average output voltage

$$V_{dc} = V\left(\frac{t_{ON}}{t_{ON} + t_{OFF}}\right) \qquad (5.1)$$

$$V_{dc} = V\left(\frac{t_{ON}}{T}\right) = V.d \qquad (5.2)$$

Where; $\left(\frac{t_{ON}}{t}\right) = d =$ duty cycle (5.3)

Average output current,

$$I_{dc} = \frac{V_{dc}}{R} \qquad (5.4)$$

$$I_{dc} = \frac{V}{R}\left(\frac{t_{ON}}{T}\right) = \frac{V}{R}d \qquad (5.5)$$

RMS value of output voltage

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} v_o^2 dt}$$

During t_{ON}, $v_o = V$

Therefore RMS output voltage

$$V_O = \sqrt{\frac{1}{T}\int_0^{t_{ON}} V^2 dt}$$

$$V_O = \sqrt{\frac{V^2}{T}t_{ON}} = \sqrt{\frac{t_{ON}}{T}}.V \qquad (5.6)$$

$$V_O = \sqrt{d}.V \qquad (5.7)$$

The Output power $P_O = V_O I_O$

Where; $I_O = \frac{V_O}{R}$

Therefore, the output power is $P_O = \frac{V_O^2}{R}$

$$P_O = \frac{dV^2}{R} \qquad (5.8)$$

Effective input resistance of chopper

$$R_i = \frac{V}{I_{dc}} \qquad (5.9)$$

$$R_i = \frac{R}{d} \qquad (5.10)$$

The output voltage can be varied by varying the duty cycle.

5.3 Control Methods

The output dc voltage can be varied by the following methods.

- Pulse width modulation control or constant frequency operation.
- Variable frequency control.

5.3.1 Pulse Width Modulation

In pulse width modulation the pulse width (t_{ON}) of the output waveform is varied keeping chopping (switching) frequency ‘f’ and hence chopping period ‘T’ constant. Therefore output voltage is varied by varying the ON time, t_{ON}. Figure 5.3 shows the output voltage waveforms for different ON times

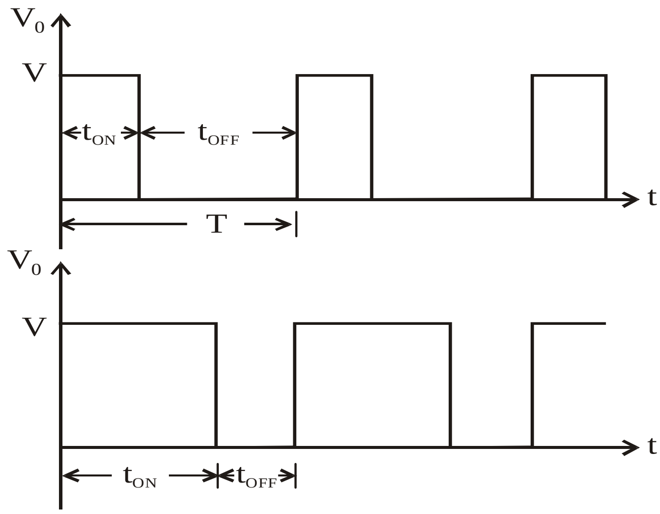
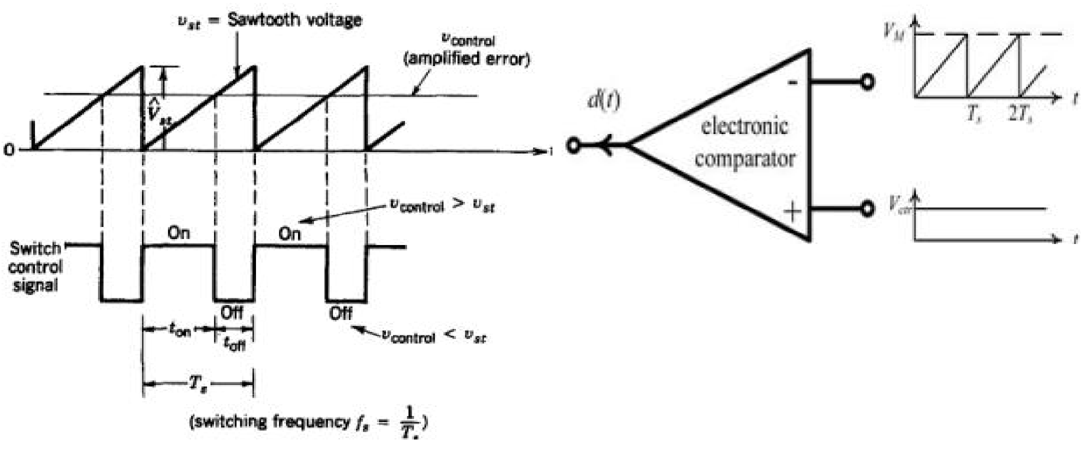


Fig. 5.3: Pulse Width Modulation Control

How a pulse-width modulator works;

The output DC voltage of DC chopper can be varied by controlling the width period (t_{on}) of the signal applied to the switch S with constant switching/chopping frequency f s . This method is called PWM method.



waveforms of PWM.

5.3.2 Variable Frequency Control

In this method of control, chopping frequency f is varied keeping either t_{ON} or t_{OFF} constant. This method is also known as frequency modulation.

Figure 5.4 shows the output voltage waveforms for a constant t_{ON} and variable chopping period T.

In frequency modulation to obtain full output voltage, range frequency has to be varied over a wide range. This method produces harmonics in the output and for large t_{OFF} load current may become discontinuous.

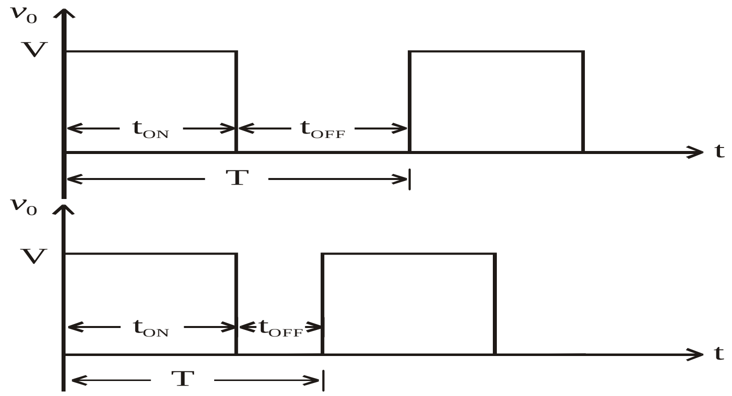


Fig. 5.4: Output Voltage Waveforms for frequency Control

5.4 Step-Down Chopper with RL load

Figure 5.5 shows a step-down chopper with R-L load and freewheeling diode. When chopper is ON, the supply is connected across the load. Current flows from the supply to the load. When chopper is OFF, the load current i_O continues to flow in the same direction through the free-wheeling diode due to the energy stored in the inductor L. The load current can be continuous or discontinuous depending on the values of L and duty cycle, d. For a continuous current operation the load current is assumed to vary between two limits $I_{\min}$ and $I_{\max}$. Figure 5.6 shows the output current and output voltage waveforms for a continuous current and discontinuous current operation.

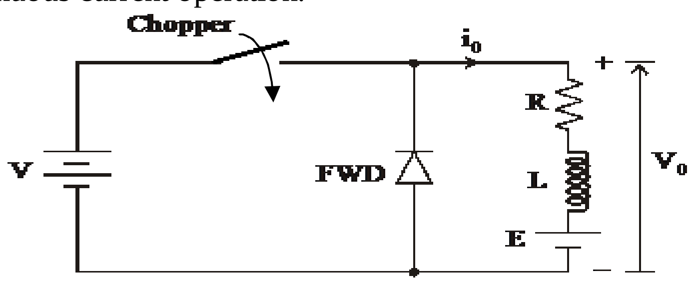


Fig. 5.5: Step Down Chopper with R-L Load

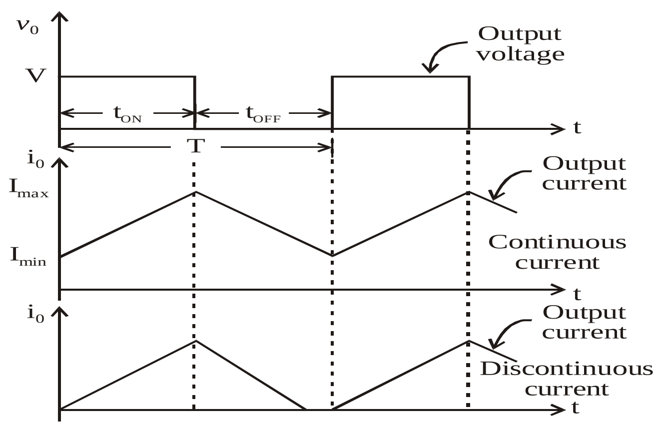


Fig. 5.6: Output Voltage and Load Current Waveforms (Continuous and discontinuous Current)

When the current exceeds $I_{\max}$ the chopper is turned-off and it is turned-on when current reduces to $I_{\min}$.

- **The expression for load current i_O for continuous current operation when;**
  - **The chopper is ON** $\left(0 \le t \le t_{ON}\right)$

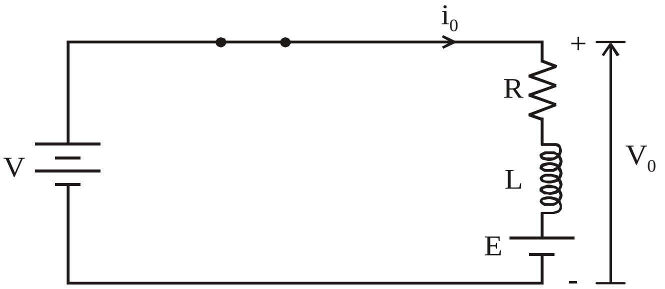


Fig. 5.5 (a)

Voltage equation for the circuit shown in figure 5.5(a) is

$$V = i_O R + L\frac{di_O}{dt} + E \qquad (5.11)$$

Taking Laplace Transform

$$\frac{V}{S} = RI_O(S) + L\left[S.I_O(S) - i_O\left(0^-\right)\right] + \frac{E}{S} \qquad (5.12)$$

At $t = 0$, initial current $i_O\left(0^-\right) = I_{\min}$

$$I_O(S) = \frac{V-E}{LS\left(S+\frac{R}{L}\right)} + \frac{I_{\min}}{S+\frac{R}{L}} \qquad (5.13)$$

Taking Inverse Laplace Transform

$$i_O(t) = \frac{V-E}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right] + I_{\min}e^{-\left(\frac{R}{L}\right)t} \qquad (5.14)$$

This expression is valid for $0 \le t \le t_{ON}$. i.e., during the period chopper is ON.

At the instant the chopper is turned off, load current is

$$i_O\left(t_{ON}\right) = I_{\max}$$

- **When Chopper is OFF** $\left(0 \le t \le t_{OFF}\right)$

Voltage equation for the circuit shown in figure 5.5(b) is

$$0 = Ri_O + L\frac{di_O}{dt} + E \qquad (5.15)$$

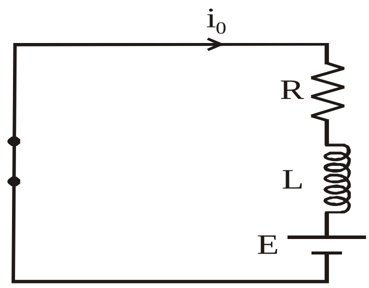


Fig. 5.5 (b)

Taking Laplace transform

$$0 = RI_O(S) + L\left[SI_O(S) - i_O(0^-)\right] + \frac{E}{S}$$

Redefining time origin we have at $t = 0$, initial current $i_O(0^-) = I_{max}$

Therefore $$I_O(S) = \frac{I_{max}}{S + \frac{R}{L}} - \frac{E}{LS\left(S + \frac{R}{L}\right)}$$

Taking Inverse Laplace Transform

$$i_O(t) = I_{max} e^{-\frac{R}{L}t} - \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right] \qquad (5.16)$$

The expression is valid for $0 \le t \le t_{OFF}$, i.e., during the period chopper is OFF. At the instant the chopper is turned ON or at the end of the off period, the load current is

$$i_O(t_{OFF}) = I_{min}$$

To find I_{max} and I_{min}

From equation (5.14),

At $t = t_{ON} = dT$, $i_O(t) = I_{max}$

Therefore $$I_{max} = \frac{V - E}{R}\left[1 - e^{-\frac{dRT}{L}}\right] + I_{min} e^{-\frac{dRT}{L}} \qquad (5.17)$$

From equation (5.16),

At $t = t_{OFF} = T - t_{ON}$, $i_O(t) = I_{min}$

$t = t_{OFF} = (1 - d)T$

Therefore $$I_{min} = I_{max} e^{-\frac{(1-d)RT}{L}} - \frac{E}{R}\left[1 - e^{-\frac{(1-d)RT}{L}}\right] \qquad (5.18)$$

Substituting for I_{min} in equation (5.17) we get,

$$I_{max} = \frac{V}{R}\left[\frac{1 - e^{-\frac{dRT}{L}}}{1 - e^{-\frac{RT}{L}}}\right] - \frac{E}{R} \qquad (5.19)$$

Substituting for I_{max} in equation (5.18) we get,

$$I_{min} = \frac{V}{R}\left[\frac{e^{\frac{dRT}{L}} - 1}{e^{\frac{RT}{L}} - 1}\right] - \frac{E}{R} \qquad (5.20)$$

$(I_{max} - I_{min})$ is known as the steady state ripple.

Therefore; the peak-to-peak ripple current is

$$\Delta I = I_{max} - I_{min}$$

Average output voltage

$$V_{dc} = d.V \tag{5.21}$$

Average output current

$$I_{dc(approx)} = \frac{I_{\max} + I_{\min}}{2} \tag{5.22}$$

Assuming load current varies linearly from $I_{\min}$ to $I_{\max}$ instantaneous load current is given by

$$i_O = I_{\min} + \frac{(\Delta I).t}{dT} \ for\ 0 \le t \le t_{ON}(dT)$$

$$i_O = I_{\min} + \left(\frac{I_{\max} - I_{\min}}{dT}\right)t \quad . \tag{5.23}$$

RMS value of load current

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT} i_0^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT}\left[I_{\min} + \frac{(I_{\max} - I_{\min})t}{dT}\right]^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT}\int_0^{dT}\left[I_{\min}^2 + \left(\frac{I_{\max} - I_{\min}}{dT}\right)^2 t^2 + \frac{2I_{\min}(I_{\max} - I_{\min})t}{dT}\right]dt}$$

RMS value of output current

$$I_{O(RMS)} = \left[I_{\min}^2 + \frac{(I_{\max} - I_{\min})^2}{3} + I_{\min}(I_{\max} - I_{\min})\right]^{\frac{1}{2}} \tag{5.24}$$

RMS chopper current

$$I_{CH} = \sqrt{\frac{1}{T}\int_0^{dT} i_0^2 dt}$$

$$I_{CH} = \sqrt{\frac{1}{T}\int_0^{dT}\left[I_{\min} + \left(\frac{I_{\max} - I_{\min}}{dT}\right)t\right]^2 dt}$$

$$I_{CH} = \sqrt{d}\left[I_{\min}^2 + \frac{(I_{\max} - I_{\min})^2}{3} + I_{\min}(I_{\max} - I_{\min})\right]^{\frac{1}{2}}$$

$$I_{CH} = \sqrt{d}\, I_{O(RMS)} \tag{5.25}$$

5.7 Principle of Step up Chopper

Figure 5.13 shows a step-up chopper to obtain a load voltage V_O higher than the input voltage *V*. The values of *L* is chosen depending upon the requirement of output current. When the chopper is *ON*, the inductor *L* is connected across the supply.

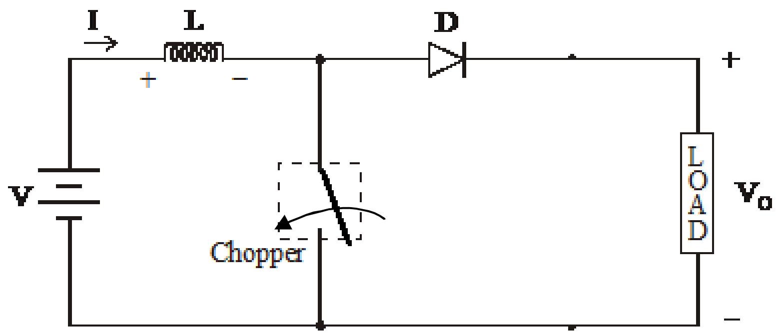


Fig. 5.13: Step-up Chopper

The inductor current *'I'* rises and the inductor stores energy during the *ON* time of the chopper, t_{ON}. When the chopper is off, the inductor current *I* is forced to flow through the diode *D* and load for a period, t_{OFF}. The current tends to decrease resulting in reversing the polarity of induced EMF in *L*. Therefore voltage across load is given by

$$V_O = V + L\frac{dI}{dt} \quad i.e., \quad V_O > V \qquad (5.27)$$

If a large capacitor 'C' is connected across the load then the capacitor will provide a continuous output voltage V_O.

- **The expression for the output voltage**

  Assume the average inductor current to be *I* during *ON* and *OFF* time of Chopper.

- **When Chopper is ON**

  Voltage across inductor $L = V$

  Therefore energy stored in inductor = $V.I.t_{ON}$ (5.28)

  where $t_{ON} = ON$ period of chopper.

- **When Chopper is OFF** (energy is supplied by inductor to load)

  Voltage across $L = V_O - V$

  Energy supplied by inductor $L = (V_O - V)It_{OFF}$, where $t_{OFF} = OFF$ period of Chopper.

  Neglecting losses, energy stored in inductor *L* = energy supplied by inductor L

  Therefore $VIt_{ON} = (V_O - V)It_{OFF}$

$$V_O = \frac{V[t_{ON} + t_{OFF}]}{t_{OFF}}$$

$$V_O = V\left(\frac{T}{T - t_{ON}}\right)$$

Where T = Chopping period or period of switching.

$T = t_{ON} + t_{OFF}$

$$V_O = V\left(\frac{1}{1-\frac{t_{ON}}{T}}\right)$$

Therefore $$V_O = V\left(\frac{1}{1-d}\right) \qquad (5.29)$$

Where $d = \frac{t_{ON}}{T} =$ duty cyle

For variation of duty cycle 'd' in the range of $0 < d < 1$ the output voltage V_O will vary in the range $V < V_O < \infty$.

The chopper switch requires a certain minimum time to turn *ON* and turn *OFF*. Hence duty cycle *d* can be varied only between a minimum and a maximum value, limiting the minimum and maximum value of the output voltage. Ripple in the load current depends inversely on the chopping frequency, *f*. Therefore to reduce the load ripple current, frequency should be as high as possible.

Problem 5.1 : For the chopper shown in figure pb 5.1, express the following variables as functions of V, R and duty cycle 'd' in case load is resistive.

- Average output voltage and current
- Output current at the instant of commutation
- Average and RMS freewheeling diode current.
- RMS value of output voltage
- RMS and average chopper currents.

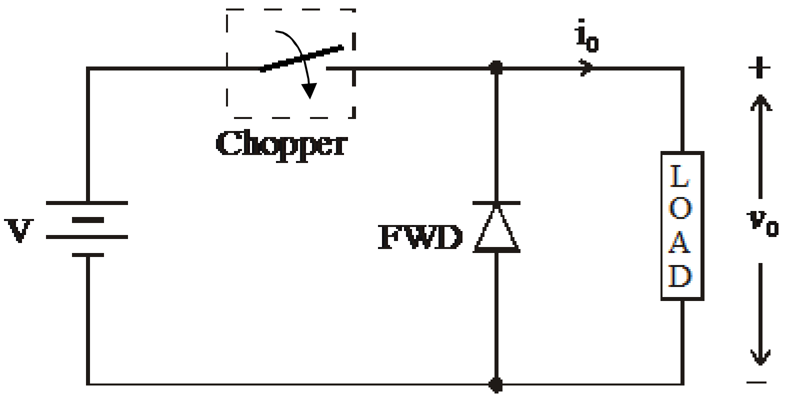


Fig. pb. 5.1

Solution

- Average output voltage, $V_{dc} = \left(\frac{t_{ON}}{T}\right)V = dV$

  Average output current, $I_{dc} = \frac{V_{dc}}{R} = \frac{dV}{R}$

- The chopper is commutated at the instant $t = t_{ON}$.

Therefore output current at the instant of commutation is $\frac{V}{R}$, since V is the output voltage at that instant.

- Freewheeling diode (FWD) will never conduct in a resistive load. Therefore average and RMS freewheeling diode currents are zero.

- RMS value of output voltage

$$V_{O(RMS)} = \sqrt{\frac{1}{T}\int_0^{t_{ON}} v_0^2 dt}$$

But $v_O = V$ during t_{ON}

$$V_{O(RMS)} = \sqrt{\frac{1}{T}\int_0^{t_{ON}} V^2 dt}$$

$$V_{O(RMS)} = \sqrt{V^2\left(\frac{t_{ON}}{T}\right)}$$

$$V_{O(RMS)} = \sqrt{d}V$$

Where duty cycle, $d = \frac{t_{ON}}{T}$

- RMS value of chopper current

= RMS value of load current

$$= \frac{V_{O(RMS)}}{R}$$

$$= \frac{\sqrt{d}V}{R}$$

Average value of the chopper (switch) current

= Average value of load current

$$= \frac{dV}{R}$$

Problem 5.2 : Input to the step up chopper is 200 V. The output required is 600 V. If the conducting time of the switch is 200 µsec. Compute;

- Chopping frequency,
- If the pulse width is halved for constant frequency of operation, find the new output voltage.

Solution

V = 200 V, $t_{ON} = 200\mu s$, $V_{dc} = 600V$

$$V_{dc} = V\left(\frac{T}{T - t_{ON}}\right)$$

$$600 = 200\left(\frac{T}{T - 200\times10^{-6}}\right)$$

Solving for T

$$T = 300\mu s$$

- Chopping frequency

$$f = \frac{1}{T}$$

$$f = \frac{1}{300 \times 10^{-6}} = 3.33KHz$$

- Pulse width is halved

Therefore $\quad t_{ON} = \frac{200 \times 10^{-6}}{2} = 100\mu s$

Frequency is constant

Therefore $\quad f = 3.33KHz$

$$T = \frac{1}{f} = 300\mu s$$

Therefore output voltage $\quad = V\left(\frac{T}{T - t_{ON}}\right)$

$$= 200\left(\frac{300 \times 10^{-6}}{(300-100)10^{-6}}\right) = 300 \text{ Volts}$$

Problem 5.3: A dc chopper in figure pb 5.3 has a resistive load of $R = 10\Omega$ and input voltage of V = 200 V. When chopper is ON, its voltage drop is 2 V and the chopping frequency is 1 kHz. If the duty cycle is 60%, determine

- Average output voltage
- RMS value of output voltage
- Effective input resistance of chopper
- Chopper efficiency.

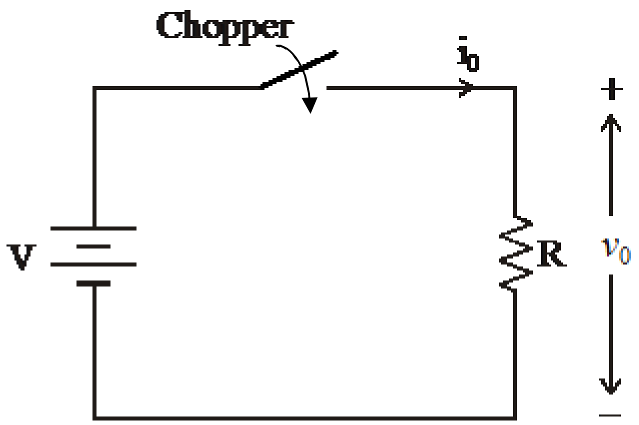


Fig.pb. 5.3

Solution

V = 200 V, $R = 10\Omega$, Chopper voltage drop, $V_{ch} = 2V$, d = 0.60, f = 1 kHz.

- Average output voltage

$$V_{dc} = d\left(V - V_{ch}\right)$$

$$V_{dc} = 0.60\left[200 - 2\right] = 118.8 \text{ Volts}$$

- RMS value of output voltage

$$V_O = \sqrt{d}\left(V - V_{ch}\right)$$

$$V_O = \sqrt{0.6}\left(200-2\right) = 153.37 \text{ Volts}$$

- Effective input resistance of chopper is

$$R_i = \frac{V}{I_S} = \frac{V}{I_{dc}}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{118.8}{10} = 11.88 \text{ Amps}$$

$$R_i = \frac{V}{I_S} = \frac{V}{I_{dc}} = \frac{200}{11.88} = 16.83\Omega$$

- Output power is

$$P_O = \frac{1}{T}\int_0^{dT} \frac{v_0^2}{R}\,dt$$

$$P_O = \frac{1}{T}\int_0^{dT} \frac{\left(V - V_{ch}\right)^2}{R}\,dt$$

$$P_O = \frac{d\left(V - V_{ch}\right)^2}{R}$$

$$P_O = \frac{0.6\left[200-2\right]^2}{10} = 2352.24 \text{ watts}$$

- Input power, $P_i = \frac{1}{T}\int_0^{dT} V i_O\,dt$

$$P_O = \frac{1}{T}\int_0^{dT} \frac{V\left(V - V_{ch}\right)}{R}\,dt$$

$$P_O = \frac{dV\left(V - V_{ch}\right)}{R} = \frac{0.6\times 200\left[200-2\right]}{10} = 2376 \text{ watts}$$

- Chopper efficiency,

$$\eta = \frac{P_O}{P_i}\times 100$$

$$\eta = \frac{2352.24}{2376}\times 100 = 99\%$$

Problem 5.4 : A chopper feeding on RL load is shown in figure 5.4. With V = 200 V, R = 5Ω, L = 5 mH, f = 1 kHz, d = 0.5 and E = 0 V. Calculate

- Maximum and minimum values of load current
- Average value of load current
- RMS load current
- Effective input resistance as seen by source
- RMS chopper current.

Solution

V = 200 V, R = 5 Ω, L = 5 mH, f = 1kHz, d = 0.5, E = 0

Chopping period is $T=\frac{1}{f}=\frac{1}{1\times10^{3}}=1\times10^{-3}$ secs

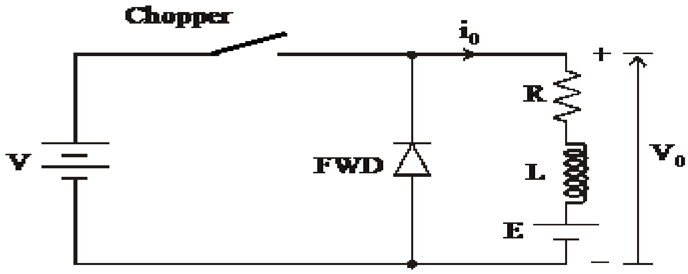


Fig.: 5.4

Refer equations (5.19) and (5.20) for expressions of $I_{\max}$ and $I_{\min}$.

Maximum value of load current

$$I_{\max}=\frac{V}{R}\left[\frac{1-e^{-\frac{dRT}{L}}}{1-e^{-\frac{RT}{L}}}\right]-\frac{E}{R}$$

$$I_{\max}=\frac{200}{5}\left[\frac{1-e^{-\frac{0.5\times5\times1\times10^{3}}{5\times10^{-3}}}}{1-e^{-\frac{5\times1\times10^{-3}}{5\times10^{-3}}}}\right]-0$$

$$I_{\max}=40\left[\frac{1-e^{-0.5}}{1-e^{-1}}\right]=24.9\text{ A}$$

Minimum value of load current is

$$I_{\min}=\frac{V}{R}\left[\frac{e^{\frac{dRT}{L}}-1}{e^{\frac{RT}{L}}-1}\right]-\frac{E}{R}$$

$$I_{\min}=\frac{200}{5}\left[\frac{e^{\frac{0.5\times5\times1\times10^{-3}}{5\times10^{-3}}}-1}{e^{\frac{5\times1\times10^{-3}}{5\times10^{-3}}}-1}\right]-0$$

$$I_{\min}=40\left[\frac{e^{0.5}-1}{e^{1}-1}\right]=15.1\text{ A}$$

Average value of load current is

$$I_{dc}=\frac{I_1+I_2}{2}\text{ for linear variation of currents}$$

Therefore $I_{dc} = \frac{24.9+15.1}{2} = 20 \text{ A}$

Refer equations (5.24) and (5.25) for RMS load current and RMS chopper current.

RMS load current from equation (5.24) is

$$I_{O(RMS)} = \left[I_{\min}^2 + \frac{\left(I_{\max} - I_{\min}\right)^2}{3} + I_{\min}\left(I_{\max} - I_{\min}\right) \right]^{\frac{1}{2}}$$

$$I_{O(RMS)} = \left[15.1^2 + \frac{\left(24.9-15.1\right)^2}{3} + 15.1\left(24.9-15.1\right) \right]^{\frac{1}{2}}$$

$$I_{O(RMS)} = \left[228.01 + \frac{96.04}{3} + 147.98 \right]^{\frac{1}{2}} = 20.2 \text{ A}$$

RMS chopper current from equation is (2.25) is

$$I_{ch} = \sqrt{d} I_{O(RMS)} = \sqrt{0.5} \times 20.2 = 14.28 \text{ A}$$

Effective input resistance is

$$R_i = \frac{V}{I_S}$$

I_S = Average source current

$$I_S = dI_{dc}$$

$$I_S = 0.5 \times 20 = 10 \text{ A}$$

Therefore effective input resistance is

$$R_i = \frac{V}{I_S} = \frac{200}{10} = 20\Omega$$