4.5 Three phase controlled rectifier

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load. The features of 3-phase controlled rectifiers are;

- Operate from 3 phase ac supply voltage
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.
- Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

4.5.1 Three phase half wave controlled rectifier with R load

The thyristor will conduct (ON state), when the anode-to-cathode voltage is positive and a firing current pulse is applied to the gate terminal. Delaying the firing pulse by an angle α controls the load voltage.

When the load is resistive, current $i_{o(\omega t)}$ has the same waveform of the load voltage. As the load becomes more and more inductive, the current flattens and finally becomes constant. The thyristor goes to the non-conducting condition (OFF state) when the following thyristor is switched ON, or the current, tries to reach a negative value.

For resistive load

- $0° \leq \alpha \leq 30°$, output voltage is continuous.
- $30° \leq \alpha \leq 120°$, output voltage is discontinuous and has some intervals in which output voltage is zero.
- $\alpha > 150°$, output voltage is zero.

1-Continuoues case $\alpha \leq 30°$

Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in the figure (7.4).

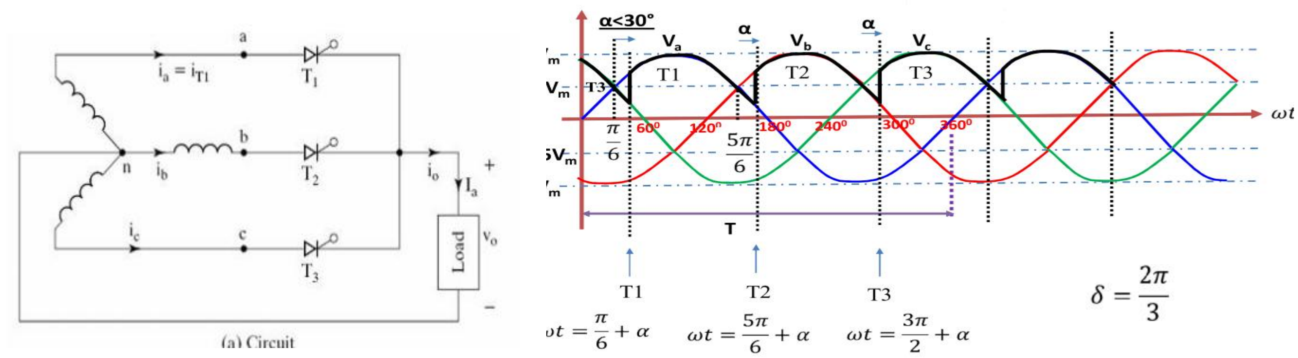
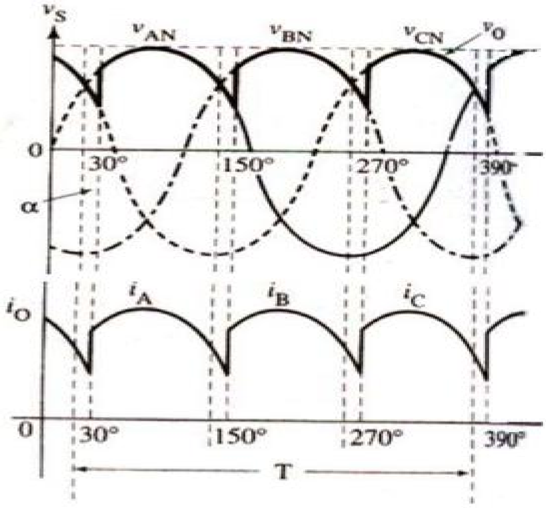


Fig4.7 three phase thyristor half wave rectifier (a) circuit ,(b) output voltage Continuous wavefom



Waveforms of three phase HWSCR Continuous Mode with R load

The average and rms load voltages are

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\left(-\cos\omega t\right)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

Note from the trigonometric relationship

$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos\left(\frac{5\pi}{6}\right)\cos(\alpha)+\sin\left(\frac{5\pi}{6}\right)\sin(\alpha) \\ \qquad +\cos\left(\frac{\pi}{6}\right).\cos(\alpha)-\sin\left(\frac{\pi}{6}\right)\sin(\alpha)\end{array}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos(150^0)\cos(\alpha)+\sin(150^0)\sin(\alpha) \\ \qquad +\cos(30^0).\cos(\alpha)-\sin(30^0)\sin(\alpha)\end{array}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos(180^0-30^0)\cos(\alpha)+\sin(180^0-30^0)\sin(\alpha) \\ \qquad +\cos(30^0).\cos(\alpha)-\sin(30^0)\sin(\alpha)\end{array}\right]$$

Note: $\cos(180^0-30^0) = -\cos(30^0)$

$\sin(180^0-30^0) = \sin(30^0)$

$$\therefore V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} +\cos(30^0)\cos(\alpha)+\sin(30^0)\sin(\alpha) \\ \qquad +\cos(30^0).\cos(\alpha)-\sin(30^0)\sin(\alpha)\end{array}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos(30^0)\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\times\frac{\sqrt{3}}{2}\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos(\alpha)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos(\alpha)$$

Where $V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\ V_m}{2\pi}$$

Where V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of output voltage is found by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

and we obtain

$$V_{O(RMS)} = \sqrt{3}V_m\left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor T1 is triggered at ωt=(30°+α) and T1 conducts up to ωt=180°. When the phase supply voltage decreases to zero, the load current falls to zero and the thyristor T1 turns off. Thus T1 conducts from ωt=(30° + α) to (180°).

2- Discontinuoues case α≥30°

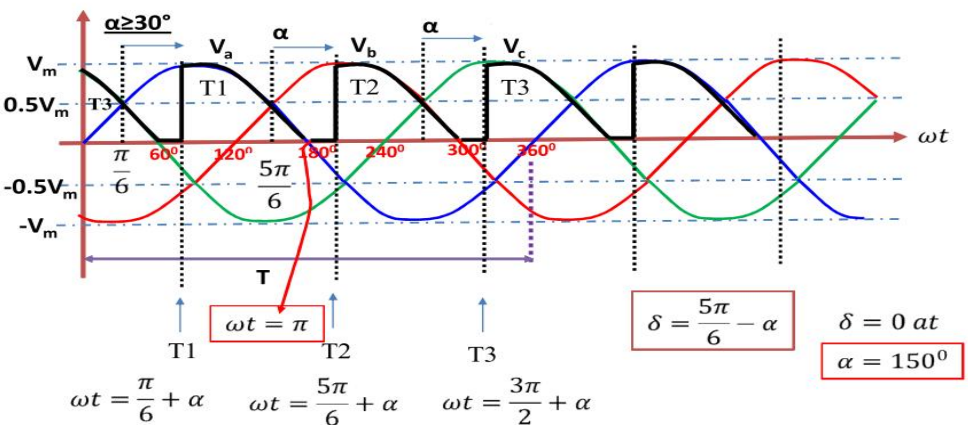
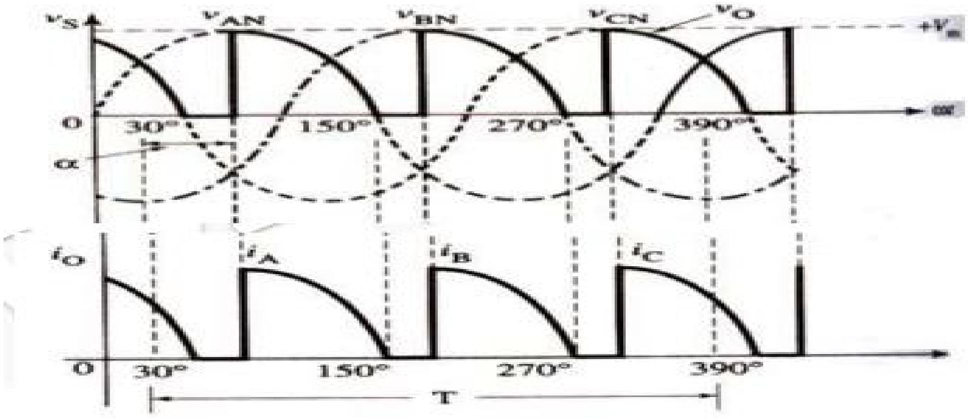


Fig4.8 Output voltage of a Three phase half wave controlled rectifier with R load (Discontinuous case α≥30°)



Waveforms of three phase HWSCR discontinuous Mode with R load

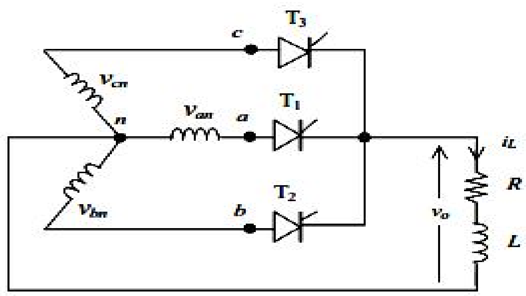
The average output voltage is given by;

$$V_{dc} = \frac{3V_{mp}}{2\pi}\left[\int_{\alpha+30^0}^{180^0} \sin\omega t.d(\omega t)\right] = \frac{3V_{mp}}{2\pi}\left[1+\cos\left(\alpha+30^0\right)\right] \quad (4.28)$$

The RMS output voltage is given by;

$$V_{or} = \left[\frac{3}{2\pi}\int_{\alpha+\pi/6}^{\pi} V_{mp}^2 \sin^2 \omega t.\, d\,(\omega t)\right]^{1/2} = \frac{\sqrt{3}.\, V_{mp}}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6}-\alpha\right)+\frac{1}{2}\sin\left(2\alpha+\pi/3\right)\right]^{1/2} \quad (4.29)$$

4.5.2 Three phase half wave controlled rectifier with RL load



Three -Phase HWSCR With RL Load

- For firing angle $\alpha<30^\circ$ the same equations for Vo(dc) and Vo(rms) when $\alpha<30^\circ$ with R Load.
- For firing angle $\alpha=30^\circ$ the same equations for Vo(dc) and Vo(rms) when $\alpha>30$ with R Load.
- For $30°<\alpha<90°$, V o(dc) and V o(rms) are different based on new "ωt" range.

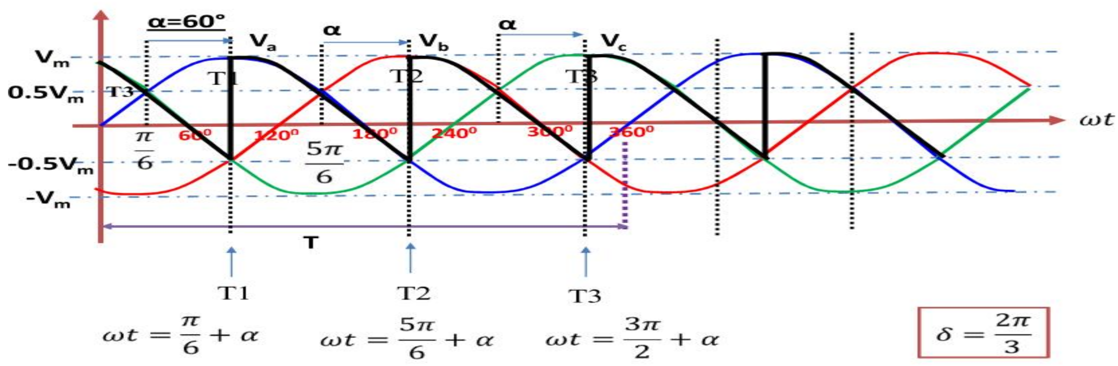


Fig4.9 Output voltage of a Three phase half wave controlled rectifier with RL load case $\alpha \geq 30^{\circ}$, $\boldsymbol{\alpha=60^{\circ}}$**;**

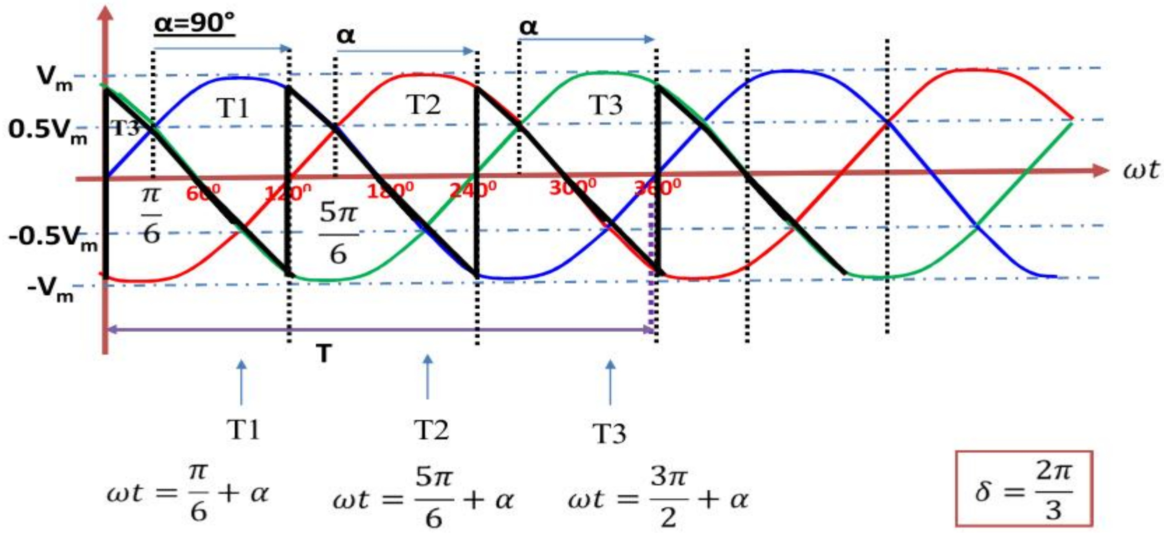
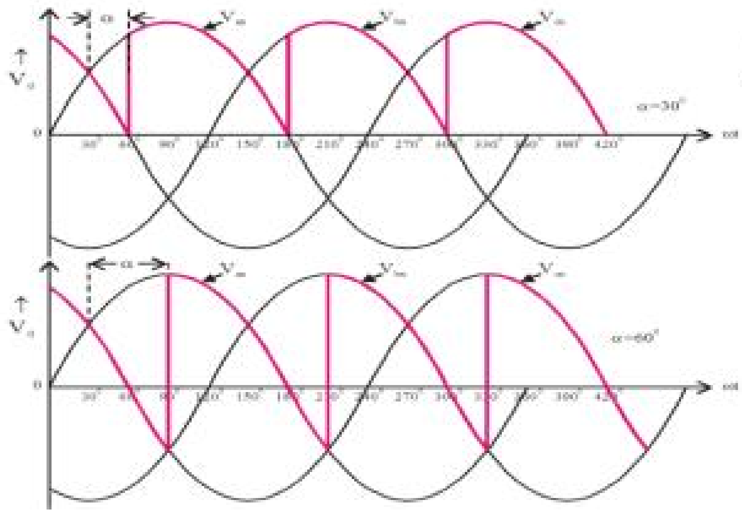


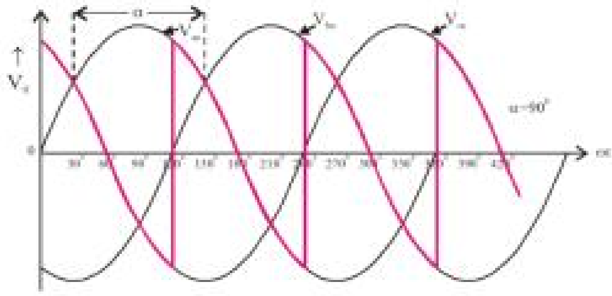
Fig4.10 Output voltage of a Three phase half wave controlled rectifier with RL load case $\alpha \geq 30^{\circ}$, $\boldsymbol{\alpha=90^{\circ}}$**;**

- Average voltage at $\boldsymbol{\alpha=90^{\circ}}$**;** $\boldsymbol{V_{o(dc)}=0V}$**.**
- In a three phase half wave controlled rectifier with RL load and freewheeling diode , Vo(dc) will be look like when R-load used for $\alpha \geq 30°$.
- Beyond $\alpha>90°$ or between $90°<\alpha<180^{\circ}$, Vo(dc) is negative, three phase 3-pulse converter operates as a line commutated inverter. for example if $\alpha=120^{\circ}$, the

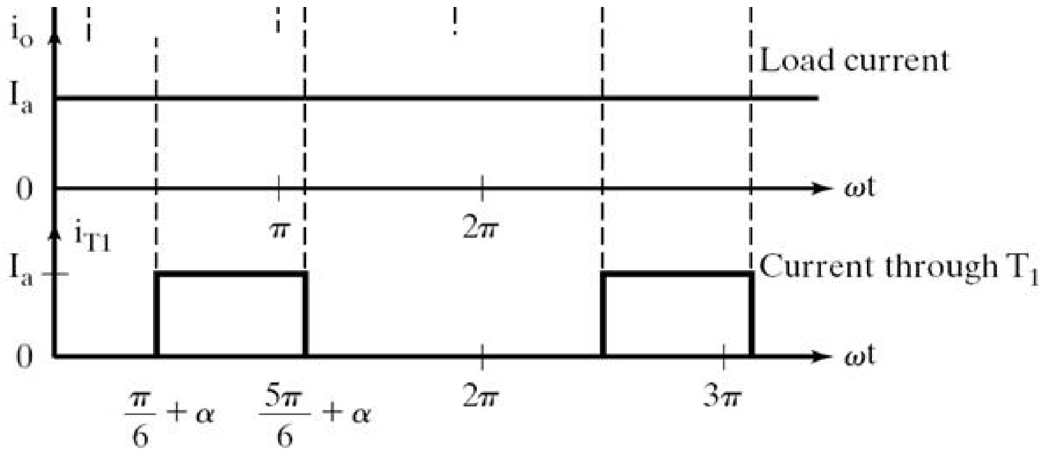
$$V_{o(dc)} = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha) \quad (4.29)$$

$$V_{o(dc)} = 0.827 * V_m * \cos(120^{\circ}) = -0.414V_m$$



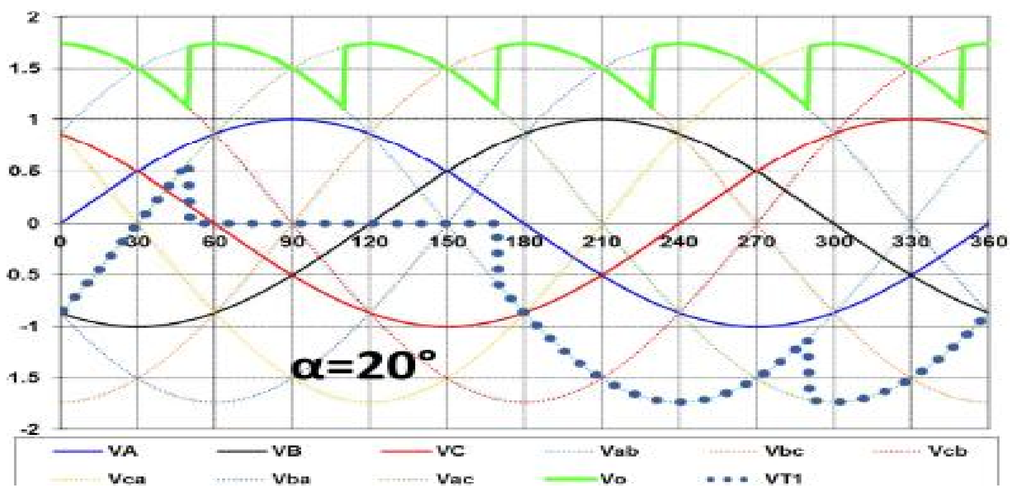


RL load

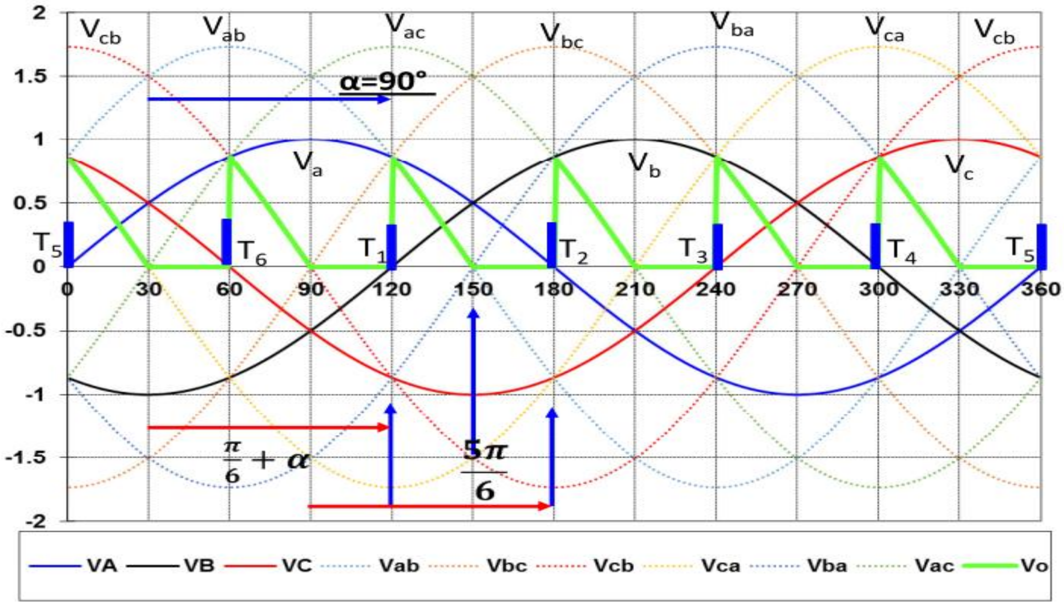


4.5.3 Three phase full wave controlled rectifier with R load

- For any α ≤60° the output current continues and no zero voltage is appear in the output voltage.



- For $\alpha > 60°$;



the average output voltage is

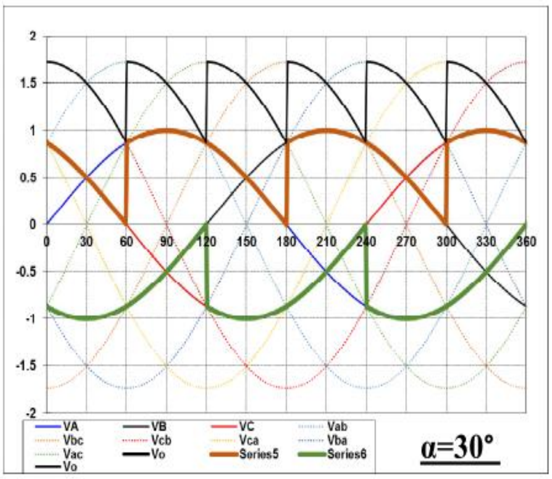
$$V_{o(av)} = \frac{1}{\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}} \sqrt{3}\, V_m \sin\left(\omega t + \frac{\pi}{6}\right) d\,\omega t$$

$$V_{o(av)} = \frac{3\sqrt{3}\, V_m}{\pi}\left[1 + \cos\left(\frac{\pi}{3}+\alpha\right)\right]$$

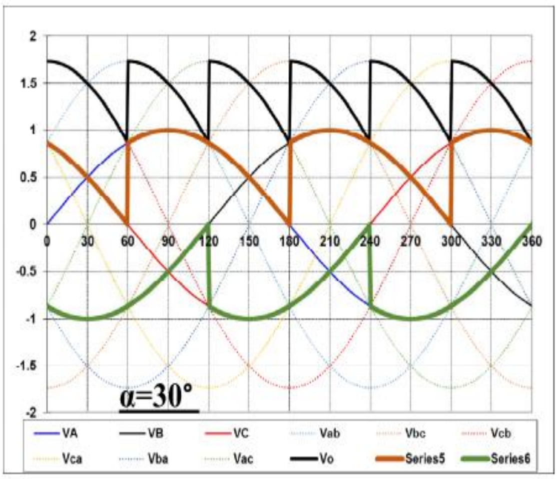
The RMS output voltage is $V_{or} = \dfrac{V_{ml}}{2\sqrt{\pi}}\sqrt{\left(4\pi - 6\alpha + 3\mathrm{Cos}(\frac{\pi}{6}+2\alpha)\right)}$

4.5.4 Three phase bridge converter with RL load:

<u>α≤60°</u>



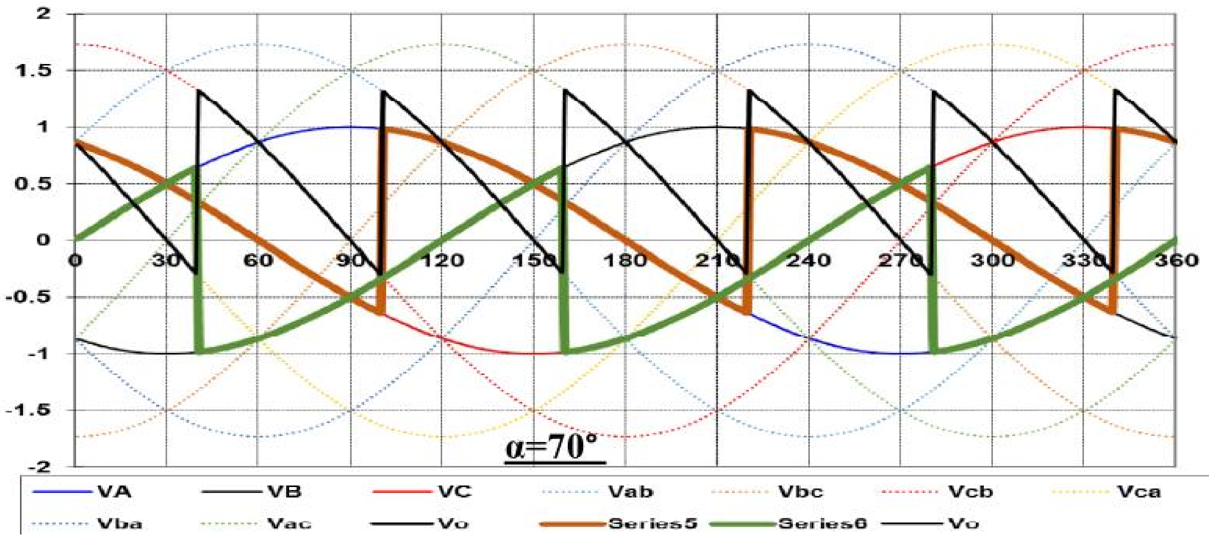
RL-Load



R-Load

For heavy inductive load, the RMS and DC output voltage equation are same one shown in R-Load for $\alpha \le 60°$.

- For $\alpha > 60°$;



Due to the magnetic field a high voltage is induced across the inductor in the direction required to keep the current flowing. This results in negative voltage appearing across a RL load.

- At α=90° the DC output voltage is equal to zero.
- at α=120° the output voltage is negative

Worked examples:

Example 2: a single phase bridge rectifier has load of R=15 Ω and, Vs=220 *sin* 314 *t* and unity transformer ratio. If it is required to obtain an average output voltage of 70% of the maximum possible output voltage, calculate:- (a) The delay angle α, (b) The efficiency, (c) Ripple factor of output voltage(d) The peak inverse voltage (PIV) of one thyristor.

Solution:

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\,(1+\cos\alpha) = 0.7\text{ , then, }\alpha = 66.42^{o}$$

$$V_m = 220\text{, then, }V_{dc} = 0.7 * V_{dm} = 0.7 * \frac{2\,V_m}{\pi} = 98.04\ V$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{98.04}{15} = 6.536\quad A$$

$$V_{rms} = \frac{V_m}{\sqrt{2\pi}}\sqrt{\pi - \alpha + \frac{\sin(2\alpha)}{2}}$$

α=66.42^{o} V_{rms}=134.638 V. Then, I_{rms}=134.638/15=8.976 A

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{dc} * I_{dc}}{V_{rms} * I_{rms}} = \quad = \frac{98.04 * 6.536}{134.638 * 8.976} = 53.04\%$$

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{134.638}{98.04} = 1.3733$$

$$RF = \frac{V_{ac}}{V_{dc}} = \sqrt{FF^2 - 1} = \sqrt{1.3733^2 - 1} = 0.9413$$

The PIV is V_m

Example 03 : A controlled full-wave gridge rectifier has pure DC load current of 50 A and, *Vs*=220 *sin* 314 *t* and unity transformer ratio. If it is required to obtain an average output voltage of 70% of the maximum possible output voltage, calculate:- (a) The delay angle α, (b) The efficiency, (c) Ripple factor (d) The peak inverse voltage (PIV) of the thyristor.

Solution:

$$V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha = 0.7$$ then, α=45.5731°= 0.7954

$$V_m = 220, \quad V_{dc} = 0.7 * V_{dm} = 0.7 * 2\,V_m / \pi = 98.04\ V, \quad V_{rms} = V_m / \sqrt{2}$$

At α=45.5731° V_{rms}=155.563 V. Then, I_{rms}=50 A

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{dc} * I_{dc}}{V_{rms} * I_{rms}} = \frac{98.04 * 50}{155.563 * 50} = 63.02\%$$

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{155.563}{98.04} = 1.587$$

$$RF = \frac{V_{ac}}{V_{dc}} = \sqrt{FF^2 - 1} = \sqrt{1.3733^2 - 1} = 1.23195$$

Example 04: Three-phase full-wave controlled rectifier is connected to 380 V, 50 Hz supply to feed a load of 10 Ω pure resistance. If it is required to get 400 V DC output voltage, calculate the following: (a) The firing angle, α (b) The rectfication effeciency (c) PIV of the thyristors.

Solution:

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha = \frac{3\sqrt{3}}{R} * \frac{\sqrt{2}}{\sqrt{3}} * 380\cos\alpha = 400V .$$

Then $\alpha = 38.79^o$, $I_{dc} = \frac{V_{dc}}{R} = \frac{400}{10} = 40A$

From (3.84) the rms value of the output voltage is:

$$V_{rms} = \sqrt{3}\,V_m\sqrt{\left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right)} = 412.412\,V$$

Then, $V_{rms} = 412.412\ V$ Then, $I_{rms} = \frac{V_{rms}}{R} = \frac{412.412}{10} = 41.24\ A$

Then, $\eta = \frac{V_{dc} * I_{dc}}{V_{rms} * I_{rms}} * 100 = \frac{400 * 40}{412.4 * 41.24} * 100 = 94.07\%$

The PIV= $\sqrt{3}\,V_m$=537.4V

Example 05: Solve the previous example if the required dc voltage is 150V.

Solution:

$$V_{dc} == \frac{3\sqrt{3} * \frac{\sqrt{2}}{\sqrt{3}} * 380}{\pi}\left[1 + \cos\left(\pi/3 + \alpha\right)\right] = 150V \qquad \alpha = 73^o$$

It is not acceptable result because the above equation valid only for $\alpha \le 60$

Then we have to use the following expression to get α:

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha = \frac{3\sqrt{3} * \frac{\sqrt{2}}{\sqrt{3}} * 380}{\pi}\cos\alpha = 150V \qquad , \quad \alpha = 75.05^o$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{150}{10} = 15A$$

The rms value of the output voltage is

$$V_{rms} = \sqrt{3}V_m\sqrt{1 - \frac{3}{4\pi}\left(2\alpha - \cos\left(2\alpha + \frac{\pi}{6}\right)\right)} = \sqrt{3} * \frac{\sqrt{2}}{\sqrt{3}} * 380 * \sqrt{\left(1 - \frac{3}{4\pi}\left(\ 2 * 75.05 * \frac{\pi}{180} - \cos\left(2 * 75.05 + 30\right)\right)\right)}$$

$$V_{rms} = 198.075\ V$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{198.075}{10} = 19.8075\ A \qquad \eta = \frac{V_{dc} * I_{dc}}{V_{rms} * I_{rms}} * 100 = \frac{150 * 15}{198.075 * 19.81} * 100 = 57.35\ \%$$

The PIV= $\sqrt{3}$ V_m=537.4V