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M'Hamad **B**ougara **University** of **B**oumerdes
Faculty of **T**echnology
Department of **Mechanical Engineering**

Course Handout

COMPOSITE MATERIALS
COURSES AND EXERCISES SOLVED

(Fundamental Unit-- Science and Technology Field – LMD Master)

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Lecturer class "A"

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FOREWORD

This handout entitled composite materials, it is intended for students of masters in the mechanical engineering field of the LMD system, it can be used as support for a course given to students. These are the courses that we taught for around ten years at M'hamad University Bougara d' Boumerdes, Faculty of Technology, UMBB. The content of this handout includes the program taught in the mechanical engineering department. It is written in the form of detailed lessons, with solved applications and additional unsolved exercises. It is presented in a very simple style which allows students to understand very quickly. The content of this handout is structured into seven chapters.

The first chapter is devoted to the bibliographic study on composite materials and their industrial applications. Chapter two concerns the constitutive relations for anisotropic materials in the first place , and in particular the behavior of orthotropic and transversely isotropic materials. Chapter three addresses the micromechanical analysis of composite materials in terms of the mechanical properties of the fiber and matrix, the relationship between them with the direction of the fibers in the privilege axis, and the effect of thermal stress. At the end of this chapter solved and other additional unsolved exercises will be given. Chapter four will be reserved for the study of the law of behavior of a unidirectional fold with any angle orientation of the fibers. It deals with the detailed relationships of stresses and strains in different orientations in a single layer, the following type of analysis is required . Finally, the last two chapters address the mechanical behavior of laminates and the failure criteria of composite materials. The main objective of this chapter is to create a structure with a rigidity that can withstand a given loading . It will be completed at the end by applying the laws of behavior in Matlab through solved exercises.

This contribution may seem meager, but we are counting on readers to help us improve it, with their criticisms, if any. We wish all our students a very good university education and a successful career.

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CHAPTER I
GENERAL INFORMATION ON COMPOSITE MATERIALS

I. INTRODUCTION

There are different families of materials: metals, plastics, composites, etc. …. Composite materials are not new, they have always been used by man, for example wood, concrete and reinforced concrete.

I.1. DEFINITION :

A composite material is made up of the assembly of two or more materials of different natures, which results in a heterogeneous material whose overall performance is superior to that of the components taken separately. We now commonly call "composite materials" arrangements of fibers, reinforcements ***which*** are embedded in a ***matrix*** whose mechanical resistance is much lower. The matrix ensures the cohesion and orientation of the fibers, it also makes it possible to transmit the stresses to which the parts are subjected.

The main interest in using composite materials comes from its excellent characteristics. They provide numerous functional advantages [1] [4]:

- ✓ Lightness, freedom of form and reduced maintenance.
- ✓ high resistance to fatigue and good electrical insulation.
- ✓ Low aging under the action of humidity, heat, corrosion.
- ✓ Insensitive to chemicals (resin).
- ✓ Their low utilization rate comes from their cost.

There are two types of composites: mass-market composites and high-performance composites.

- ➢ **Wide distribution:**

Widespread composite materials offer essential advantages, which are: optimization of costs by reduction in cost prices, its composition of polyester with long or short glass fibers (in the form of mat or fabric) and the simplicity of the principle of material development (contact molding, SMC and injection).

- ➢ **High performance:**

High-performance composite materials are used in the aeronautics field where the need for high performance comes from high added values. The reinforcements are rather long fibers. The reinforcement rate is greater than 50%.

These composites are produced by the following procedures:

- autoclave draping, filament winding, RTM.

I.2. CONSTITUENTS OF COMPOSITE MATERIALS:

I.2.1. The reinforcements :

The reinforcements help to improve the mechanical tensile strength and rigidity of composite materials and are in filament form (organic or inorganic fibers).

I.2.1.1. Different types of reinforcement: [1], [7]

The most commonly used reinforcements are in the form of fibers or derived forms and constitute a volume fraction of composite material generally between 0.3 and 0.7. Fiber reinforcements come in various commercial forms (Fig.1).

Aramid | Carbon | Glass fabric | Fiberglass L | Glass mat

Figure. (I.1): Different types of fibers

I.2.1.2. Different types of reinforcement:

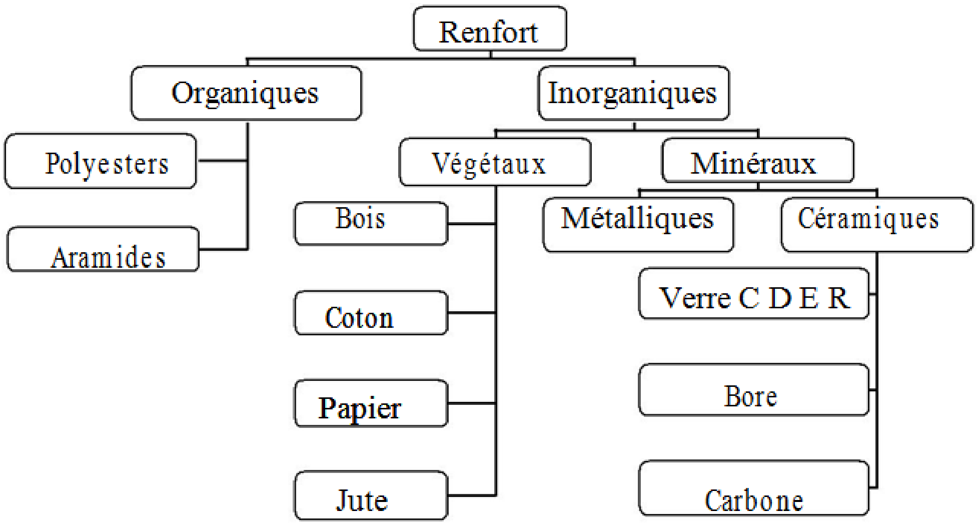


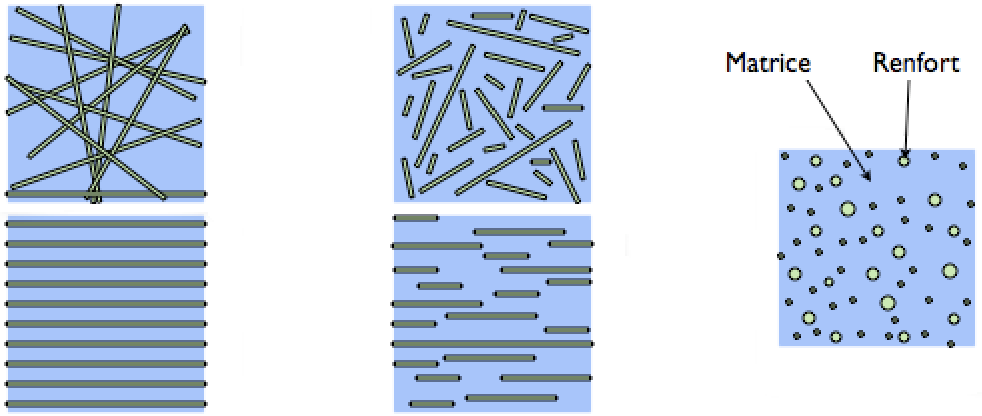
Fig. (I.2): Different families of reinforcement.

I.2.1.3. Geometry of reinforcements:

From a geometric point of view, we can distinguish three main types of reinforcements:

- Long fibers (ie length comparable to the dimensions of the part, figure 4a);
- Short fibers (ie of short length compared to the dimensions of the part, figure 4b);
- Particles, or reinforcing fillers (figure 4c).

All these reinforcements are included within a matrix which distributes the efforts between them and protects them from external attacks, as indicated above. In addition, when the reinforcements are fibers, they can either be oriented in a precise direction or arranged randomly.



(has) (b) (vs)

Figure (I.3): The geometric structures of the composites: (a) long fibers, (b) short fibers, (c) particles.

I.2.1.3. Main mechanical characteristics of the basic fibers: [20]

Table I.1: Mechanical properties of fibers

fiber	density	σ_r (MPa) in traction	Elongation at the breakup in %	E_L (MPa) Longitudinal	Diameter of filament elementary µm
Glass E	2.54	3400	4.8	73000	3-30
Glass R	2.48	4400	5.4	86000	3-30
Aramid BM	1.45	3100	2	70000	12
Aramid HM	1.45	3100	1	130000	12
Kevlar	-	2900	2.3	130000	1.45
Aluminum	-	1380	0.7	380000	3.9
Carbon HT	1.78	2800	0.5	200000	8
HM Carbon	1.80	2200	-	400000	8
Boron	2.63	3500	0.8	400000	100-200

I.2.2. The matrix :

The matrix is the element that binds and maintains the fibers. It distributes the forces (resistance to compression or bending) and provides chemical protection of the fibers.

I.2.2.1. Different natures of the matrix:

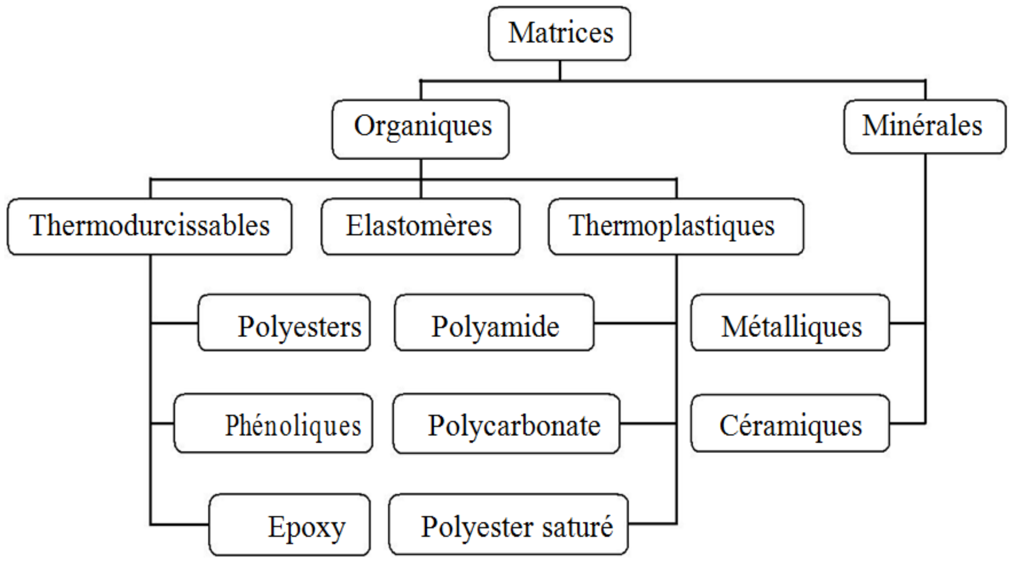


Fig. (I.4): Different matrix families.

- ✓ Thermosetting resin: are shaped and polymerize into the desired shape. Irreversible transformation.
- ✓ Thermoplastic resin: shaped by heating, hardens upon cooling. The transformation is reversible.

Table I.2: Main properties of Thermosetting and Thermoplastic matrices:

Dies	Thermosets	Thermoplastics
Basic state	Viscous liquid to polymerize	Solid ready to use
Storage	Reduced	Unlimited
Wettability of reinforcements	Well-off	Difficult
Molding	Continuous heating	Heating + cooling
Cycle	Long (polymerization)	Short
Shock resistance	Limited	Pretty good
Thermal hold	Best	Reduced (except new TP)
Scraps and waste	Lost or used in charges	Recyclable
Working conditions	Solvent fumes	Cleanliness

I.2.2.2. Main mechanical characteristics of resins:

Table I.3: Mechanical properties of resins. [20] [4]

	Polyester	Epoxy	Phenolic	Polyamide	Aluminum
ε_r in traction (%)	2 - 5	2 - 5	2.5	-	-
ρ (Kg/m^3)	1200	1100-1500	1200	1130	2630
σ_r in tension (MPa)	50 - 80	60 - 80	40	70	358
E in traction (GPa)	2.8 – 3.5	3 – 5	-	3	69

I.2.3. Charges: [1]

The purpose of the reinforcing filler is to improve the mechanical characteristics of the resin, or reduce the cost of the resins while maintaining the performance of the resins. In general, these fillers are microbeads (glass, carbon, epoxy, phenolic, polystyrene, etc.) or particles (carbon black powder).

I.2.4. Additives: [1]

They are necessary to ensure sufficient adhesion between the fibrous reinforcement and the matrix and to modify the appearance or characteristics of the material to which they are added.

I.2.5. The interface:

In addition to these fibers and the matrix, it is necessary to add: an interface which ensures reinforcement/matrix compatibility Fig. I.5, which transmits the constraints from one to the other without relative displacement. Chemical products are also included in the composition of the composite, the interphase, etc. ... which can affect the mechanical behavior, but are practically never involved in the calculation of composite structures. [1].

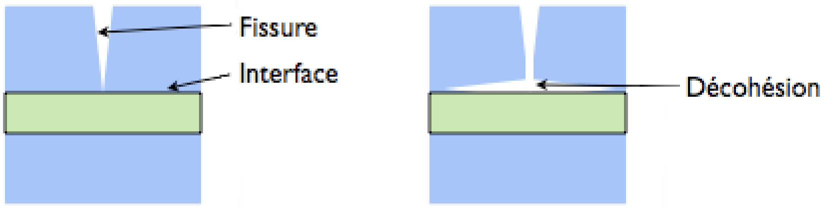


Figure (I.5) : The role of the interface on the toughness of composites: when a crack reaches the interface (a), it is deflected (b).

I.3 Structure of composite parts: fabrics, laminates, etc.

The design and manufacture of parts using these materials do not present any particular distinctive signs. On the other hand, long fiber composite parts generally have very specific structures. These structures allow the designer to “put the material where it is needed”, by optimizing the orientation and arrangement of the reinforcements according to the loading undergone by the part. In practice, the diameter of the fibers of a “modern” composite being microscopic, the composite parts are generally made from “ready to shape” structural elements.

I.3.1 Woven composites

Many composite parts are made using woven fabrics or composites. In these structures, the fibers are braided or aligned into “cables” called strands or simply wires, each with a few hundred or thousands of fibers. As shown in Figure 8. The fabrics in Figure 9 are balanced, that is, they have the same number of threads in both weaving directions. They therefore have the same resistance and the same rigidity in these two directions, but it should be noted that they do not have an isotropic behavior: they a priori resist better in traction in the direction of the wires than in traction in the direction of the wires. 45° or in shear

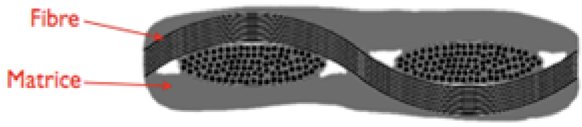
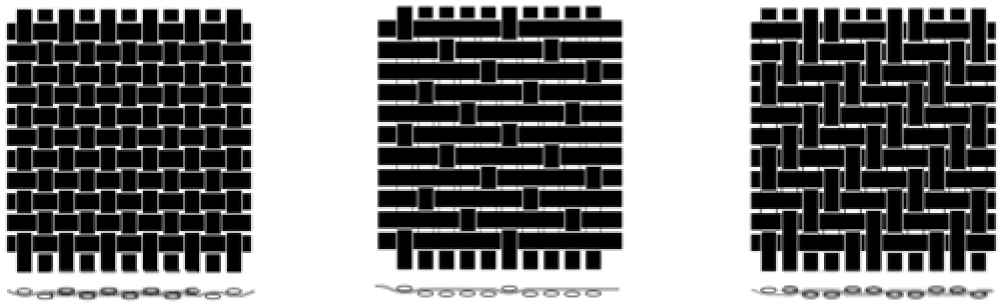


Figure (I.6) : Structure of a woven composite: fibers (grouped into threads) and matrix (intra- and inter-threads). Image by Martin Genet.



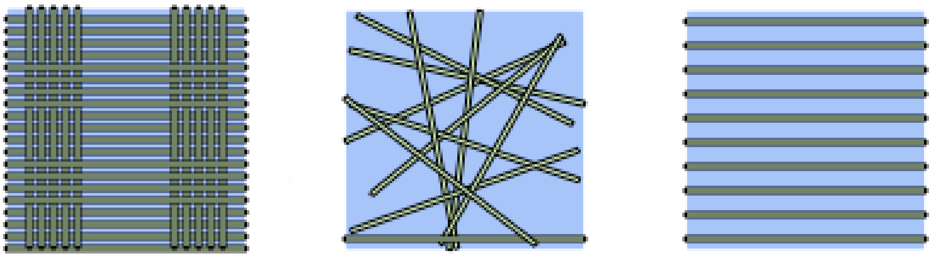
(a) (b)(c)

Figure (I.7): Examples of common planar fabrics. Image [2].

I.3.2 Laminated composites

Lamination is another structure commonly encountered in composite parts. In a laminate, the fibers are arranged in thin layers (a few tenths of a millimeter) called **plies** , which are stacked on top of each other; after shaping, these layers will be linked together via the matrix. Within a ply, the reinforcements can have any type of arrangement, as long as it is flat:

- Consider one of the flat fabrics described above (figure I.8 a), which gives a more or less anisotropic behavior depending on the proportion of threads in the two directions;
- Consider a mat : the fibers are arranged "in bulk" without preferred orientation on a few layers (figure I.8 b), which gives quasi-isotropic behavior in the plane;
- Consider a unidirectional fabric : the fibers are arranged parallel to each other on a few layers (figure I.8 c) and only held in this arrangement by a few weft threads, which gives a strongly anisotropic behavior.



(b) (b) (vs)

Figure (I.8): Possible arrangements of fibers in a ply: (a) flat fabric (here unbalanced), (b) mat, (c) unidirectional.

The orientation of the plies (figure I.9), it is possible to finely adapt the mechanical properties of the laminate to external stresses, and therefore to achieve a high level of optimization by putting the material where it is most useful. . At the scale of the structure, the mechanical behaviors thus obtained can be very complex, and range from quasi-isotropic to marked anisotropy when the application requires it (see resource: Modeling the behavior of composites: laminated beams).

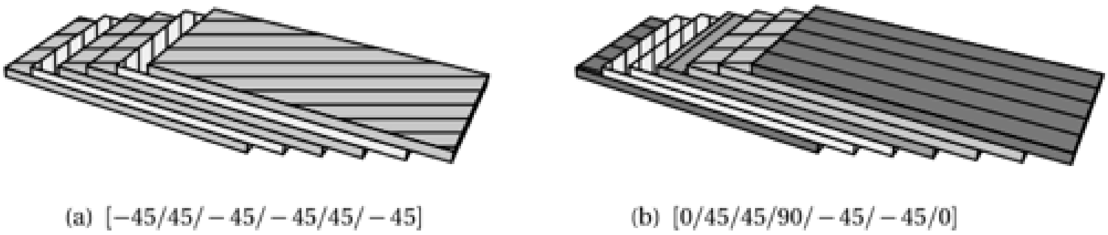


Figure (I.9): Examples of laminates based on unidirectional plies. The numbers in brackets designate the angle of each fold (in degrees) relative to a reference direction.

I.3.3 Sandwich structures:

To design parts that are rigid in bending and torsion, a commonly used technique is that of sandwich structures, made up of two skins (generally laminated) glued to a thick but light core, such as a polymer foam or a honeycomb, using adhesives (figure I.10).

Figure (I.10) : A sandwich structure (NASA image) and its constituents. Image [2]

I. 4. THE IMPLEMENTATION OF COMPOSITE MATERIALS: [3] [7]

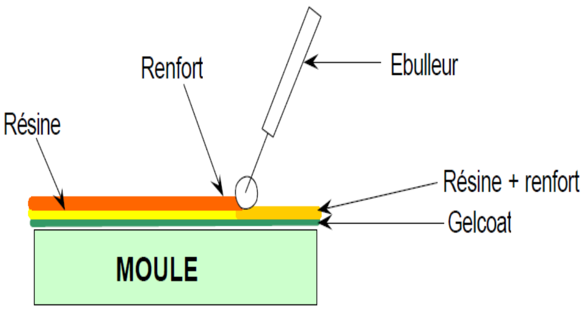
Generally, the implementation of composite materials is done according to certain technologies and parameters, namely:

- ✓ **So-called open mold technologies**
- ✓ **Technologies for large series**

I.4.1. So-called open mold technologies:

I.4.1.1 Contact molding

- **Principle** Manual process for producing parts from thermosetting resins, at room temperature and without pressure.
- **Raw materials** - Reinforcements: mats, fiberglass, carbon or aramid fabrics (weight reinforcement rate of 25 to 45% in the case of glass)
  - Resins: polyesters, epoxy, phenolics (resols),
  -Miscellaneous: catalyst, accelerator, fillers, pigments, release agent
- **Material** - Mould: simple shell generally in composite, possibly in several assembled elements
  - Hand tools: scissors, brushes, bubblers, slicers, gun
- **Application areas** - Boating, swimming pool
  - Chemical engineering
  - Transport, bodywork (small series)
  - Building, public works (formwork)
- **Development procedure**



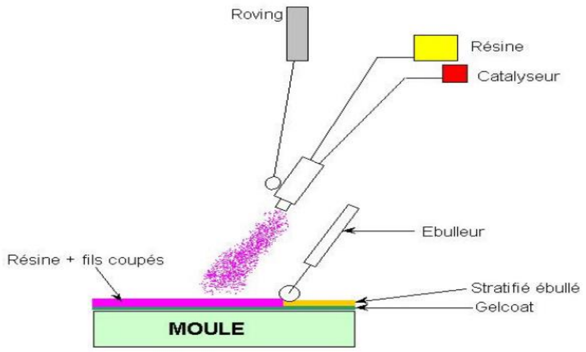
I.4.1.2. Simultaneous projection molding

- **Principle** Manual or robotic process allowing the production of parts from thermosetting resins at room temperature and without pressure. The raw materials are used using a so-called “projection” machine.
- **Raw materials** - Reinforcement: fiberglass in the form of assembled roving, reinforcement rate of 25 to 34% by weight

- Resins: mainly polyesters but also phenolic or hybrids

- Miscellaneous: catalysts, accelerator, pigments, fillers, release agent, solvent.

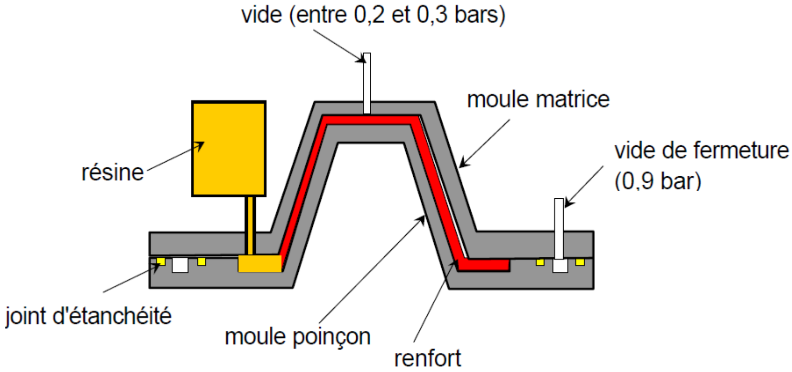
- **Material**
  - Composite molds (single shell)
  - Projection machine (internal or external mixing)
  - Ventilation device, extraction of styrene vapors
  - Small hand lamination tools
- **Application areas**
  - Boat production
  - Coatings
  - Buildings: facade, sanitary items
  - Public works: formwork
  - Industrial cowling
  - Sandwich panels for insulated trucks
- **Development procedure**



I.4.1.3. Vacuum molding

- **Principle** Vacuum molding is carried out between mold and rigid, semi-rigid or flexible counter-mold depending on the characteristics of the parts. The reinforcement (mat, fabric, preform) is placed inside the mold; the catalyzed resin is poured onto the reinforcement. We use the pressure exerted on the mold during vacuuming to distribute the resin and impregnate the reinforcement.
- **Raw materials**
  - Reinforcements: cut wire or continuous wire mats, preforms, fabrics (reinforcement rate: 20 – 30% by weight for mats and % for fabrics)
  - Resins: polyester, vinyl ester , phenolic, epoxy
  - Miscellaneous: catalyst, accelerator, pigments, fillers, release agent, solvent
- **Material**
  - Molds and counter-molds in composites or flexible films
  - Vacuum pumps with buffer tank

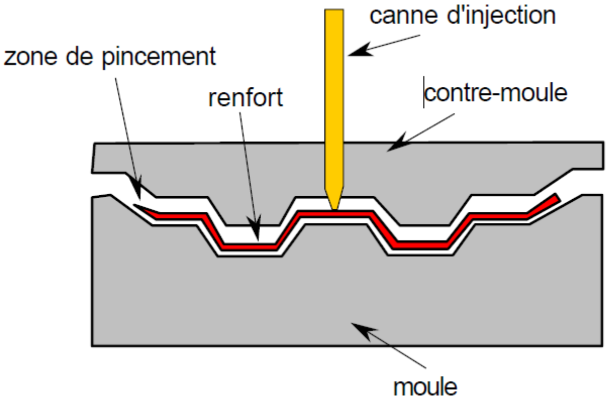
- **Application areas**
  - Building: overhead lighting domes
  - Transport: sandwich panels for insulated boxes , containers
  - Miscellaneous parts: enveloping protective helmets, cowlings, etc.
- **Development procedure**



I.4.1.4. Low pressure resin injection molding - RTM

- **Principle**
  Liquid resin injection molding RTM (Resin Transfer Molding) is carried out between rigid mold and counter-mold. The reinforcement (mats, preform, possibly fabrics) is placed in the air gap of the mold. Once it is firmly closed, the resin, accelerated and catalyzed, is injected under low pressure (1.5 to 4 bars) through the reinforcement until the impression is completely filled. After the resin has hardened, the mold is opened and the part is unmolded.
- **Material**
  - Rigid and resistant composite mold and counter-mold with quick closing system. Metal-composite and metallic variant
  - Resin injection equipment: pressure pot or metering pump system
  - Mold handling equipment (opening/closing)
- **Application areas**
  - Bodywork elements for passenger or utility vehicles
  - Small sanitary items
  - Small and medium sized tanks, covers
  - Various industrial parts
  - Bicycle forks, tennis rackets

- **Development procedure**



I.4.1.5. Low pressure "wet process" cold press molding

- **Principle** Molding using a compression press between rigid composite mold and counter-mold, initially without external heat input. Open mold, the reinforcement (mat) is placed on the lower part of the mold and the resin, equipped with a very reactive catalytic system, is poured in bulk onto the reinforcement.

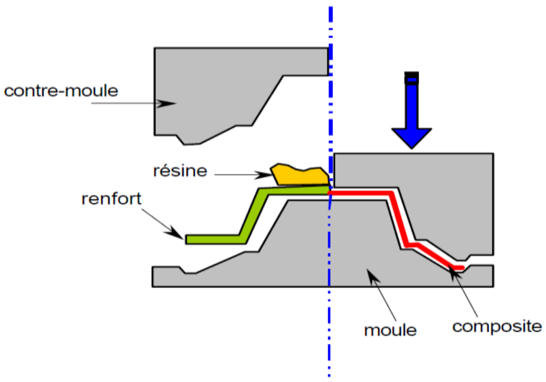
  Closing the mold under pressure (2 to 4 bars) causes the distribution of the resin in the cavity and the impregnation of the reinforcement. The hardening of the resin is gradually accelerated by the rise in temperature of the mold due to the exotherm of the reaction, which allows rapid demoulding.
- **Raw materials** - Reinforcement: continuous thread mat, low solubility binder

  - Resin: polyester

  - Miscellaneous: Catalysts, accelerators, fillers, pigments, release agents
- **Material** - Low pressure compression press (30 useful t/m²) with adjustable closing speeds

  - Mold and counter-mold: composite and resin concrete boxes
- **Application areas** - Various cowlings

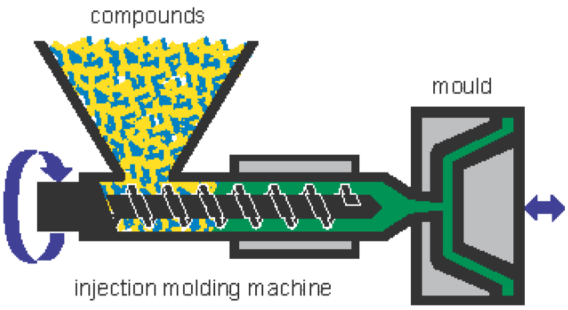
  - Handling bins
- **Development procedure**



I. 4.2. Technologies for large series:

I.4.2.1. Compound injection molding - BMC

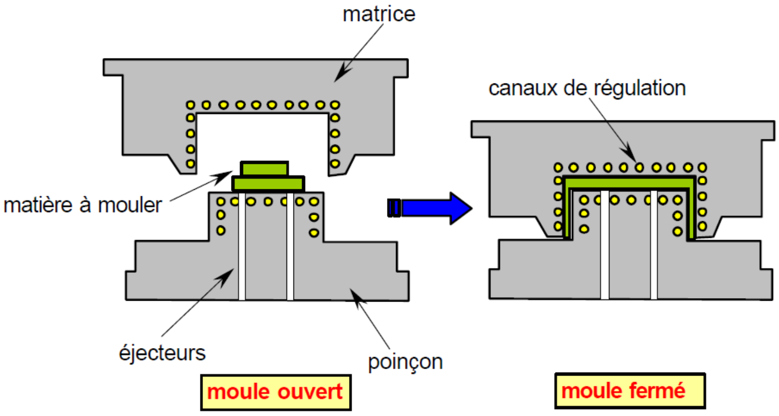
- **Principle** The BMC compound (Bulk Molding Compound) prepared in a mixer is a molding mass made up of resin, fillers and various adjuvants, reinforced by cut glass threads. The compound is hot molded (130 - 150 °C) by injection (mainly) between mold and machined steel counter-mold. The closing pressure (50 to 100 bars) of the mold causes the previously measured material to flow and the cavity to fill.
- **Raw materials** Commercial compound or compound prepared in-house:
  - Polyester resin (mainly), hot catalyst, release agent, fillers, pigments, possibly anti-shrinkage and curing agents
  - Reinforcement: cut glass wires (6 to 25 mm) at a rate of 10 to 28%
- **Material** - Preparation: weighing means, mixer for preparing the compound
  thermoregulated chrome-plated machined steel mold , injection or compression press with adjustable closing speed
- **Application areas**
  - Parts for electrical equipment
  - Automotive parts under the hood
  - Various industrial parts
- **Development procedure**



I.4.2.2. Compression molding of prepreg mat - SMC

- **Principle** prepreg mat (Sheet Molding Compound) is made up of a sheet of cut or continuous threads, impregnated between films by mixing polyester resin, fillers and various specific adjuvants. After a certain time, the viscosity of the mixture becomes such that the product can be unfilmed and handled without sticking. Cut into blanks of determined mass and dimensions, the prepreg mat is molded hot (140 to 160°C) by compression between mold and machined steel counter-mold. The pressure (50 to 100 bars) causes the material to flow and the impression to fill. The very short hardening time (depending on the thickness) allows rapid demoulding.

- **Raw materials** - Commercial prepreg mat or compound prepared in-house:
  - Impregnation mixture: polyesters, shrinkage compensating agents, fillers, catalysts, inhibitors, curing agents, mold release agents, pigments
  - Reinforcement: specific glass wires in the form of roving (reinforcement rate 25 to 50% by weight)
- **Material** prepreg mat production line
  - High pressure compression press, with adjustable closing speeds
  Thermoregulated chrome steel molds
- **Application areas** - Automotive industry (tourism and utility): body parts under hoods, protective parts
  - Electrical industry: metering boxes, lighting strip
  - Various industrial parts
- **Development procedure**



I.4.2.3. Filament Winding Molding

- **Principle** Molding process limited to revolution shapes. Initially, production of envelopes of revolution requiring high mechanical performance by progressive winding on a mandrel, at a determined angle of glass wires impregnated with resin; unwinding after hardening of the resin. Subsequently, extension to less efficient structures by combining wound rovings with other types of reinforcement (cut wire, mat, fabric) applied in an appropriate manner.
- **Raw materials** - Reinforcements: specific winding rovings (possibly prepreg), cut roving ,

cut wire mats, uni/bidirectional fabrics, surface mats

- Resins: polyester, epoxy, vinyl ester , phenolics

- Miscellaneous: catalysts, accelerators

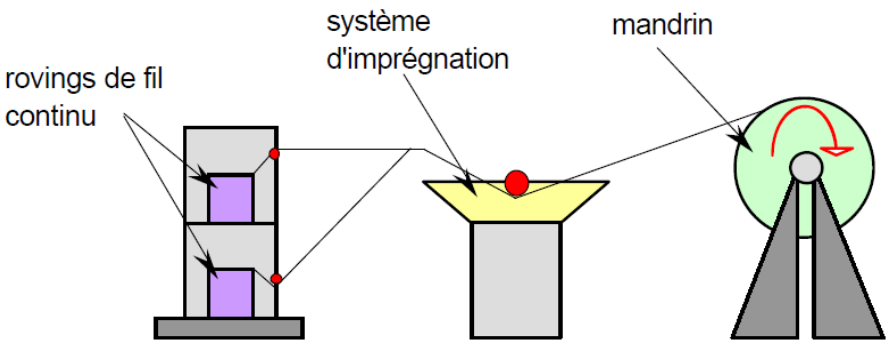
- **Material** - Winding machines, many types adapted to each application case

  - Chucks

- **Application areas** - Pipes that must withstand high pressures, transport tanks, storage tanks

  - Chemical engineering equipment
  - Electrical industry
  - Armament (rocket launcher tubes, etc.)
  - Automotive: suspension springs
  - Sport: pole, fishing rod, diving tank

- **Development procedure**



I.4.2.4. Centrifugal casting

- **Principle** Molding process limited to cylindrical envelopes. Inside a cylindrical mold rotating at low speed, wires cut from roving (or mat), catalyzed and accelerated resin and possibly granular fillers are deposited. Then, we increase the rotation speed of the mold to densify and debubble the material. After hardening of the resin, possibly accelerated by a thermal input, the part can be very easily extracted from the mold.

- **Raw materials** - Resins: polyesters, vinyl esters , possibly epoxy

  - Reinforcements: roving cut in situ (masts arranged stationary for small diameters)
  - Miscellaneous: catalytic systems, pigments, sand

- **Material** - Centrifuge machine, roving feeding/cutting device, resin, sand

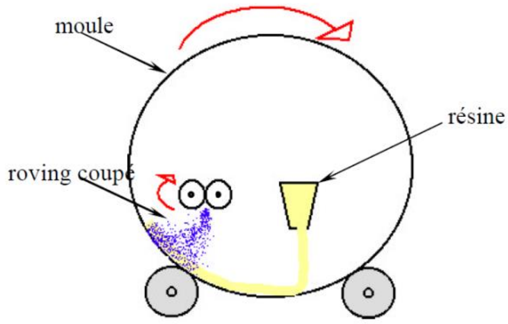
  - Mussels: diameter 0.1 to 0.5 meters

- **Application areas** - Pipes: up to 2 m in diameter

  - Tanks (diameter 1 to 2 m)
  - Silos (diameter 4 to 5 m, length 10 to 12 m)

- Wine press cages

- **Development procedure**



I.4.2.5. Pultrusion molding

- **Principle**
The process is intended for the continuous production of profiles with constant sections. Continuous reinforcements, various rovings , mats and fabrics in strips of appropriate widths, pulled by a traction bench located at the end of the production line, are successively prearranged in a precise manner, impregnated with resin and brought to the desired shape by passing through a heated die in which the resin hardens.

- **Raw materials**
- Reinforcements: direct rovings , textured yarns, continuous yarn mats (insoluble binder), fabrics
- Resins: polyester, epoxy, vinyl ester , acrylic, phenolic
- Miscellaneous: hardening systems, fillers, pigments, lubricants

- **Material**
- Pultrusion line: creels (reinforcement supply), guide elements, impregnation system, die with heating system, traction bench, cutting bench.

- **Application areas**
- Anchor bolts, cores of high voltage electrical insulators , fishing rods
- Structural tubes
- All electrical insulating or corrosion resistant profiles
- Luggage racks for buses

I.4.2.6 . Crushed Fiber Reinforced Reactive Resin Injection Molding (RRIM)

- **Principle**
This mainly concerns the molding of rigid polyurethane foams.
The reinforcement, crushed fiberglass, is incorporated into the polyol, at a rate of 10-20% by weight (on the final product). The molding process remains the same as for unreinforced foams: metered supply under pressure of each of the two components (polyol and isocyanate), mixing, injection

into a closed mold, reaction, hardening, demolding.

- **Raw materials** - Resins: polyols and isocyanate
  - Reinforcement: crushed fiberglass (0.1 to 0.3 mm)
- **Material** - Polyurethane injection machine
  - Mold / counter-mold: composite, light alloy, steel (depending on the series to be produced), mold closing system
- **Application areas** - Automobile industry
  - Wing extension
  - Interior interior design elements

I.4.2.7. Long fiber reinforced reactive resin injection molding (SRIM)

- **Principle** This is a molding between mold and counter-mold. The reinforcement in the form of mats or fabrics (20 to 60% by weight) is placed beforehand in the heated mold (100 - 150 °C). The highly reactive two-component resin system is injected under pressure (20 - 30 bars). After hardening (1 to 3 min), the part can be unmolded.
- **Raw materials** - Injection machine for two-component mixtures
  - Machined steel mold, polished, with heating system
  Press for opening/closing the mold under pressure
- **Material** - Polyurethane injection machine
  - Mold / counter-mold: composite, light alloy, steel (depending on the series to be produced),
  mold closing system
- **Application areas** - Automobile industry
  - Bumper beam
  - Crossmember, spar, chassis

CHAPTER II
CONSTITUTIVE RELATIONS OF ANISOTROPIC MATERIALS IN LINEAR ELASTICITY

I. CONSTITUTIVE RELATIONS OF ANISOTROPIC MATERIALS IN LINEAR ELASTICITY

II.1 Introduction

Stress-strain or constitutive relationships for anisotropic materials will be treated first, and in particular the behavior of orthotropic and transversely isotropic materials, the latter including unidirectional composites. Then, using tensor notation based on four indices and conventional notation with two indices, the expressions for changing the axes in terms of stresses, strains, stiffnesses and flexibility will be detailed.

II.2 The anisotropy of materials according to the properties of the C and S matrices

The reference state is a natural state without stress and deformation, that is to say:

$$\sigma = 0 \text{ And } \varepsilon = 0$$

In linear elasticity theory, the stress tensor σis given as a function of the strain tensor εby the tensor relation:

$$\sigma = E\varepsilon \qquad \text{II.1}$$

II.2.1. Linear elasticity:[2]

The linear elasticity relation can be written in the following matrix form:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \qquad \text{I I.2}$$

With : $\begin{matrix} \sigma_{ii} = \sigma_\alpha & \varepsilon_{ii} = \varepsilon_\alpha & \alpha = 1 \\ \sigma_{ij} = \sigma_\beta & \varepsilon_{ij} = \varepsilon_\beta & \beta = 9 - i - j \end{matrix}$ II.3

Or in condensed form:

$$\sigma = C\varepsilon \qquad \text{II.4}$$

This law, generally called generalized Hooke's law, introduces the symmetrical stiffness matrix [C].

The linear behavior of a material is therefore described in the general case by 21 independent coefficients. The elasticity relation can be written in the inverse form, as follows:

$$\varepsilon = S\sigma \quad \text{with} \quad S = C^{-1} \qquad \text{II.5}$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \quad \text{II.6}$$

S: Matrix of flexibility or suppleness (symmetric).

C: Rigidity matrix (symmetric).

C is the stiffness matrix; Sthe flexibility matrix.C And Sare symmetric matrices: there are therefore 21 rigidity constants C_{ij}or flexibility constants S_{ij}. This case corresponds to a material having no symmetry properties. Such a material is called **triclinic material** or anisotropic material.

II.2.2 monoclinic materials:

The monoclinic material studied at the (M) plane as plane of mirror symmetry.

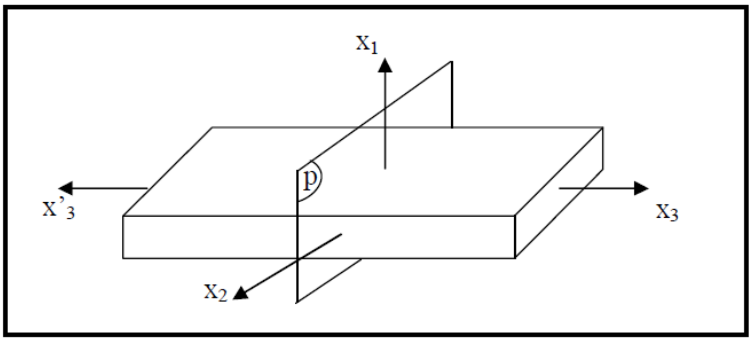


Fig. (II.1): Plane of symmetry

The shape of the rigidity (or flexibility) matrix must be such that a change of base carried out by symmetry with respect to this plane does not modify the matrix. In the case where the plane of symmetry is the plane (1,2), the exploitation of the base changes leads to a rigidity matrix of the form:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \quad \text{II. 7}$$

The flexibility matrix has the same shape. The number of independent elasticity constants is reduced to 13.

I I.2.3 orthotropic materials:

An orthotropic material has three planes of symmetry, perpendicular in pairs. It should be noted that the existence of two perpendicular planes of symmetry implies the existence of the third.

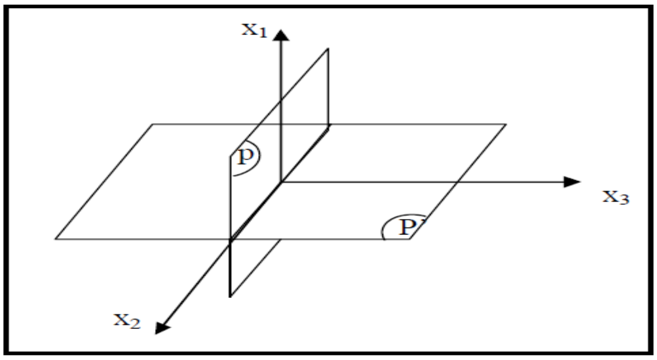


Fig. (II.2): Orthotropic material

The shape of the rigidity matrix is therefore obtained by adding to the monoclinic material a plane of symmetry perpendicular to the previous one. The invariance of the matrix in a change of base carried out by symmetry with respect to this second plane leads to a rigidity matrix of the form:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad \text{II.8}$$

The flexibility matrix has the same shape. The number of independent elasticity constants is reduced to 9.

II.2.4 transverse isotropic materials:

The material therefore behaves like an orthotropic material which also has an axis of revolution. The material is then called orthotropic material of revolution or transverse isotropic material. It follows that a change of base carried out by any rotation around this axis must leave the rigidity (or flexibility) matrix unchanged.

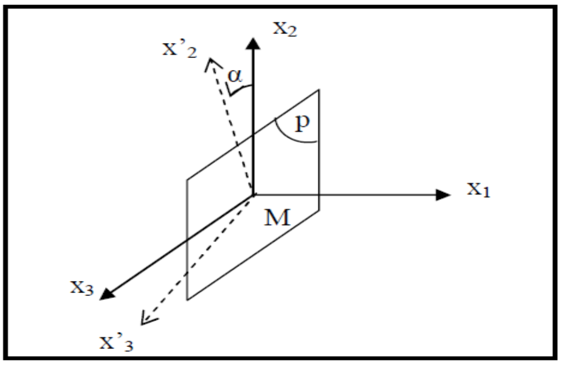


Fig. (II.3): Isotropy plane

The rigidity matrix is therefore written as follows:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}\,(C_{22} - C_{23}) & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad \text{II. 9}$$

The flexibility matrix has the same shape. The orthotropic to transverse isotropic material properties are determined by independent elastic constants.

II.2.5 isotropic materials:

A material is isotropic if its properties are independent of the choice of reference axes. There is then no preferred direction, and the rigidity (or flexibility) matrix must be invariant in any change of orthonormal bases.

The number of independent elasticity constants is therefore reduced to 2, and leads to the rigidity matrix:

$$C_{22} = C_{11};\ C_{23} = C_{12};\ C_{66} = \frac{1}{2}(C_{11} - C_{22})$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}\,(C_{11} - C_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}\,(C_{11} - C_{22}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\,(C_{11} - C_{22}) \end{bmatrix} \quad \text{II. 10}$$

The flexibility matrix has the same shape. Generally, stiffness constants are expressed by introducing the lamé coefficients **λ** and ***μ*:**

$$C_{12} = \lambda;\ \frac{1}{2}\,(C_{11} - C_{22}) = \mu \text{then: } C_{11} = \lambda + 2\mu$$

The constants of the flexibility matrices for monoclinic, orthotropic, transverse isotropic and isotropic materials are mentioned in Table II.1

Table II.1 engineering constants of flexibility matrices for materials.

Materials	Flexibility matrix
Monoclinic	$S = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & 0 & 0 & \frac{v_{61}}{G_{23}} \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & 0 & 0 & \frac{v_{62}}{G_{23}} \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & \frac{v_{63}}{G_{23}} \\ 0 & 0 & 0 & \frac{1}{G_{23}} & \frac{v_{54}}{G_{23}} & 0 \\ 0 & 0 & 0 & \frac{v_{45}}{G_{23}} & \frac{1}{G_{13}} & 0 \\ \frac{v_{16}}{E_1} & \frac{v_{26}}{E_2} & \frac{v_{36}}{E_3} & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$
Orthotropic	$S = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$
Transverse isotropic	$S = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{21}}{E_2} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_2} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+v_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix}$

Isotropic	$S = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix}$

Table II.2 engineering constants of flexibility matrices for materials.

Materials	Mechanical characteristics of materials	
	Independent	**Depend**
Monoclinic	E_1, E_2, E_3 G_{23}, G_{13}, G_{12} $\nu_{23}, \nu_{13}, \nu_{12}$ $\nu_{16}, \nu_{26}, \nu_{45}, \nu_{36}$	
Orthotropic	E_1, E_2, E_3 G_{23}, G_{13}, G_{12} $\nu_{23}, \nu_{13}, \nu_{12}$	
Transverse isotropic	E_1, E_2 G_{12} ν_{23}, ν_{12}	$E_3 = E_{2,}, G_{13} =, G_{12}$ $G_{23} = \frac{E_2}{2(1+\nu_{23})}$ $\nu_{13} = \nu_{12}$
Isotropic	$E_1 \quad (= E)$ $\nu_{12} \quad (= \nu)$	$E_3 = E_2 = E,\ \nu_{13} = \nu_{12} = \nu$ $G_{23} = G_{13} = G_{12} = \frac{E_2}{2(1+\nu_{23})}$

CHAPTER III
MICROMECHANICAL ANALYSIS OF ELASTIC BEHAVIOR OF COMPOSITE MATERIALS

III. MICROMECHANICAL ANALYSIS OF COMPOSITE MATERIALS:

III.1 Introduction:

III.1 Introduction:

The first step in a composite calculation is to determine the mechanical characteristics of the material as a function of those of its components (fiber , matrix) Fig III.1. In most cases, these calculations are reduced solely to calculating the Young's modulus. In our case study , the transverse isotropic material .

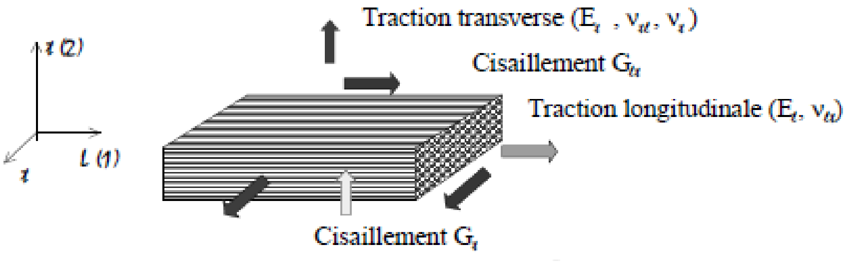


Fig. III.1 Mechanical properties of a unidirectional fold

Determination of unknown properties of the composite based on known properties of the fiber and matrix

Fig. III.1 Mechanical properties of a unidirectional ply

Determination of unknown composite properties based on known fiber and matrix properties

Fibre properties E_f, ν_f, G_f		**Matrix properties** E_m, ν_m, G_m

Micromechanical analysis

Mechanical properties of a unidirectional ply

Fig. III.2 Micromechanical analysis procedure for composites

III.2 Composite homogenization calculations:

orthotropy reference (ℓ , t) (Fig.III.3), made up of fibers embedded in a polymer matrix. Consider an elementary cell of volume fraction V = 1 made up of fibers and matrix with:

III.2 Composite homogenization calculations:

Consider a UD composite material with orthotropy (l ,t) (Fig. III.3), consisting of fibers embedded in a polymer matrix. Let be an elementary cell of volume fraction V = 1 made up of fibers and matrix with:

- **Volume fraction**

- V_f : Fiber volume fraction, with

$$V_f = \frac{v_f}{v_c} \quad \text{III. 1}$$

- V_m: Matrix volume fraction, with

$$V_m = \frac{v_m}{v_c} \quad \text{III. 2}$$

- V_c: Total volume fraction of composite, with

$$V_c = V_f + V_m = 1 \quad \text{III. 3}$$

- **Mass fraction**

- W_f : Mass fraction of fiber, with

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} \quad \text{I II. 4}$$

- W_m: Mass fraction of matrix, with

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m \quad \text{III. 5}$$

- W_c: Total mass fraction of composite, with

$$W_c = W_f + W_m = 1 \quad \text{III. 6}$$

- **Density**

- Composite density in terms of volume fraction **(mixing law**)

$$\rho_c = \rho_f V_f + \rho_m V_m \quad \text{III. 7}$$

- Composite density in terms of mass fraction

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m} \quad \text{III. 8}$$

- **Empty content**

- Experimental density of a composite

$$\rho_{ce} = \frac{w_c}{v_c} => v_c = \frac{w_c}{\rho_{ce}} \quad \text{III. 9}$$

- Theoretical density of a composite

$$\rho_{ct} = \frac{w_c}{v_c} => \; v_c = \frac{w_c}{\rho_{ct}} = v_f + v_m$$
III. 10

- Void volume

$$v_v = \frac{w_c}{\rho_{ce}}(\frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}})$$
III. 11

- Volume fraction of voids

$$V_v = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$
III. 12

The terminology used in micromechanical analysis of a unidirectional fold:

- E_f, E_mYoung's modulus of the fiber and matrix
- G_f, G_m- modulus of elasticity of the fiber and the matrix
- ν_f, ν_mPoisson's ratio of fiber and matrix
- V_f, V_m- Volume fraction of the fiber and the matrix

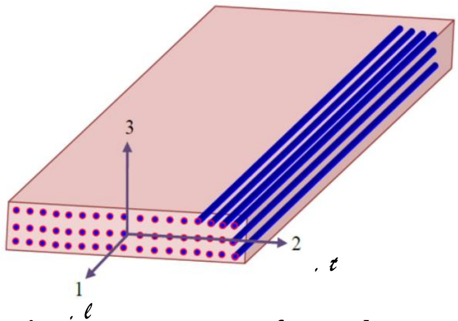


Fig. III.3 structure of a unidirectional fold

Purpose: To determine the relationships existing betweenE_l, E_t, ν_{lt}, G_{lt}, E_f, E_m, ν_m, ν_f, G_f, G_m

Hypotheses :

- We work in linear elasticity.

- We assume that the fiber/matrix connection is perfect.

- Locally, we have: $\sigma_f = E_f \varepsilon_f$, :$\sigma_m = E_m \varepsilon_m$

III.3 Determination of the elastic properties of an orthotropic fold of revolution

a) Longitudinal Young's modulus $E_l$orE_1

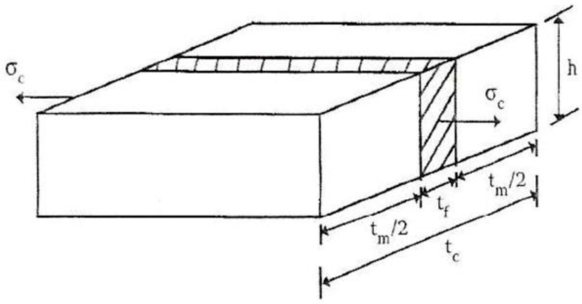


Fig.III.4 longitudinal stress applied to representative elementary volume VER

Hypothesis 1: the deformation is constant in a straight section, that is to say that:

$$\varepsilon_1 = \varepsilon_c = \varepsilon_f = \varepsilon_m \quad \text{III. 13}$$

The total force applied to the UD ply:

$$F_1 = F_f + F_m \quad \text{III. 14}$$

- F_f, F_mForces of the fiber and matrix respectively
- the definition of the constraint$\sigma_1 = F_1/S_1$

$$F_1 = \sigma_1 S_1 = \sigma_f S_f + \sigma_m S_m \quad \text{III. 15}$$

- S_f, S_mAre the cross-sectional areas of the fibers and the matrix, respectively.

Hook 's law :

$$\sigma_1 = E_1\varepsilon_1 \text{ with } \sigma_1 = F_1 S_1,$$
$$\sigma_f = E_f\varepsilon_f \text{ with } \sigma_f = F_f S_f,$$
$$\sigma_m = E_m\varepsilon_m \text{ with } \sigma_m = F_m S_m \quad \text{III. 16}$$

By substituting (III.16) into the relation (III.15) we obtain:

$$E_1\varepsilon_1 S_1 = E_f\varepsilon_f S_f + E_m\varepsilon_m S_m \quad \text{III. 17}$$

After simplification the longitudinal module E_1equal

$$E_1 = E_f V_f + E_m V_m \quad \text{III. 18}$$

b) Transverse Young's modulus $E_t$orE_2

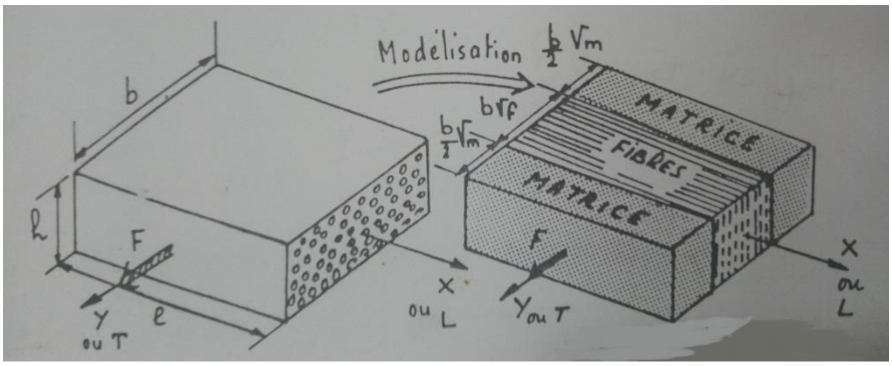


Fig.III.5 transverse stress applied to representative elementary volume VER

$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_t & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sigma_t = F/hl \qquad \text{III.19}$$

$$\sigma_t^c = E_t \, \varepsilon_t^c \qquad \text{III. 20}$$

Hypothesis 2: the constraint is constant in a straight section, that is to say that:

$$\sigma_t^c = \sigma_t^f = \sigma_t^m \text{III. 21}$$

σ_t and the same in the matrix and in the fiber:

$$\begin{aligned} \varepsilon_t^m &= \sigma_t^m / E_m \\ \varepsilon_t^f &= \sigma_t^f / E_f \end{aligned} \qquad \text{III.22}$$

If b and the composite width:

$$\varepsilon_t^c = \Delta b / b \qquad \text{III.23}$$

Δb : Total elongation of composite.

Δb^m, : Δb^f total matrix and fiber elongation

Likewise by definition of relative elongations:

$$\begin{aligned} \varepsilon_t^m &= \Delta b^m / b \\ \varepsilon_t^f &= \Delta b^f / b \end{aligned} \qquad \text{III.24}$$

On the other hand, the total elongation $\Delta b$of the composite is the sum of the elongations of the matrix and the fiber, that is to say:

$$\Delta b = \Delta b^m + \Delta b^f \qquad \text{III.25}$$

From where

$$\varepsilon_t^c = \varepsilon_t^m V_m + \varepsilon_t^f V_f$$

By substitution of (III.21) and (III.22) we obtain the equation (III.26):

$$\frac{1}{E_1} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

c) **Slip module**G_{lt}

We carry out the same modeling as in the previous paragraph

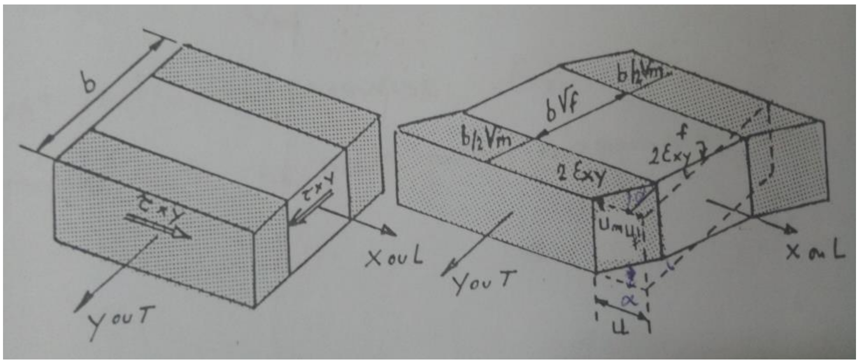


Fig.III.6 Pure shear applied to representative elementary volume VER

The loading is pure shear. Macroscopic point of view the stress field is worth:

$$[\sigma] = \begin{bmatrix} 0 & \sigma_{lt} & 0 \\ \sigma_{lt} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For an orthotropic material:

$$\sigma_{lt} = 2G_{lt}\varepsilon_{lt} \qquad \text{III.27}$$

If $u$is the displacement of the end of the section, by definition of the slip we have:

$$2\varepsilon_{lt} = u/b/2 \text{III.28}$$

Let ε_{lt}^{f}, $\varepsilon_{lt}^{m}$the slips in the matrix and in the fiber. $\sigma_{lt}$is the same in both components.

$$\sigma_{lt} = 2G_f\varepsilon_{lt}^{f} \,;\, \sigma_{lt} = 2G_m\varepsilon_{lt}^{m} \text{III.29}$$

G_f, : $G_m$sliding modules in the matrix and in the fiber.

According to figure III.6 we must have:

$$2\varepsilon_{lt} = u_f/\frac{b}{2}.V_f; 2\varepsilon_{lt}^{m} = u_m/\frac{b}{2}.V_m \text{III.30}$$

Since the total displacement $u$equal

$$u = u_f + u_m \qquad \text{III.31}$$

$$\varepsilon_{lt} = \varepsilon_{lt}^{f}V_f + \varepsilon_{lt}^{m}V_m \text{III.32}$$

And so :

$$\frac{1}{G_{lt}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

Noticed :

Generally $G_f \gg G_m$and the sliding module $G_{lt}$is directly connected to the sliding module of the die

$$G_{lt} \simeq \frac{G_m}{V_m}$$

Since the material is transverse isotropic $G_{lt} = G_{lz}$, this reasoning cannot be applied to determine G_{lz}.

d) Poisson coefficientν_{lt}

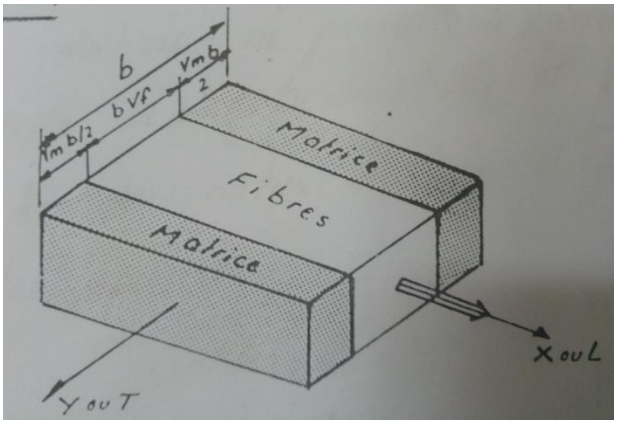


Fig.III.7 Simple traction on representative elementary volume VER

Still using the same modeling, consider a simple traction state along the axis (x, l):

ε_l : Relative elongation along the axis l, identical the relative elongations in the transverse direction will be (ε_t^f or ε_y^f) , (ε_t^m or ε_y^m) such that:

$$\varepsilon_t^m = -\nu_m \varepsilon_l$$
$$\varepsilon_t^f = -\nu_f \varepsilon_l \qquad \text{III.33}$$

overall Δb)thickness variation of the material is:

$$\Delta b = -\nu_m \varepsilon_l V_m b - \nu_f \varepsilon_l V_f b \qquad \text{III.34}$$

The Poisson's ratio $\nu_{lt}$of the material is equal to (III.35):

$$\nu_{lt} = -\frac{\varepsilon_t}{\varepsilon_l}$$

And since :$\varepsilon_t = \Delta b / b$

We obtain

$$\nu_{lt} = \nu_f V_f + \nu_m V_m \qquad \text{III.35}$$

Noticed :

The transverse isotrope leads to:$\nu_{lt} = \nu_{tl}$

We will obtain ν_{tl}($\nu_{tl} = \nu_{zl}$) using the symmetry property of the flexibility matrix.

III.4 Thermoelastic properties of a unidirectional composite

When the influence of temperature variations must be taken into account, the so-called Hooke's law of behavior and recalled below:

$$\varepsilon = \frac{1+\nu}{E}[\sigma] - \frac{\nu}{E} trace[\sigma] \mathrm{I}$$

Is replaced by the so-called Hooke-Duhamel law:

$$\varepsilon = \frac{1+\nu}{E}[\sigma] - \frac{\nu}{E} trace[\sigma] \mathrm{I} + \alpha \Delta T \qquad \text{III.36}$$

With notations: εstrain tensor

$[\sigma]$stress tensor

I unit tensor

ν, E elastic coefficients of the material considered

α thermal expansion coefficient

ΔT algebraic temperature deviation by contribution to the so-called neutral state for which stresses and deformation are zero.

a) Coefficient of longitudinal thermal expansion $\alpha_1$orα_l

The composite is not subjected to any external force but undergoes a temperature variation of:

$$\Delta T = T - T_0 \qquad \text{III.37}$$

In the model considered the longitudinal deformation is constant for the fiber and the ctd matrix:

$$\varepsilon_1^c = \varepsilon_1^f = \varepsilon_1^m \qquad \text{III.38}$$

Only the longitudinal stress can exert the difference between the thermal coefficients of the fiber $\alpha_f$and the matrix α_m.

The longitudinal expressions of the deformation for the fiber and the matrix equal:

$$\begin{aligned} \varepsilon_1^f &= \frac{\sigma_1^f}{E_f} + \alpha_f \Delta T \\ \varepsilon_1^m &= \frac{\sigma_1^m}{E_m} + \alpha_m \Delta T \end{aligned} \qquad \text{III.39}$$

The longitudinal expressions of the deformation for the fiber and the matrix equal:

$$\begin{aligned} \sigma_1^f &= E_f(\varepsilon_1 - \alpha_f \Delta T) \\ \sigma_1^m &= E_m(\varepsilon_1 - \alpha_m \Delta T) \end{aligned} \qquad \text{III.40}$$

The resulting force for longitudinal traction:

$$F_1 = \sigma_1 S_1 = \sigma_1^f S_f + \sigma_1^m S_m \qquad \text{III.41}$$

We obtain for a thermomechanical test

$$\sigma_1^f V_f + \sigma_1^m V_m = 0 \qquad \text{III.42}$$

By substituting equation (III.40) into (III.42) we obtain equation (III.43):

$$V_f E_f(\varepsilon_1 - \alpha_f \Delta T) + V_m E_m(\varepsilon_1 - \alpha_m \Delta T) = 0 \qquad \text{III.43}$$

So we have :

$$\left(V_m E_m + V_f E_f\right)\varepsilon_1 = \left(V_m E_m \alpha_m + V_f E_f \alpha_f\right)\Delta T$$

The longitudinal thermal expansion coefficient $\alpha_1$of the composite being defined by:

$$\alpha_1 = \frac{\varepsilon_1}{\Delta T}$$

We obtain the following expression (III.44):

$$\alpha_1 = \frac{V_m E_m \alpha_m + V_f E_f \alpha_f}{\left(V_m E_m + V_f E_f\right)}$$

b) Transverse thermal expansion coefficient $\alpha_2$orα_t

The transverse expressions of the fiber and the matrix are:

$$\begin{aligned} \varepsilon_2^f &= -\frac{\nu_f}{E_f}\sigma_1^f + \alpha_f \Delta T \\ \varepsilon_2^m &= -\frac{\nu_m}{E_m}\sigma_1^m + \alpha_m \Delta T \end{aligned} \qquad \text{III.45}$$

By replacing the longitudinal stresses with their values determined in the previous paragraph we obtain:

$$\begin{aligned} \varepsilon_2^f &= -\nu_f(\varepsilon_1 - \alpha_f \Delta T) + \alpha_f \Delta T \\ \varepsilon_2^m &= -\nu_m(\varepsilon_1 - \alpha_m \Delta T) + \alpha_m \Delta T \end{aligned} \qquad \text{III.46}$$

The variation of the splice for a transverse tensile test given by:

$$\Delta e = \Delta e_f + \Delta e_m = e_f \varepsilon_2^f + e_m \varepsilon_2^m$$

The expression for transverse deformation of the composite is:

$$\varepsilon_2 = \frac{\Delta e}{\mathrm{e}} = V_f \varepsilon_2^f + V_m \varepsilon_2^m \qquad \text{III.47}$$

By substituting equation (III.46) into (III.47) we obtain equation (III.48):

$$\varepsilon_2 = -\left(V_m \nu_m - V_f \nu_f\right)\varepsilon_1 + \left(V_m \nu_m \alpha_m + V_f \nu_f \alpha_f\right)\Delta T + \left(V_m \alpha_m + V_f \alpha_f\right)\Delta T$$

With

$$\varepsilon_1 = \frac{\left(V_m E_m \alpha_m + V_f E_f \alpha_f\right)}{\left(V_m E_m + V_f E_f\right)}\Delta T$$

We therefore have the expression (III.48):

$$\varepsilon_2 = \left[\frac{V_m V_f\left(\nu_m E_f - \nu_f E_m\right)\left(\alpha_m - \alpha_f\right)}{V_m E_m + V_f E_f} + V_m \alpha_m + V_f \alpha_f\right]\Delta T$$

The transverse thermal coefficient $\alpha_2$of composite given by:

$$\alpha_2 = \frac{\varepsilon_2}{\Delta T}$$

From equation (III.48) we obtain the expression (III.49) of transverse thermal coefficient:

$$\alpha_2 = V_f \alpha_f + V_m \alpha_m + \frac{\nu_m E_f - \nu_f E_m}{\frac{E_f}{V_m} + \frac{E_m}{V_f}}\left(\alpha_m - \alpha_f\right)$$

Conclusion

The combined effect of stresses and temperature variation leads to the following “thermodynamic” law of behavior of the unidirectional:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -v_{21}/E_2 & 0 \\ -v_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} + \Delta T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \text{III.49}$$

With the notation of the axes (***ℓ,t)*** by (1,2)

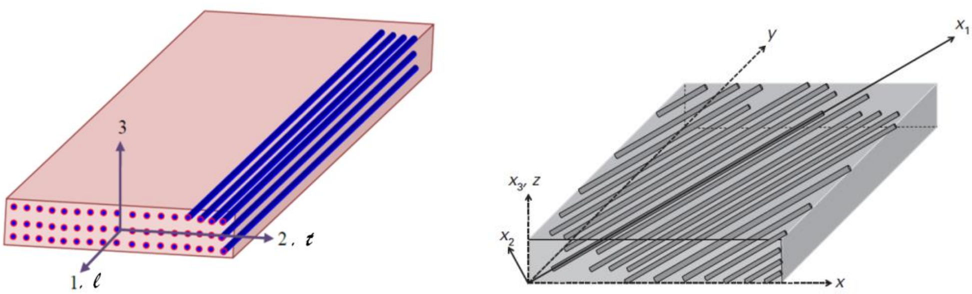
CHAPTER IV
ELASTIC CONSTANTS OF A UNIDIRECTIONAL COMPOSITE & OFF THE FIBER AXIS

IV.1 Introduction:

In the previous chapter we illustrated how to obtain the elastic properties of a unidirectional ply from the mechanical properties in all the components (Fiber, Matrix). In this chapter, we will refer the response of each layer (Ply) of material to the same overall system. To obtain the details of stresses and strains in different orientations in a single layer, the following type of analysis is required. We accomplish this by transforming the stress-strain relationships for the coordinate system of a unidirectional fold (1-2-3) into the global coordinate system (xyz). This transformation will be done for the plane stress state using the standard transformation relations for stresses and strains.

IV.2 law of behavior in the plane of a unidirectional fold:

Consider a composite material UD with orthotropic reference (ℓ , t) (Fig.VI.1 (a)), made up of fibers embedded in a polymer matrix.



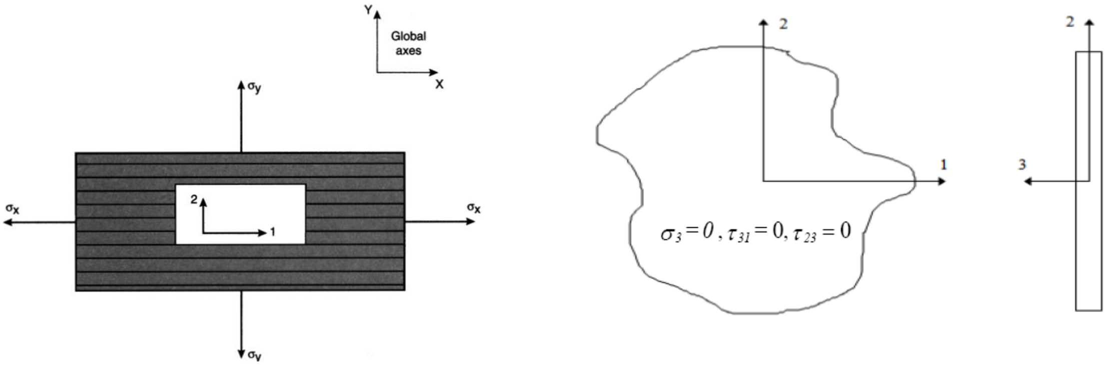
(has) **(b)**

Fig. IV.1 Structure of a unidirectional fold, (a) the local reference coordinates (ℓ ,t,z) in the preferred axis (b) the global reference coordinates in off-axis (x,y,z)

The flexibility matrix of an isotropic material of revolution is written as follows:

$$S = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{21}}{E_2} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_2} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+v_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix}$$

VI. 2.1 law of behavior in the plane of a unidirectional fold:



(has) **(b)**

Fig. IV.2 Structure of a unidirectional fold, (a) UD fold under a plane stress (***ℓ ,t,z***) in the preferred axis (b) thin UD fold splice $\sigma_3 = 0, \tau_{23} = 0, \tau_{31} = 0$

The flexibility matrix of a unidirectional fold in the plane written:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \text{IV.1}$$

The stiffness matrix of a unidirectional fold in the plane written:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & \varrho_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \text{IV.2}$$

With

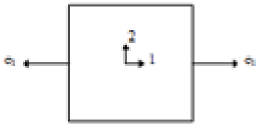
$$C_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2}$$
$$C_{12} = -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2}$$
$$C_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2}$$
$$C_{66} = -\frac{1}{S_{66}}$$

VI.2.2 Determination of the elastic constants of a unidirectional composite

(a) Longitudinal traction in the privilege axis *ℓ*

we apply a constraint in axis 1 then we$\sigma_1 \neq 0, \sigma_2 = 0, \tau_{12} \neq 0$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} \varepsilon_1 = S_{11}\sigma_1 \\ \varepsilon_2 = S_{12}\sigma_1 \\ \gamma_{12} = 0 \end{matrix}$$



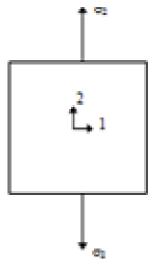
$$E_1 = \frac{\sigma_1}{\varepsilon_1} = \frac{1}{S_{11}} \rightarrow S_{11} = \frac{1}{E_1}$$

$$E_1 = -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{S_{12}}{S_{11}} \rightarrow S_{12} = -\frac{v_{21}}{E_1}$$

(b) *t* axis

we apply a constraint in the t axis then we $\sigma_1 = 0, \sigma_2 \neq 0, \tau_{12} = 0$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} 0 \\ \sigma_2 \\ 0 \end{bmatrix} \to \begin{matrix} \varepsilon_1 = S_{11}\sigma_2 \\ \varepsilon_2 = S_{12}\sigma_2 \\ \gamma_{12} = 0 \end{matrix}$$



$$E_2 = \frac{\sigma_2}{\varepsilon_2} = \frac{1}{S_{22}} \to S_{22} = \frac{1}{E_2}$$

$$v_{12} = -\frac{\varepsilon_1}{\varepsilon_2} = -\frac{S_{12}}{S_{22}} \to S_{12} = -\frac{v_{12}}{E_2}$$

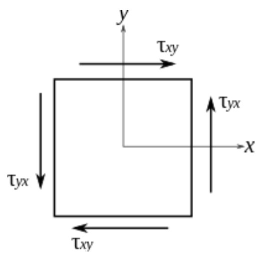
Such that the following relationship

$$S_{12} = -\frac{v_{12}}{E_2} \qquad\qquad S_{12} = -\frac{v_{12}}{E_1}$$

$$\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}$$

(c) Pure shear in plane of 1-2

We apply a pure Shear in plane then we $\sigma_1 = 0, \sigma_2 = 0, \tau_{12} \neq 0$



$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \tau_{12} \end{bmatrix} \to \begin{matrix} \varepsilon_1 = 0 \\ \varepsilon_2 = 0 \\ \gamma_{12} = S_{66}\tau_{12} \end{matrix}$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{1}{S_{66}} \to S_{66} = \frac{1}{G_{12}}$$

- **Flexibility matrix**

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

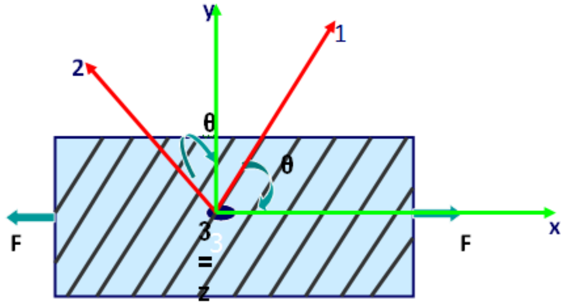
- **Stiffness matrix coefficients**

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1 - v_{21}v_{12}} & \frac{v_{12}E_2}{1 - v_{21}v_{12}} & 0 \\ \frac{v_{12}E_2}{1 - v_{21}v_{12}} & \frac{E_1}{1 - v_{21}v_{12}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

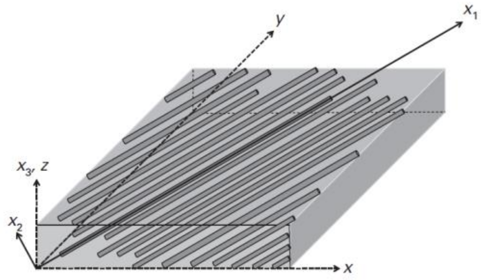
IV. 3 Elastic constants of a fold in any direction

The preceding analysis is a preparation for the more interesting and practical situation where the applied loading axis does not coincide with the fiber axis. This is illustrated in Fig.IV.3 .

Hypothesis: very thin fold $\sigma_{33} = \sigma_{zz} = 0$(flat stress state)



(has)



(b)

Fig.IV.3 Fold structure in any direction, (a) fold coordinates in any direction (b) global reference coordinates in off-axes (x ,y, z)

According to equation (IV.36) (flexibility matrix in the transverse isotropic orthotropic case) the behavior of the material is described, in the orthotropy axes as follows:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -v_{21}/E_2 & 0 \\ -v_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

The transformation matrix (direction cosine matrix) for an angle of rotation of an axis (3 ,z) is described as follows:

$$[T_\sigma] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix}$$

Or in abbreviated form

$$[T_\sigma] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$

With c= cos, s= sin

The relationship between the local frame (ℓ ,t,z) and the global frame (x,y,z) for a constraint transformation state is as follows:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T_\sigma] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T_\sigma]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

In abbreviated form, this matrix equation can be written:

$$\{\sigma\}_{12} = [T_\sigma]\{\sigma\}_{xy} \qquad \text{II.4}$$

$$\{\sigma\}_{xy} = [T_\sigma]^{-1}\{\sigma\}_{12} \qquad \text{II.4}$$

Likewise for a deformation transformation state:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{bmatrix} \rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [T_\varepsilon] \begin{bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy}/2 \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} \rightarrow \begin{bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy}/2 \end{bmatrix} = [T_\sigma]^{-1} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix}$$

In abbreviated form, this matrix equation can be written:

$$\{\varepsilon\}_{12} = [T_\varepsilon]\{\varepsilon\}_{xy} \qquad \text{II.4}$$

$$\{\varepsilon\}_{xy} = [T_\sigma]^{-1}\{\varepsilon\}_{12} \qquad \text{II.4}$$

Now, in order to determine the stiffness matrix in global directions (x, y) for a ply in which the fibers are aligned at an angle $\theta$to the global direction x, it is necessary to go through the following three steps:

1- Determine the strains in the local directions (1,2) by transforming the applied strains across the angle $\theta$from the global directions (x, y).
2- Calculate the stresses in the local directions using the on-axis stiffness matrix $[C]$derived previously.
3- Transform constraints into global directions through an angle of – θ.

Using matrix notation, these three steps are performed as follows:

Step 1 :

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix}$$

Note the change to $[T_\sigma]$to give the deformation transformation matrix $[T_\varepsilon]$, so that we can use γ_{12}instead of $\frac{1}{2}\gamma_{12}$.

2nd step :

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Step 3:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

Connect these steps together to obtain the global stresses in terms of the global strains:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}c^2 & s^2 & -2sc\\ s^2 & c^2 & 2sc\\ sc & -sc & c^2-s^2\end{bmatrix}\begin{bmatrix}C_{11} & C_{12} & 0\\ C_{21} & C_{22} & 0\\ 0 & 0 & \varrho_{66}\end{bmatrix}\begin{bmatrix}c^2 & s^2 & sc\\ s^2 & c^2 & -sc\\ -sc & sc & c^2-s^2\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_x\\ \gamma_{xy}\end{Bmatrix}$$

Which provides an overall stiffness matrix$[\overline{C}\,]$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}}\\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}}\\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_x\\ \gamma_{xy}\end{Bmatrix}$$

In abbreviated form, this matrix equation can be written:

$$\{\sigma\}_{xy}=[\overline{C}\,]\{\varepsilon\}_{xy}$$

With :

$$\overline{C_{11}}=C_{11}c^4+C_{22}s^4+2(C_{12}+2C_{66})s^2c^2$$

$$\overline{C_{12}}=(C_{11}+C_{22}-4C_{66})s^2c^2+C_{12}(s^4+c^4)$$

$$\overline{C_{16}}=(C_{11}-C_{12}-2C_{66})sc^3-2(C_{11}-C_{12}-2c_{66})cs^3$$

$$\overline{C_{22}}=C_{11}s^4+C_{22}c^4+2(C_{12}+2C_{66})s^2c^2$$

$$\overline{C_{26}}=(C_{11}-C_{12}-2C_{66})cs^3-2(C_{11}-C_{12}-2C_{66})sc^3$$

$$\overline{C_{66}}=(C_{11}-C_{22}-2C_{12}-2C_{66})s^2c^2+C_{66}(s^4+c^4)$$

Moreover :

$$C_{11}=\frac{E_1}{1-v_{21}v_{12}}$$

$$C_{12}=\frac{v_{12}E_2}{1-v_{21}v_{12}}=\frac{v_{12}E_1}{1-v_{21}v_{12}}=C_{21}$$

$$C_{22}=\frac{E_2}{1-v_{21}v_{12}}$$

$$C_{66}=C_{12}$$

$$C_{16}=C_{26}=C_{61}=C_{62}=0$$

By a similar analysis, it can be shown that, for applied stresses (rather than applied strains) in the global directions, the global flexibility matrix $\overline{[S]}$has the form:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix} = [T]^T[S][T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} \overline{S_{11}} & \overline{S_{12}} & \overline{S_{16}} \\ \overline{S_{12}} & \overline{S_{22}} & \overline{S_{26}} \\ \overline{S_{16}} & \overline{S_{26}} & \overline{S_{66}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{S_{11}} & \overline{S_{12}} & \overline{S_{16}} \\ \overline{S_{12}} & \overline{S_{22}} & \overline{S_{26}} \\ \overline{S_{16}} & \overline{S_{26}} & \overline{S_{66}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

In abbreviated form, this matrix equation can be written:

$$\{\varepsilon\}_{xy} = [\overline{S}]\{\sigma\}_{xy}$$

Or :

$\overline{S_{11}} = S_{11}c^4 + S_{22}s^4 + (2S_{12} + S_{66})s^2c^2$

$\overline{S_{12}} = (S_{11} + S_{22} - S_{66})s^2c^2 + S_{12}(s^4 + c^4)$

$\overline{S_{16}} = (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})cs^3$

$\overline{S_{22}} = S_{11}s^4 + S_{22}c^4 + (2S_{12} + S_{66})s^2c^2$

$\overline{S_{26}} = (2S_{11} - 2S_{12} - S_{66})cs^3 - (2S_{22} - 2S_{12} - S_{66})sc^3$

$\overline{S_{66}} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4)$

With

$S_{11} = \dfrac{1}{E_1}$

$S_{12} = -\dfrac{v_{21}}{E_2} = -\dfrac{v_{22}}{E_1} = S_{21}$

$S_{22} = \dfrac{1}{E_2}$

$S_{66} = \dfrac{1}{G_{12}}$

$S_{16} = S_{26} = S_{61} = S_{62} = 0$

VI.3.1 Determination of the elastic constants of a fold in any direction

- **Flexibility matrix**

The new constitutive law takes the following technical form

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{v_{yx}}{E_y} & \frac{\eta_{xy}}{G_{xy}} \\ -\frac{v_{xy}}{E_x} & \frac{1}{E_y} & \frac{\mu_{xy}}{G_{xy}} \\ \frac{\eta_x}{E_x} & \frac{\mu_y}{E_y} & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

With :

$$E_x(\theta) = \frac{1}{\overline{\overline{S_{11}}}} = \frac{1}{\frac{c^4}{E_1} + \frac{s^4}{E_2} + (-2\frac{v_{12}}{E_2} + \frac{1}{G_{12}})s^2c^2}$$

$$E_y(\theta) = \frac{1}{\overline{\overline{S_{22}}}} = \frac{1}{\frac{s^4}{E_1} + \frac{c^4}{E_2} + (-2\frac{v_{12}}{E_2} + \frac{1}{G_{12}})s^2c^2}$$

$$G_{xy}(\theta) = \frac{1}{\overline{\overline{S_{66}}}} = \frac{1}{4s^2c^2\left(\frac{1}{E_1} + \frac{1}{E_2} + 2\frac{v_{12}}{E_2}\right) + \frac{(c^2 - s^2)^2}{G_{12}}}$$

$$\frac{v_{yx}}{E_y}(\theta) = \frac{v_{21}}{E_2}(c^4 + s^4) - s^2c^2\left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)$$

$$\frac{\eta_{xy}}{G_{xy}}(\theta) = -2cs\left\{\frac{c^2}{E_1} + \frac{s^2}{E_2} + (c^2 - s^2)\left(\frac{v_{21}}{E_2} - \frac{1}{2G_{12}}\right)\right\}$$

$$\frac{\mu_{xy}}{G_{xy}}(\theta) = -2cs\left\{\frac{s^2}{E_1} + \frac{c^2}{E_2} - (c^2 - s^2)\left(\frac{v_{21}}{E_2} - \frac{1}{2G_{12}}\right)\right\}$$

- **Stiffness matrix**

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix}$$

With :

$$\bar{E}_{11}(\theta) = \bar{E}_1c^4 + \bar{E}_2s^4 + 2s^2c^2(v_{21}\bar{E}_1 + 2G_{12})$$

$$\bar{E}_{22}(\theta) = \bar{E}_2c^4 + \bar{E}_1s^4 + 2s^2c^2(v_{21}\bar{E}_1 + 2G_{12})$$

$$\bar{E}_{66}(\theta) = s^2c^2(\bar{E}_1 + \bar{E}_2 - v_{21}\bar{E}_1) + G_{12}(s^4 + c^4)$$

$$\bar{E}_{12}(\theta) = s^2c^2(\bar{E}_1 + \bar{E}_2 - 4G_{12}) + (s^4 + c^4)v_{21}\bar{E}_1$$

$$\bar{E}_{16}(\theta) = -cs\{c^2\bar{E}_1 - s^2\bar{E}_2 - (s^2 - c^2)(v_{21}\bar{E}_1 + 2G_{12})\}$$

$$\bar{E}_{26}(\theta) = -cs\{s^2\bar{E}_1 - c^2\bar{E}_2 + (s^2 - c^2)(v_{21}\bar{E}_1 + 2G_{12})\}$$

Expressions in which:

$$\bar{E}_1 = E_1/1 - v_{21}v_{12} \ ;\bar{E}_2 = E_2/1 - v_{21}v_{12}$$

VI.4 thermomechanical behavior of the unidirectional for any direction of the fibers

We seek to express the global equation (III.49) of the president chapter in the axes (x ,y) other than those of orthotropy .

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -v_{21}/E_2 & 0 \\ -v_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} + \Delta T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \quad \text{III.49}$$

We have :

$$\{\sigma\}_{12} = [T_\sigma]\{\sigma\}_{xy} \quad \text{IV.4}$$

$$\{\varepsilon\}_{xy} = [T_\sigma]^{-1}\{\varepsilon\}_{12} \quad \text{IV.4}$$

By substitution in equation (III.49) we obtain:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix} = [T]^T \begin{bmatrix} 1/E_1 & -v_{21}/E_2 & 0 \\ -v_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} + \Delta T [T]^T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}$$

$$[T]^T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} c^2\alpha_1 + s^2\alpha_2 \\ s^2\alpha_1 + c^2\alpha_2 \\ 2sc\,(\alpha_2 - \alpha_1) \end{Bmatrix} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}$$

We finally obtain:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1/E_x & -v_{yx}/E_y & \eta_{xy}/G_{xy} \\ -v_{xy}/E_x & 1/E_y & \mu_{xy}/G_{xy} \\ \eta_x/E_x & \mu_y/E_y & 1/G_{xy} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \quad \text{IV.4}$$

With :

$$\begin{cases} \alpha_x = c^2\alpha_1 + s^2\alpha_2 \\ \alpha_y = s^2\alpha_1 + c^2\alpha_2 \\ \alpha_{xy} = 2sc\,(\alpha_2 - \alpha_1) \end{cases} \quad \text{IV.4}$$

CHAPTRE V
MECHANICAL BEHAVIOUR OF LAMINATES

V.1 Introduction:

A laminated material is stacking of unidirectional folds with different orientations of the fibers. The laminate pet be designed by isotropic plates of different materials. In the present context the laminate is made by layers of fibrous materials.

The main purpose of this design and to achieve a structure with rigidity that can withstand a given load. Indeed, the main directions of the layers of the laminate can be oriented differently; the rigidity of the material thus obtained increases considerably. However, if the architecture does not check a symmetry by contribution to the median plane we will have a coupling between extension and distortion. It is noted that in the present study the phenomenon of inter laminar slip is ignored. (Fig.1)

V.2 strain relationship in the case of a laminate:

The stress relationship deformation in the main directions of an orthotropic material is written in matrix form as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} == \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \qquad \text{V.1}$$

In a plane (x, y) other than that of orthotropy, we will have:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad \text{V.2}$$

$\overline{C_{ij}}$ The equations of Chapter IVC_{ij} are determined according to the constants.

For a laminate, equation (V.2) will be relative to $K^{i\,éme}$ layers ($K^{i\,éme}$ fold) and it is written:

$$\{\sigma\}_k = [\bar{C}\,]_k \{\varepsilon\}_k \qquad \text{V.3}$$

The aim is to study the variation of stresses and deformations along the thickness of the laminate. The resultant forces and resultant moments will be determined by integrating the equations (V.3) through the thickness.

The knowledge of stress and deformation variation and essential for the study of the stresses of extension and bending of laminated plates.

The displacements are continuous of port and other of the fold and there is no relative sliding, the laminate behaves as a single plate with variable material properties. After deformation (bending) the cross sections remain normal at the mean plane and under these conditions the shear deformations in planes perpendicular to the mean plane are ignored ($\gamma_{xz} = \gamma_{yz} = 0$) in addition, normal directions will also be ignored ($\varepsilon_z = 0$)Kirchoff assumptions for plates and

Kirchoff-Love assumptions for shells are used to deduce displacements and therefore deformations in the laminate.

V.2.1 Expression of deformations as a function of displacements:

The point displacementB (Fig.2) after deformation isu .

The lineABCD remains normal at the neutral axis:

$$\begin{cases} u_c = u_0 - z_c\beta \\ \beta = \frac{\partial w_0}{\partial x} \end{cases} \qquad \text{V.4}$$

β: Slope of the laminate section in the direction"x".

The displacementu at any pointz of the thickness of the laminate is:

$$u = u_0 - z\frac{\partial w_0}{\partial x} \qquad \text{V.5}$$

By similar reasoning, we obtain the displacementv in the directiony :

$$v = v_0 - z\frac{\partial w_0}{\partial y} \qquad \text{V.6}$$

The deformations are determined according to the displacements by: (small deformations)

$$\varepsilon_x = \frac{\partial u}{\partial x}\ ; \qquad \varepsilon_y = \frac{\partial v}{\partial y}\ ; \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \text{V.7}$$

Based on (5) and (6):

$$\begin{cases} \varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} \\ \varepsilon_y = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_0}{\partial x^2} \\ \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x\,\partial y} \end{cases} \qquad \text{V.8}$$

Or, in matrix form:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \qquad \text{V.9}$$

The curvatures (arrows) of the median surface are:

$$\begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial x^2} \\ 2\frac{\partial^2 w_0}{\partial x\,\partial y} \end{Bmatrix} \qquad \text{V.10}$$

V.2.2 Stress relationship layer deformation:$K^{i\ \text{éme}}$

For substitution of equation (V.10) in (V.2), the stresses in the$K^{i\ \text{éme}}$ fold can be expressed as a function of the deformations (numbers and curvatures) of the median surface:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}} \end{bmatrix}_k \left[\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \right] \qquad \text{V.11}$$

Since the elastic constants$\overline{C_{ij}}$ can be different for each layer of the laminate, the variation of deformations across the material thickness is not necessarily linear although the stresses follow a linear variation (Fig.3)

V.3 Resulting forces and resulting moments:

We consider in the following a total thickness plane laminate (t) it consists of (n) folds. The pli nemek has a thickness tk .

The laminate is subjected to stresses in its plane notedN_x , $N_y$and$T_{xy} = T_{yx}$ (Fig.4) (resulting from stresses on membranes)

N_x : Resulting effort in x direction, per unit width in y direction.

T_{xy}: (Or T_{yx}) membrane shear per unit width in y direction (or in x direction).

$$N_x = \int_{-t/2}^{t/2} \sigma_x \, dz \quad ; N_y = \int_{-t/2}^{t/2} \sigma_y \, dz ; \quad T_{xy} = \int_{-t/2}^{t/2} \tau_{xy} \, dz \qquad \text{V.12}$$

To the membrane forces are added momentsM_x , $M_y$andM_{xy} (Fig.4b) (bending and torsional moment)

M_x : Bending moment of axis y, from stressσ_x, per unit width in y direction

M_{xy} : (Or M_{yx}) torsion moment of axis x (or y), of the stress τ_{xy}, per unit of width according to the direction y (or following x).

$$M_x = \int_{-t/2}^{t/2} \sigma_x \, .zdz; \, M_y = \int_{-t/2}^{t/2} \sigma_y \, .zdz \, ; \, M_{xy} = \int_{-t/2}^{t/2} \tau_{xy} \, .zdz \qquad \text{V.13}$$

Equations (V.12) and (V.13) may take the following form (Fig.5):

$$\begin{Bmatrix} N_x \\ N_y \\ T_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz \qquad \text{V.14}$$

And

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} . z \, dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} . z \, dz \qquad \text{V.15}$$

z_k andz_{k-1} are defined in Figure 5. Note that$z_0 = -t/2$. The resulting forces and moments are independent of the variable z, they are a function of x and y the coordinates in the median plane surfaces.

By substitution of equation (V.11) in equations (V.14) and (V.15) one obtains:

$$\begin{Bmatrix} N_x \\ N_y \\ T_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}} \end{bmatrix}_k \left[\int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} . z\, dz \right] \qquad \text{V.16}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}} \end{bmatrix}_k \left[\int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} . z\, dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} . z^2\, dz \right] \qquad \text{V.17}$$

One can easily see that$\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0, K_x, K_y$ etK_{xy}ne are not function of$z$ but take only values on the surface of the medium plane.

(V.16) and (V.17) read as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \qquad \text{V.18}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \qquad \text{V.19}$$

With:

$$\begin{cases} A_{ij} = \sum_{k=1}^{n} \left(\overline{C_{ij}}\right)_k (z_k - z_{k-1}) \\ B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{C_{ij}}\right)_k (z_k^2 - z_{k-1}^2) \\ D_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{C_{ij}}\right)_k (z_k^3 - z_{k-1}^3) \end{cases} \qquad \text{V.20}$$

Tell que :

A_{ij} : Stiffness in tension.

B_{ij} : Coupling coefficients.

D_{ij} : Bending coefficients (Bending stiffness).

Application: 1. Demonstrate that the force per unit of width generated by an extension of a laminate manufactured by 2 folds of the same thickness with an orientation$+\alpha$ and$-\alpha$ for each, is expressed by:

$$N_x = A_{11}\varepsilon_x^0 + A_{12}\varepsilon_y^0 + B_{16}K_{xy}$$

Consequence:

The resulting forceN_x causes, in addition to normal deformationsε_x^0 andε_y^0 , distortion (torsion)K_{xy}.

V.4 Studies of specific cases:

Layer stacking (layer natures, orientation, stack sequence, etc.) determines the structure of the stiffness matrix, the general shape of which is given by the relation (V.20).

The study of the few particular cases of usual laminate will allow to obtain more simplified forms of the stiffness matrix.

Quite often, laminates are made from layers, which have the same characteristics (same constituents, same geometric configurations, same thicknesses, etc.), but have different orientations of their main axes compared to the reference axes of the laminate.

V.4.1 Case of an isotropic layer:

In the case of a plate of homogeneous isotropic material, the elastic behavior is described by elastic constants$E$. The matrix of rigidity reduced in plane stresses is:ϑ

$$[C]=\begin{bmatrix}\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0\\ \frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0\\ 0 & 0 & \frac{E}{2(1-v)}\end{bmatrix} \qquad \text{V.21}$$

The equations (V.20) give:

$A_{11}=\frac{E.t}{1-v^2}=A$; $\qquad D_{11}=\frac{E.t^3}{12(1-v^2)}=D$

$A_{12}=vA$; $\qquad D_{12}=vD$

$A_{22}=A$;$D_{22}=D$ V.21 $B_{ij}=0$

$A_{16}=A_{26}=0$; $\qquad D_{16}=D_{26}=0$

$A_{66}=\frac{E.t}{2(1-v)}=\frac{(1-v)}{2}A$; $\qquad D_{66}=\frac{E.t^3}{24(1-v)}=\frac{(1-v)}{2}D$

Membrane resultants (, N_x andN_y) depend solely onT_{xy} membrane deformations (andε_x^0, ε_y^0); the bending and torsional moments (γ_{xy}^0 ,M_x andM_y) depend only on the curvatures of the middle plane (M_{xy},K_x andK_y).K_{xy}

Consequence: in the case of an isotropic plate, there is no membrane-bending/torsion coupling.

$$\begin{cases}\begin{Bmatrix}N_x\\ N_y\\ T_{xy}\end{Bmatrix}=\begin{bmatrix}A & vA & 0\\ vA & A & 0\\ 0 & 0 & \frac{(1-v)}{2}A\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}\\ \begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\begin{bmatrix}D & vD & 0\\ vD & D & 0\\ 0 & 0 & \frac{(1-v)}{2}D\end{bmatrix}\begin{Bmatrix}K_x\\ K_y\\ K_{xy}\end{Bmatrix}\end{cases} \qquad \text{V.22}$$

Note: $D=\frac{At^2}{12}$

V.4.2 Orthotropic layer relative to its main axes:

In the orthotropy axes, the reduced stiffness matrix of an orthotropic layer is written:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix}$$

With:

$$C_{11} = \overline{E_1}, C_{22} = \overline{E_2}.. ; C_{12} = v_{12}\overline{E_1}, C_{66} = G_{12}$$
$$\overline{E_1} = \frac{v_{12}E_2}{1 - v_{21}v_{12}} \overline{E_2} = \frac{E_2}{1 - v_{21}v_{12}}$$

The stiffness coefficients of the laminate will be:

$A_{11} = C_{11}.t$; $\quad D_{11} = \frac{C_{11}.t^3}{12}$

$A_{12} = C_{12}.t$; $\quad D_{12} = \frac{C_{12}.t^3}{12}$

$A_{22} = C_{22}.t$;$D_{22} = \frac{C_{22}.t^3}{12}$ $\quad B_{ij} = 0$ V.23

$A_{16} = A_{26} = 0$; $\quad D_{16} = D_{26} = 0$

$A_{66} = C_{66}.t$; $\quad D_{66} = \frac{C_{66}.t^3}{12}$

Conclusion:

As for the isotropic case, in the case of an orthotropic plate there is no membrane-bending/torsion coupling.

$$\begin{cases} \begin{Bmatrix} N_x \\ N_y \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \end{cases} \quad \text{V.22}$$

V.4.3 Orthotropic layer relative to axes other than those of orthotropy:

Under these conditions the matrix of reduced rigidity is written:

$$[\bar{C}] = \begin{bmatrix} \overline{C_{11}} & \overline{C_{12}} & \overline{C_{16}} \\ \overline{C_{12}} & \overline{C_{22}} & \overline{C_{26}} \\ \overline{C_{16}} & \overline{C_{26}} & \overline{C_{66}} \end{bmatrix}$$

And the stiffness coefficients of the plate will be:

$$A_{ij} = \overline{C_{ij}}.t \ ;D_{ij} = \frac{\overline{C_{ij}}.t^3}{12} \qquad B_{ij} = 0 \quad \text{V.25}$$

Note: $D = A_{ij}\frac{t^2}{12}$

$$\begin{cases} \begin{Bmatrix} N_x \\ N_y \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \end{cases} \quad \text{V.26}$$

Note that in this case too; there is no membrane-bending/torsion coupling. This during it notes that the normal stress ($N_x$or) depends on the shear deformation as it depends on the normal deformations, and the shear stressN_y depends onT_{xy} andhowε_x^0 it depends onε_y^0 (tensile-shear couγ_{xy}^0pling).

V.5 Symmetrical laminates:

A laminate is symmetrical if the middle plane is a plane of symmetry. Two symmetrical layers have:

- Same matrix of reduced rigidity $(\overline{C_{ij}})_k$
- Same thicknesst_k
- Opposite odds)z_k $(-z_k$

The coefficientsB_{ij} of the laminate stiffness matrix will be zero. The constitutive equation takes the following general form:

$$\begin{Bmatrix} N_x \\ N_y \\ T_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{26} & 0 & 0 & 0 \\ A_{16} & A_{26} & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & D_{16} \\ 0 & 0 & 0 & D_{21} & D_{22} & D_{26} \\ 0 & 0 & 0 & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_x \\ K_y \\ K_{xy} \end{Bmatrix} \quad \text{V.27}$$

- No coupling flexion- membrane. (simpler behavior)
- Symmetrical laminates are widely used. Specific conditions require a non-symmetrical laminate (heat shield).

V.5.1 Stacking of several isotropic layers (Fig.7):

The layer stiffness matrix gives:$K^{i\ éme}$

$$(\overline{C_{11}})_k = (\overline{C_{22}})_k = \frac{E_k}{1-\nu_k^2}\ ;\ (\overline{C_{16}})_k = (\overline{C_{26}})_k = 0$$

$$(\overline{C_{12}})_k = \frac{\nu_k E_k}{1-\nu_k^2}\ ;\ (\overline{C_{66}})_k = \frac{E_k}{2(1-\nu_k)}$$

The resulting efforts and moments will always be:

$$\begin{cases} \begin{Bmatrix} N_x \\ N_y \\ T_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} \end{cases} \qquad \text{V.28}$$

V.6 antisymmetric laminates (Fig.9-10):

Symmetrical laminates are used to eliminate membrane-flexion coupling. This coupling may be necessary in some designs; example: wing turbine with left profile.

The case of better shear stiffness is sought, it is necessary to have layers with defirent orientation. Symmetrical laminate, orientation of antisymmetric axes by contribution to the medium plane.

2 antisymmetric layers have:

- Opposite sides (z_k,$-z_k$);
- Same thicknesst_k ;
- Orientationsθ and$-\theta$ by contribution to the reference axes of the laminate.

CHAPTER VI

FAILURE CRITERIA OF COMPOSITE MATERIALS

VI.1 Introduction:

There are several failure manes of composite materials. The most commonly used (especially in the case of orthotropic materials) pre-design calculations is the so-called "Hill" criterion.

This criterion is an extension of that of Von Mises for isotropic materials working in the elastic domain.

Tsai -Hill criterion :

Hill proposed the following criterion for anisotropic materials:

$$(G+H)\sigma_1^2+(F+H)\sigma_2^2+(F+G)\sigma_3^2-2H\sigma_1\sigma_2-2G\sigma_1\sigma_3-2F\sigma_2\sigma_3+2L\tau_{23}^2+2M\tau_{13}^2+2N\tau_{12}^2=1 \quad \text{VI.1}$$

F, G, H, L, M and Nanisotropy parameters.

These parameters can be determined based on the breaking stresses $X, Y$and Sa unidirectional fold (Tsai) thanks to the results provided by the following tests:

- $\tau_{12}=S$and all other constraints are zero.$2N=\frac{1}{S^2}$
- $\sigma_1=X, G+H=\frac{1}{X^2}$
- $\sigma_2=Y, F+H=\frac{1}{Y^2}$ VI.2
- $\sigma_3=Z, F+G=\frac{1}{Z^2}$

A combination of equations (VI.2) leads to:

$$\begin{cases} 2H=\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2} \\ 2G=\frac{1}{X^2}+\frac{1}{Z^2}-\frac{1}{Y^2} \\ 2F=\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2} \end{cases} \quad \text{VI.3}$$

For a plane stress state of plane 1-2 of the fold: $\sigma_3=\tau_{13}=\tau_{23}=0$as we can easily see $y=z$(Fig.1).

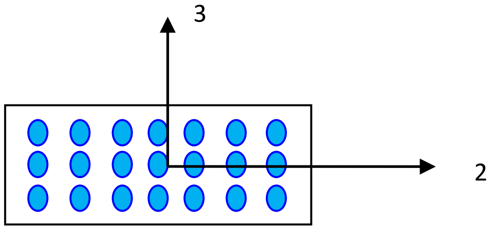


Figure VI.1 . State of plane stresses

By substituting (2) and (3) into (1) we obtain:

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \qquad \text{VI.4}$$

Equation (4) represents the Tsai -Hill failure criterion for a fold working in its plane.

Note: the breaking stresses of the fiber/matrix plies are different in tension and compression, see table.

VI.3 Evolution of resistance following the direction of stress:

VI.3.1 Tensile or compressive stress:

According to Figure 2, the constraints $\sigma_1, \sigma_2$and τ_{12}(in the orthotropy axes) will be given by:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}c^2 & s^2 & -2sc\\ s^2 & c^2 & 2sc\\ sc & -sc & c^2-s^2\end{bmatrix}\begin{bmatrix}\sigma_x\\ 0\\ 0\end{bmatrix} \qquad \text{VI.5}$$

$$\sigma_1 = c^2\sigma_x$$

$\sigma_2 = s^2\sigma_x$ VI.6

$$\tau_{12} = sc\sigma_x$$

By substituting equation (6) into (4) we obtain:

$$\overline{\sigma_x} = \frac{1}{\sqrt{\frac{c^4}{X^2} + \frac{s^4}{Y^2} + c^2s^2\left(\frac{1}{S^2} - \frac{1}{X^2}\right)}} \qquad \text{VI. 7}$$

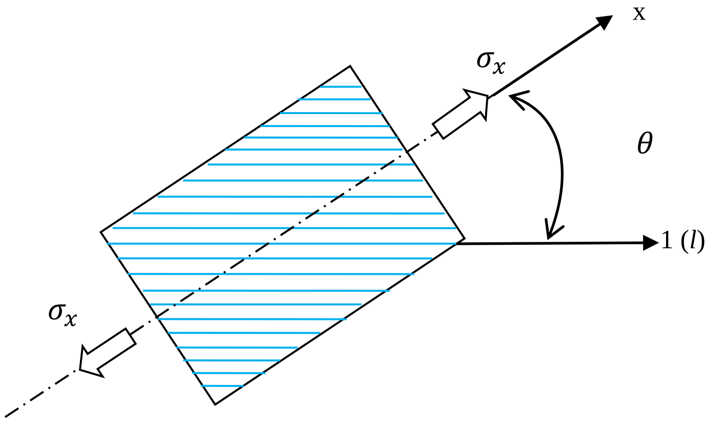


Figure VI.2 . Tensile (or compressive **) stress.**

Figure 3 represents a comparison of the criterion results with the experimental one.

VI.3.2 Shear stress:

Relation (5) written for the aces in Figure 4 allows us to obtain:

$$\sigma_1 = -2sc^2\tau_{xy}$$

$$\sigma_2 = 2sc\tau_{xy} \quad \text{VI.8}$$

$$\tau_{12} = (c^2 - s^2)\tau_{xy}$$

By substituting equation (8) into (4) we obtain:

$$\overline{\tau_{xy}} = \frac{1}{\sqrt{4c^2s^2\left(\frac{2}{X^2} + \frac{1}{Y^2}\right) + \frac{(c^2 - s^2)^2}{{\tau_{12}}^2}}} \quad \text{VI. 9}$$

Note: here, taking into account the figure ($\tau_{xy} > 0$) and the relationships (VI.9), $\sigma_{l\ rupture}$will be the limit stress in compression, and $\sigma_{t\ rupture}$the limit stress in tension, and this for $0^\circ \leq \theta \leq 90^\circ$.

VI. 4 Components of the Tsai -Hill breakup theory :

Apply $\sigma_1 = (\sigma_1^T)_{ult}$,to a one-way crease, then the damaged crease. Therefore, the equation reduces to

$$(G_2 + G_3)(\sigma_1^T)_{ult}^2 = 1$$

Apply $\sigma_1 = (\sigma_1^T)_{ult}$,to a one-way crease, then the damaged crease. Therefore, the equation reduces to

$$(G_1 + G_3)(\sigma_2^T)_{ult}^2 = 1$$

Apply $\sigma_2 = (\sigma_2^T)_{ult}$to a one-way crease, then the damaged crease. Therefore, the equation reduces to

$$(G_2 + G_3)(\sigma_2^T)_{ult}^2 = 1$$

Apply $\sigma_3 = (\sigma_2^T)_{ult}$, to a unidirectional ply, and assuming that the normal tensile breaking strength is the same in directions (2) and (3), and that the ply is damaged. Therefore, the equation reduces to

$$(G_1 + G_2)(\sigma_2^T)_{ult}^2 = 1$$

Apply $\tau_{12} = (\tau_{12})_{ult}$to a one-way crease, then the damaged crease. Therefore, the equation reduces to

$$2G_6(\tau_{12})_{ult}^2 = 1$$

$$(G_2+G_3)(\sigma_1^T)_{ult}^2=1 \qquad G_1=\frac{1}{2}\left(\frac{2}{[(\sigma_2^T)_{ult}]^2}-\frac{1}{[(\sigma_2^T)_{ult}]^2}\right)$$

$$(G_1+G_3)(\sigma_2^T)_{ult}^2=1 \qquad G_2=\frac{1}{2}\left(\frac{1}{[(\sigma_2^T)_{ult}]^2}\right)$$

$$(G_2+G_3)(\sigma_2^T)_{ult}^2=1 \qquad G_3=\frac{1}{2}\left(\frac{1}{[(\sigma_2^T)_{ult}]^2}\right)$$

$$2G_6(\tau_{12})_{ult}^2=1 \qquad G_6=\frac{1}{2}\left(\frac{1}{[(\tau_{12})_{ult}]^2}\right)$$

VI.4.1 Tsai -Hill fracture theory in plane stress

Because the unidirectional layer is assumed to be under plane stress, i.e.$\sigma_3=\tau_{31}=\tau_{23}=0$

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3-2G_1\sigma_2\sigma_3+2G_5\tau_{23}^2 + 2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2-\left[\frac{\sigma_1\sigma_2}{(\sigma_2^T)_{ult}^2}\right]+\left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2+\left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2<1$$

- Tsai - Hill fracture theory considers the interaction between the three strength parameters of the unidirectional
- Tsai -Hill failure theory does not distinguish between compressive and tensile forces in its equation. This may result in an underestimate of the maximum loads that can be applied compared to other failure theories.
- Tsai -Hill failure theory underestimates the failure stress because the transverse strength of a unidirectional blade is generally less than its transverse compressive strength.

VI.4.2 Modified Tsai -Hill breakup theory

$$\left[\frac{\sigma_1}{X_1}\right]^2-\left[\left(\frac{\sigma_1}{X_2}\right)\left(\frac{\sigma_2}{X_2}\right)\right]+\left[\frac{\sigma_2}{Y}\right]^2+\left[\frac{\tau_{12}}{S}\right]^2<1$$

$$X_1=(\sigma_1^T)_{ult},\ \text{if}\sigma_1>0$$
$$=(\sigma_1^C)_{ult},\ \text{if}\sigma_1<0$$
$$X_2=(\sigma_1^T)_{ult},\ \text{if}\sigma_2>0$$
$$=(\sigma_1^C)_{ult},\ \text{if}\sigma_2<0$$
$$Y=(\sigma_2^T)_{ult},\ \text{if}\sigma_2>0$$
$$=(\sigma_2^C)_{ult},\ \text{if}\sigma_2<0$$
$$S=(\tau_{12})_{ult}$$

VI.5 Tsai -Wu breakup theory

Tsai -Wu applied the fracture theory to a fold in plane stress. A fold is considered to have broken if

$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2$Eq-1

Violated, this theory of failure is more general than the Tsai -Hill theory of failure because it distinguishes between the compressive and tensile forces of a blade.

- Components$H_1 - H_\infty$ of fracture theory are found using the five strength parameters of a unidirectional blade.

VI.5.1 Tsai -Wu criterion components

- Apply $\sigma_1 = (\sigma_1^T)_{ult}, \sigma_2 = 0, \tau_{12} = 0$to the unidirectional, then the damaged unidirectional fold. Therefore, equation (1) reduces to

$$H_1(\sigma_1^T)_{ult} + H_{11}(\sigma_1^T)_{ult}^2 = 1 \qquad \text{Eq-2}$$

- Apply $\sigma_1 = -(\sigma_1^T)_{ult}, \sigma_2 = 0, \tau_{12} = 0$to the unidirectional, then the damaged unidirectional fold. Therefore, the equation reduces to

$$-H_1(\sigma_1^C)_{ult} + H_{11}(\sigma_1^C)_{ult}^2 = 1\text{Eq-3}$$

From equations (2) and (3),

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} + \frac{1}{(\sigma_1^C)_{ult}}$$

$$H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}$$

- Apply $\sigma_1 = 0, \sigma_2 = (\sigma_2^T)_{ult}, \tau_{12} = 0$to the unidirectional, then the damaged unidirectional fold. Therefore, the equation reduces to

$$H_2(\sigma_2^T)_{ult} + H_{22}(\sigma_2^T)_{ult}^2 = 1 \qquad \text{Eq-4}$$

- Apply $\sigma_1 = 0, \sigma_2 = (\sigma_2^T)_{ult}, \tau_{12} = 0$to the unidirectional, then the damaged unidirectional fold. Therefore, the equation reduces to

$$-H_2(\sigma_2^C)_{ult} + H_{22}(\sigma_2^C)_{ult}^2 = 1\text{Eq-5}$$

From equations (4) and (5),

$$H_2 = \frac{1}{(\sigma_2^T)_{ult}} + \frac{1}{(\sigma_2^C)_{ult}}$$

$$H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}$$

- Apply $\sigma_1 = 0, \sigma_2 = 0, \tau_{12} = (\tau_{12})_{ult}$to the unidirectional, then the damaged unidirectional fold. Therefore, equation (1) reduces to

$$H_6(\tau_{12})_{ult} + H_{66}(\tau_{12})_{ult}^2 = 1 \qquad \text{Eq-6}$$

- Apply $\sigma_1 =, \sigma_2 = 0, \tau_{12} = -(\tau_{12})_{ult}$to the unidirectional, then the damaged unidirectional fold. Therefore, the equation reduces to

$$-H_6(\tau_{12})_{ult} + H_{66}(\tau_{12})^2_{ult} = 1 \text{Eq-7}$$

From equations (6) and (7),

$$H_{66} = 0$$

$$H_6 = \frac{1}{(\tau_{12})^2_{ult}}$$

VI.5.1.1 Determination of H12

- Apply **equal tensile forces along both** material axes in a unidirectional composite. If $\sigma_x = 0, \sigma_y = 0, \tau_{xy} = 0$, is the load at which the ply damaged, then

$$(H_1 + H_2)\sigma + (H_{11} + H_{22} + +2H_{12})\sigma^2 = 1 \text{Eq-8}$$

The solution to equation (8) gives

$$H_{12} = \frac{1}{2\sigma^2}[1 - (H_1 + H_2)\sigma - (H_{11} + H_{22})\sigma^2]$$

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$

Take bend at 45° under uniaxial tension σ_x. The $\sigma_x$breaking stress is noted. If this constraint is $\sigma_x = 0$the use equation, the local stresses at rupture are

$$\sigma_1 = \frac{\sigma}{2}$$

$$\sigma_2 = \frac{\sigma}{2}$$

$$\tau_{12} = -\frac{\sigma}{2}$$

Substitute the local constraints above into equation 8,

$$(H_1 + H_2)\frac{\sigma}{2} + (H_{11} + H_{22} + H_{66} + 2H_{12})\frac{\sigma^2}{4} = 1$$

$$H_{12} = \frac{2}{\sigma^2} - \frac{(H_1 + H_2)}{\sigma} - \frac{1}{2}(H_{11} + H_{22} + H_{66})$$

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1$$

VI .5.1.2 Empirical models of H $_{12}$

- Tsai -Hill theory

$$H_{12} = \frac{1}{2(\sigma_1^T)^2_{ult}}$$

- According to the Hoffman criterion

$$H_{12} = \frac{1}{2(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}$$

- According to the Mises- Hencky criterion

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}$$

CHAPTER VII
MECHANICS OF COMPOSITE MATERIALS WITH MATLAB CODE

1) - THE FUNCTION WHICH GIVES THE FLEXIBILITY MATRIX IN THE LOCAL REFERENCE:

```
function y = ReducedCompliance (E1, E2, NU12, G12)
y = [1/E1 -NU12/E1 0 ; -NU12/E1 1/E2 0 ; 0 0 1/G12];
```

2) - THE FUNCTION WHICH GIVES THE RIGIDITY MATRIX IN THE LOCAL REFERENCE:

```
function y = ReducedStiffness (E1,E2,NU12,G12)
NU21 = NU12*E2/E1;
y = [E1 /( 1-NU12*NU21) NU12*E2/(1-NU12*NU21) 0 ;NU12*E2/(1-
NU12*NU21) E2/(1-NU12*NU21) 0 ; 0 0 G12];
```

3) - THE FUNCTION WHICH GIVES THE FLEXIBILITY MATRIX IN THE GLOBAL REFERENCE WITH A WELL-DETERMINED ANGLE:

```
function y = Sbar ( S,theta )
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
T = [m*mn*n 2*m*n ; n*nm*m -2*m*n ; -m*nm*nm*mn*n];
Tinv = [m*mn*n -2*m*n ; n*nm*m 2*m*n ; m*n -m*nm*mn*n];
y = Tinv *S*T;
```

4) - THE FUNCTION WHICH GIVES THE RIGIDITY MATRIX IN THE GLOBAL REFERENCE WITH A WELL-DETERMINED ANGLE:

```
function y = Qbar ( Q,theta )
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
T = [m*mn*n -m* n ; n*nm*mm*n ; 2*m*n -2*m*nm*mn*n];
Tinv = [m*mn*n 2*m* n ; n*nm*m -2*m*n ; -m*nm*nm*mn*n];
y = Tinv *Q*T;
```

5) - THE FUNCTION WHICH GIVES THE THERMAL DELATATION COEFICIENTS ALPHA1 AND ALPHA2:

```
function y = Alpha1(Vf,E1f,Em,Alpha1f,Alpham)
Vm = 1 - Vf ;
y = ( Vf *E1f*Alpha1f + Vm * Em * Alpham ) /( E1f* Vf + Em *
Vm );

function y =
Alpha2(Vf,Alpha2f,Alpham,E1,E1f,Em,NU1f,NUm,Alpha1f)
Vm = 1 - Vf ;
y = (Alpha2f - ( Em /E1)*NU1f*( Alpham - Alpha1f)* Vm )* Vf +(
Alpham + (E1f/E1)* NUm *( Alpham - Alpha1f)* Vf )* Vm ;
```

6) - THE FUNCTION WHICH GIVES THE THERMAL EXPANSION COEFFICIENTS IN THE CASE OF ANY ORIENTATION OF THE FIBERS:

```
function y = AlphaXY (alpha1,alpha2,theta)
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
y =[
m*m*alpha1+n*n*alpha2;n*n*alpha1+m*m*alpha2;2*m*n*(alpha2-
alpha1)];
```

7) - THE FUNCTION WHICH GIVES THE THERMOMECHANICAL DEFORMATIONS IN THE CASE OF ANY ORIENTATION OF THE FIBERS:

```
function y =
epsilontherm(E1,E2,NU12,NU21,G12,theta,delta,segma1,segma2,to1
2,alpha1,alpha2)
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
S = [1/E1 -NU12/E1 0 ; -NU12/E1 1/E2 0 ; 0 0 1/G12];
T = [m*mn*nm*n ; n*nm*m -m*n ; -2*m*n 2*m*nm*mn*n];
Tinv = [m*mn*n -2*m*n ; n*nm*m 2*m*n ; m*n -m*nm*mn*n];
segmaXY = [segma1; segma2 ; to12];
alphaxy = delta*[alpha1; alpha2; 0];
Q =[ E1/(1-NU12*NU21) NU12*E2/(1-NU12*NU21) 0 ;NU12*E2/(1-
NU12*NU21) E2/(1-NU12*NU21) 0 ; 0 0 G12];
y = T*S* Tinv * segmaXY +T*Q* alphaxy ;
```

8) - THE FUNCTION WHICH GIVES THE THERMOMECHANICAL CONSTRAINTS IN THE CASE OF ANY ORIENTATION OF THE FIBERS:

```
function y =
constranttherm(E1,E2,NU12,G12,theta,delta,epsX,epsY,gamaXY,al
pha1,alpha2,P)

NU21 = NU12*E2/E1;
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
Q = [E1 /( 1-NU12*NU21) NU12*E2/(1-NU12*NU21) 0 ; NU12*E2/(1-
NU12*NU21) E2/(1-NU12*NU21) 0 ; 0 0 G12];
T = [m*mn*n 2*m*n ; n*nm*m -2*m*n ; -m*nm*nm*mn*n];
Tinv = [m*mn*n -2*m*n ; n*nm*m 2*m*n ; m*n -m*nm*mn*n];
epsilon = [ epsX ; epsY ; gamaXY ];
alphaxy = delta*[alpha1; alpha2; 0];
if P == 1
y = Tinv *Q*T*epsilon- Tinv *Q* alphaxy ;
elseif P == 2
y = Tinv *Q*T;
end
```

9) - THE FUNCTION WHICH GIVES THE CONSTRAINTS IN THE LOCAL REFERENCE (FIBER AXES):

```
function y = sigma12( segmax,segmay,toxy,theta )
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
T = [m*mn*n 2*m*n ; n*nm*m -2*m*n ; -m*nm*nm*mn*n];
sigmaXY =[ segmax;segmay;toxy ];
y= T* sigmaXY ;
```

10) - THE FUNCTION WHICH GIVES THE DEFORMATIONS IN THE LOCAL REFERENCE (FIBER AXES):

```
function y = epsilon12( epsx,epsy,gamaxy,theta )
m = cos ( theta*pi/180);
n = sin( theta*pi/180);
T1 = [m*mn*n -m*n ; n*nm*mm*n ; 2*m*n -2*m*nm*mn*n];
epsilonXY =[ epsx;epsy;gamaxy ];
y=T1* epsilonXY ;
```

11) - EXECUTION AND GRAPHS:

Note: the example we used is above.

- **CALLS FUNCTIONS ON THE WORKSPACE:**

```
>> S = ReducedCompliance ( 145.2, 15.33, 0.279, 4.185)
S =

0.0069 -0.0019 0
-0.0019 0.0652 0
0 0 0.2389
>> Sbar (S ,0 )

years =

0.0069 -0.0019 0
-0.0019 0.0652 0
0 0 0.2389

>> Sbar (S ,10 )

years =

0.0204 -0.0137 -0.0746
-0.0137 0.0752 0.0546
-0.0373 0.0273 0.2154

>> Sbar (S ,20 )

years =

0.0552 -0.0434 -0.1177
-0.0434 0.0999 0.0802
-0.0589 0.0401 0.1559
```

```
>> Sbar (S .30 )

years =

0.0968 -0.0773 -0.1123
-0.0773 0.1260 0.0618
-0.0561 0.0309 0.0882

>> Sbar (S .40 )

years =

0.1284 -0.0994 -0.0631
-0.0994 0.1386 0.0056
-0.0315 0.0028 0.0440

>> Sbar (S ,41 )

years =

0.1305 -0.1005 -0.0566
-0.1005 0.1387 -0.0012
-0.0283 -0.0006 0.0419

>> Sbar (S .42 )

years =

0.1324 -0.1013 -0.0499
-0.1013 0.1385 -0.0081
-0.0250 -0.0041 0.0402

>> Sbar (S .43 )

years =

0.1340 -0.1019 -0.0431
-0.1019 0.1381 -0.0151
-0.0215 -0.0076 0.0390

>> Sbar (S .44 )

years =

0.1354 -0.1023 -0.0362
-0.1023 0.1374 -0.0221
-0.0181 -0.0111 0.0382

>> Sbar (S .45 )
```

```
years =

0.1365 -0.1024 -0.0292
-0.1024 0.1365 -0.0292
-0.0146 -0.0146 0.0380

>> Sbar (S .46 )

years =

0.1374 -0.1023 -0.0221
-0.1023 0.1354 -0.0362
-0.0111 -0.0181 0.0382

>> Sbar (S .47 )

years =

0.1381 -0.1019 -0.0151
-0.1019 0.1340 -0.0431
-0.0076 -0.0215 0.0390

>> Sbar (S .48 )

years =

0.1385 -0.1013 -0.0081
-0.1013 0.1324 -0.0499
-0.0041 -0.0250 0.0402

>> Sbar (S .49 )

years =

0.1387 -0.1005 -0.0012
-0.1005 0.1305 -0.0566
-0.0006 -0.0283 0.0419

>> Sbar (S .50 )

years =

0.1386 -0.0994 0.0056
-0.0994 0.1284 -0.0631
0.0028 -0.0315 0.0440

>> Sbar (S .60 )

years =

0.1260 -0.0773 0.0618
```

```
-0.0773 0.0968 -0.1123
0.0309 -0.0561 0.0882

>> Sbar (S .70 )

years =

0.0999 -0.0434 0.0802
-0.0434 0.0552 -0.1177
0.0401 -0.0589 0.1559

>> Sbar (S .80 )

years =

0.0752 -0.0137 0.0546
-0.0137 0.0204 -0.0746
0.0273 -0.0373 0.2154

>> Sbar (S .90 )

years =

0.0652 -0.0019 0.0000
   -0.0019 0.0069 -0.0000
0.0000 -0.0000 0.2389
```

- **The curve of Ex :**

```
>> x = [0 10 20 30 40 41 42 43 44 45 46 47 48 49 50 60 70 80
90]

x =

Columns 1 through 15

0 10 20 30 40 41 42 43 44 45 46 47 48 49 50

Columns 16 through 19

60 70 80 90
>> y1 =[ 0.0069 0.0204 0.0552 0.0968 0.1284 0.1305 0.1324
0.1340 0.1354 0.1365 0.1374 0.1381 0.1385 0.1387 0.1386 0.1260
9 0.0752 0.0652 ]

y1 =

Columns 1 through 17

0.0069 0.0204 0.0552 0.0968 0.1284 0.1305 0.1324 0.1340 0.1354
0.1365 0.1374 0.1381 0.1385 0.1387 0.1386 0.1260 0.0999
```

```
Columns 18 through 19

    0.0752 0.0652

>> plot(x ,1 ./y1,'-.go')
>> xlabel ( '\theta(degrees)');
>> ylabel ( 'S^{-}_{11} GPa ');
>> ylabel ( 'Ex GPa ');
```

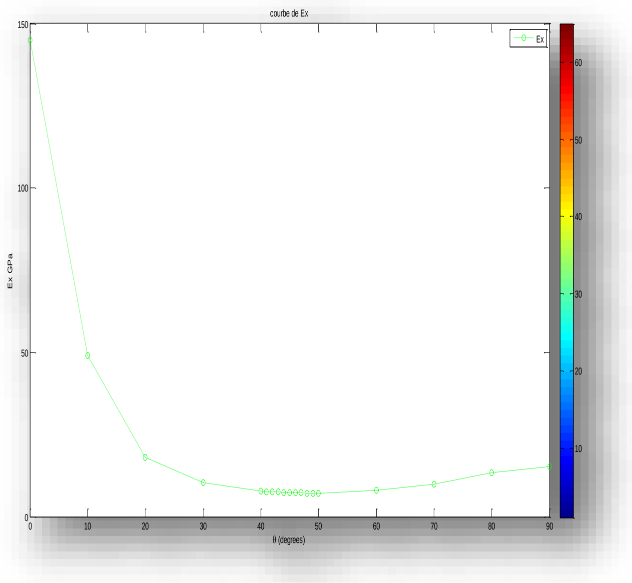


Fig -1- Curve of Ex

- **Ey's curve :**

```
>> y2 =[ 0.0652 0.0752 0.0999 0.1260 0.1386 0.1387 0.1385
0.1381 0.1374 0.1365 0.1354 0.1340 0.1324 0.1305 0.1284 0.0968
2 0.0204 0.0069 ]

y2 =

Columns 1 through 17

0.0652 0.0752 0.0999 0.1260 0.1386 0.1387 0.1385 0.1381 0.1374
0.1365 0.1354 0.1340 0.1324 0.1305 0.1284 0.0968 0.0552

Columns 18 through 19

    0.0204 0.0069

>> plot(x ,1 ./y2,'-.go')
>> ylabel ( ' Ey GPa ');
>> xlabel ( '\theta(degrees)');
```

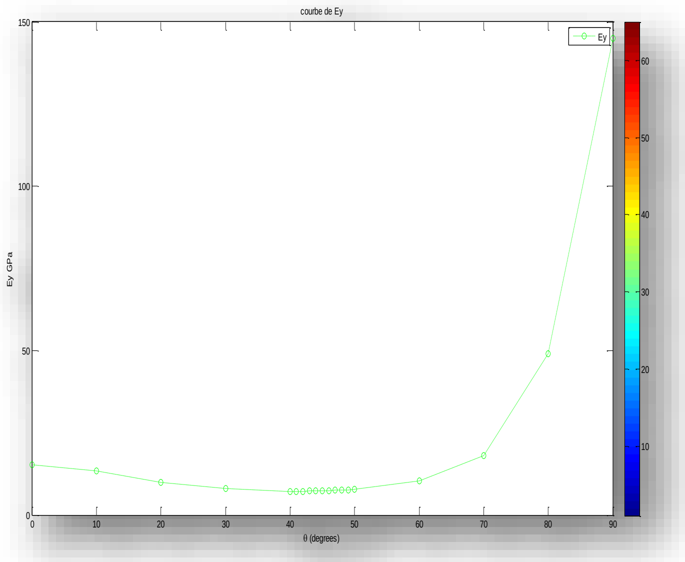


Fig -2- Ey curve

➢ **The Gxy curve :**

```
>> y3 =[ 0.2389 0.2154 0.1559 0.0882 0.0440 0.0419 0.0402
0.0390 0.0382 0.0380 0.0382 0.0390 0.0402 0.0419 0.0440 0.0882
9 0.2154 0.2389]

y3 =

Columns 1 through 17

0.2389 0.2154 0.1559 0.0882 0.0440 0.0419 0.0402 0.0390 0.0382
0.0380 0.0382 0.0390 0.0402 0.0419 0.0440 0.0882 0.1559

Columns 18 through 19

0.2154 0.2389

>> plot(x ,1 ./y3,'-.go')
>> xlabel ( '\theta(degrees)');
>> ylabel ( ' Gxy GPa ');
```

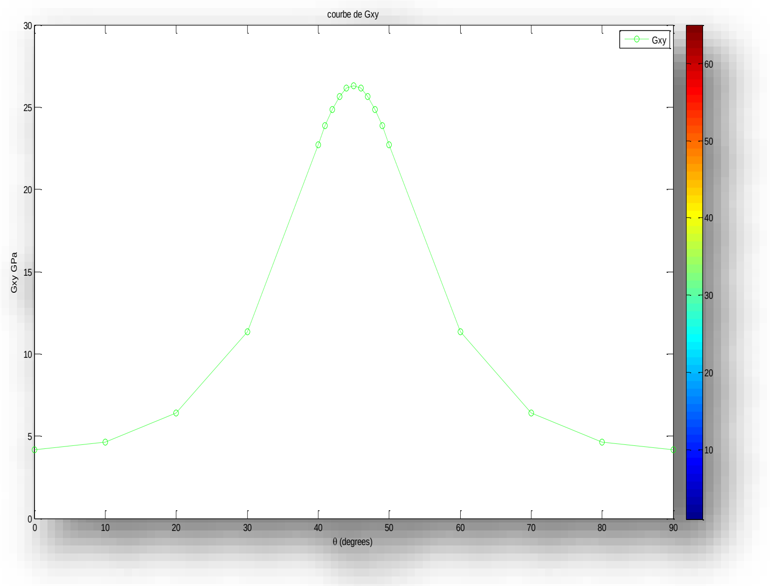


Fig -3- Gxy curve

- **The ratio curve –(NUyx /Ey):**

```
>> y4 =[ -0.0019 -0.0137 -0.0434 -0.0773 -0.0994 -0.1005 -
0.1013 -0.1019 -0.1023 -0.1024 -0.1023 -0.1019 -0.1013 -0.1005
-0.0994 -0.0773 -0.043 4 -0.0137 -0.0019]

y4 =

Columns 1 through 17

-0.0019 -0.0137 -0.0434 -0.0773 -0.0994 -0.1005 -0.1013 -
0.1019 -0.1023 -0.1024 -0.1023 -0.1019 -0.1013 -0.1005 -0.0994
-0.0773 -0.0434

Columns 18 through 19

   -0.0137 -0.0019
>> plot(x ,y4 ,'-.go')
>> xlabel ( '\theta(degrees)');
>> ylabel ( 'ratio1GPa ' );
```

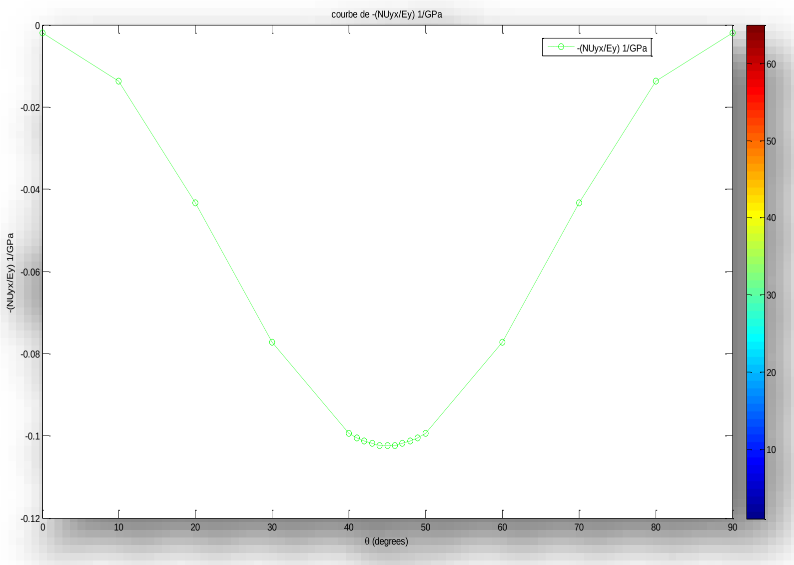


Fig -3- Ratio curve – (Nyx /Ey)

- **The curve of the first mutual influence coefficient1:**

```
>> y5 =[ 0 -0.0746 -0.1177 -0.1123 -0.0631 -0.0566 -0.0499 -
0.0431 -0.0362 -0.0292 -0.0221 -0.0151 -0.0081 -0.0012 0.0056
0.0618 0.0802 0.0 546 0.0000]

y5 =

Columns 1 through 17

0 -0.0746 -0.1177 -0.1123 -0.0631 -0.0566 -0.0499 -0.0431 -
0.0362 -0.0292 -0.0221 -0.0151 -0.0081 -0.0012 0.0056 0.0618
0.0802

Columns 18 through 19

    0.0546 0
>> plot(x ,y5 ,'-.go')
>> xlabel ( '\theta(degrees)');
>> ylabel ( ' mutual 1');
```

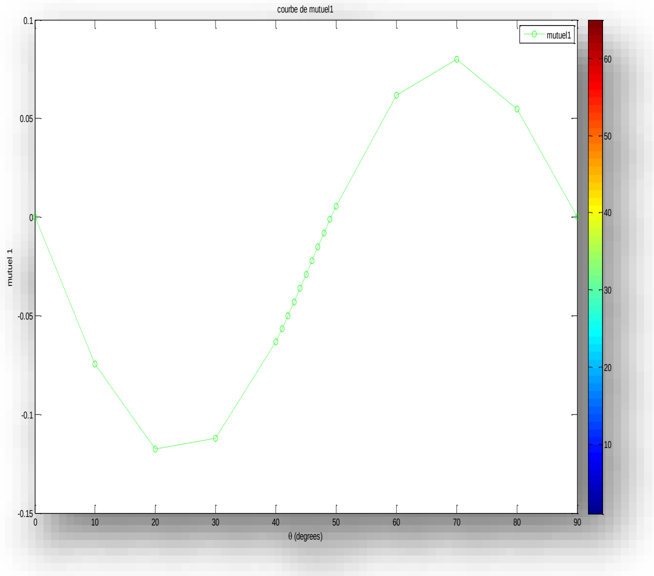
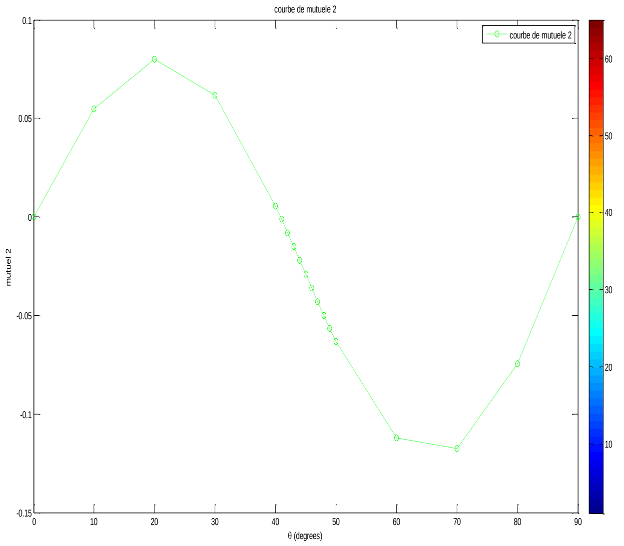


Fig -3- Curve of the first mutual influence coefficient1

➢ **The graph of the second coefficient of mutual influence2:**

```
>> y6 =[ 0 0.0546 0.0802 0.0618 0.0056 -0.0012 -0.0081 -0.0151
-0.0221 -0.0292 -0.0362 -0.0431 -0.0499 -0.0566 -0.0631 -
0.1123 -0.1177 -0.0 746 -0.0000]

y6 =

Columns 1 through 17

0 0.0546 0.0802 0.0618 0.0056 -0.0012 -0.0081 -0.0151 -0.0221
-0.0292 -0.0362 -0.0431 -0.0499 -0.0566 -0.0631 -0.1123 -
0.1177

Columns 18 through 19

   -0.0746 0

>> plot(x ,y6 ,'-.go')
>> xlabel ( '\theta(degrees)');
>> ylabel ( ' mutual 2');
```



Example uses:

Example 3.6 A unidirectional composite consisting of carbon fibres in a PEEK matrix has the fibres aligned at 25° to the loading axis. If the fibres and matrix have the properties indicated below, calculate E_x, E_y, G_{xy}, ν_{xy}, and ν_{yx}.

$$\begin{array}{ll} E_f = 240 \ \text{GN/m}^2 & E_m = 3 \ \text{GN/m}^2 \\ \nu_f = 0.23 & \nu_m = 0.35 \\ V_f = 0.6 & \\ \rho_f = 1700 \ \text{kg/m}^3 & \rho_m = 1300 \ \text{kg/m}^3 \\ G_f = 80 \ \text{GN/m}^2 & G_m = 1.1 \ \text{GN/m}^2 \end{array}$$

Solution The lamina properties in the local (1–2) directions are calculated as follows:

$$E_1 = E_f V_f + E_m.(1 - V_f), \qquad E_1 = 145.2 \ \text{GN/m}^2$$

$$E_2 = \frac{E_m \cdot (1 + 2 \cdot \beta \cdot V_f)}{(1 - \beta \cdot V_f)}, \qquad E_2 = 15.33 \ \text{GN/m}^2$$

Halpin–Tsai Equation

$$G_{12} = G_m \cdot \frac{(G_f + G_m) + V_f(G_f - G_m)}{(G_f + G_m) - V_f(G_f - G_m)},$$

$$G_{12} = 4.185 \ \text{GN/m}^2 \qquad \text{Halpin–Tsai Equation}$$

$$\nu_{12} = \nu_f \cdot V_f + \nu_m \cdot (1 - V_f) \qquad \nu_{12} = 0.278$$

$$\nu_{21} = \nu_{12} \cdot \left(\frac{E_2}{E_1}\right) \qquad \nu_{21} = 0.029$$

- **CALCULATE THE THERMAL EXPANSION COEFFICIENTS ALPHA1 AND ALPHA2:**

```
>> Alpha1( 0.23,240,3,25,30)

years =

25.2008
>> Alpha2( 0.6,30,20,145.2,240,0.23,0.35,15,20)

years =

   26
```

- **CALCULATE THE THERMAL EXPANSION COEFFICIENTS IN THE AXES OF THE FOLD FOR θ=40:**

```
>> y = AlphaXY ( 25.2008,26,40)

y =

25.5310
25.6698
0.7871
```

- **THERMOMECHANICAL DEFORMATIONS IN THE CASE OF ANY FIBER ORIENTATION:**

```
>> y =
epsilontherm(145.2,15.33,0.278,0.029,4.185,40,50,100,200,50,25
.2008,26)

y =

1.0e+005 *

1.2205
0.9350
-1.6201
```

- **THERMOMECHANICAL CONSTRAINTS IN THE CASE OF ANY FIBER ORIENTATION:**

```
>> y = constrainttherm (145.2,15.33,0.278,4.185,40,50,1.0e+005
*1.2205,1.0e+005 * 0.9350, 1.0e+005 * -1.6201,25.2008,26,1)

y =

1.0e+006 *

-1.9114
-0.5010
-5.0161
```

- **CALCULATE STRESS IN THE FIBER AXES:**

```
>> sigma12( 100,-90,200,0)

years =

100
-90
200
```

SECOND METHOD FOR PLOTTING THE EX EY GXY PARAMETERS…:

- **Ex curve :**

```
clc
e1=150;
e2=100;
```

```
nu12=0.32;
g12=22;
nu21 = nu12*e2/e1;
theta= 0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
y= m.^4*e1+n.^4*e2+2.*m.^2.*n.^2*(nu21*e1+2*g12);
plot( theta , y);
```

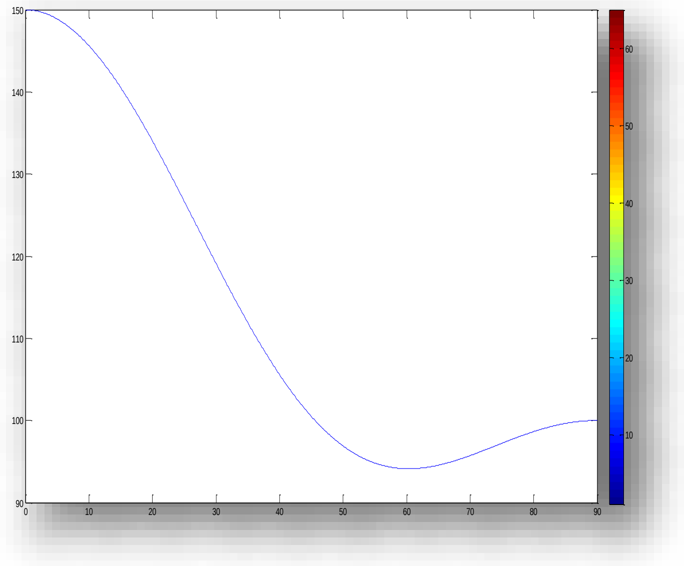


Fig -1- Curve of Ex

- **Ey curve :**

```
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
nu21 = nu12*e2/e1;
theta= 0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
y= n.^4*e1+m.^4*e2+2.*m.^2.*n.^2*(nu21*e1+2*g12);
plot( theta , y);
```

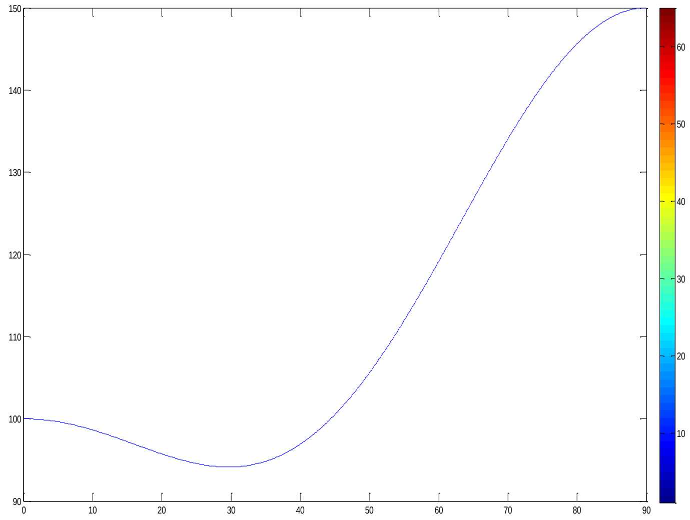


Fig -2- Ey curve

- **Gxy curve :**

```
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
nu21 = nu12*e2/e1;
theta= 0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
y=m.^2.*n.^2*(e1+e2-(2*nu21*e1) )+ ((m.^2-n.^2).^2)*g12;
plot( theta , y);
```

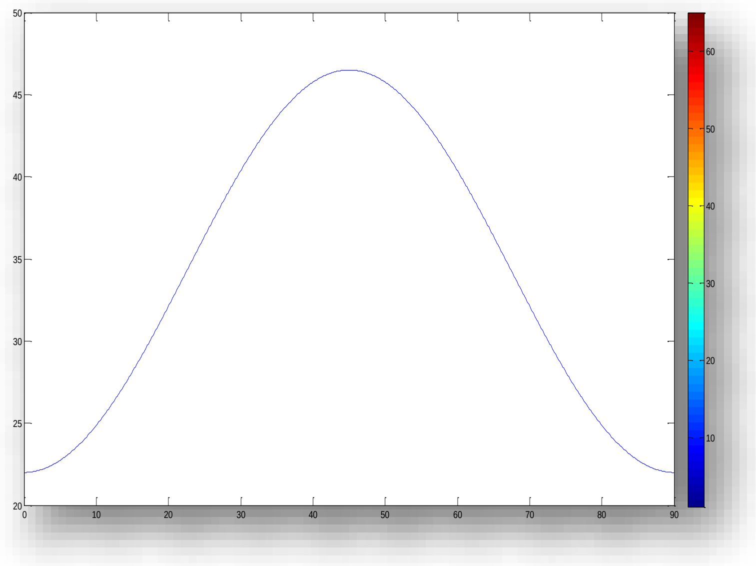
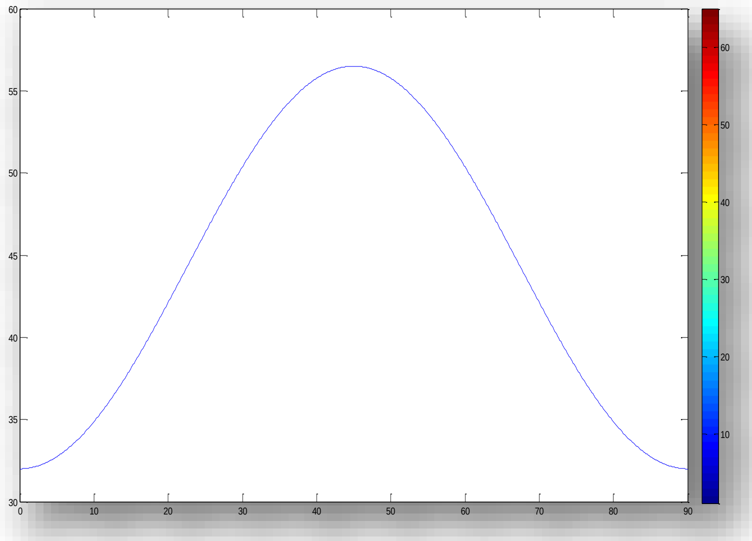


Fig -3- Gxy curve

- **The ratio curve –(NUyx /Ey):**

```
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
nu21 = nu12*e2/e1;
theta= 0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
y=m.^2.*n.^2*(e1+e2-4*g12 )+ (m.^4+n.^4)*nu21*e1;
plot( theta , y);
```



ratio curve –(NUyx /Ey)

- **The curve of the first mutual influence coefficient1:**

```
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
nu21 = nu12*e2/e1;
theta= 0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
y=-m.*n.*((m.^2*e1-n.^2*e2)-(m.^2-n.^2)*(nu21*e1+2*g12));
                       plot( theta , y);
```

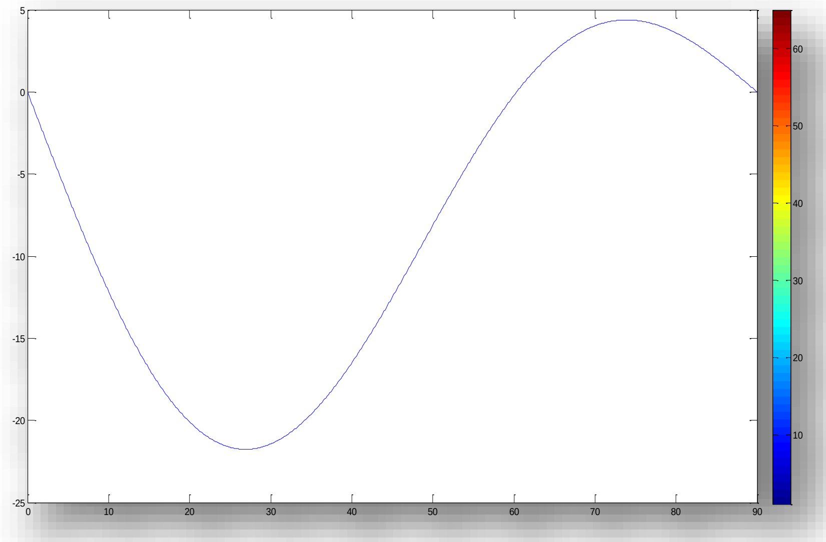


Fig -3- Curve of the first mutual influence coefficient1

- **The curve of the first mutual influence coefficient1:**

```
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
nu21 = nu12*e2/e1;
theta= 0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
y=-m.*n.*((n.^2*e1-m.^2*e2 )+ (m.^2-n.^2)*(nu21*e1+2*g12));
plot( theta , y);
```

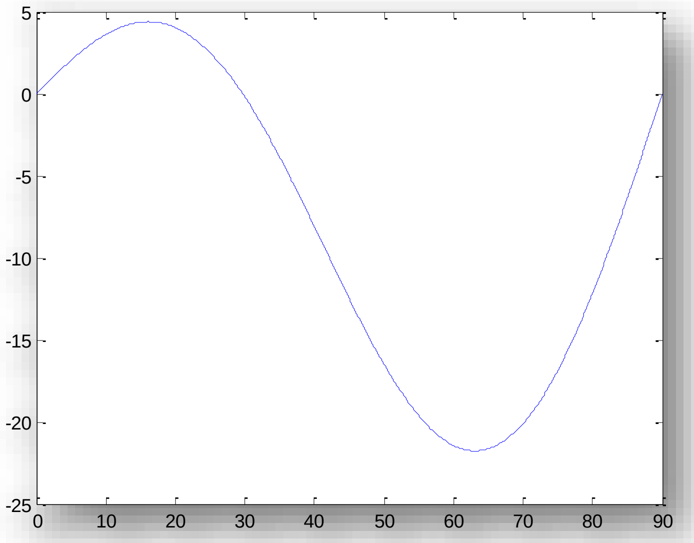
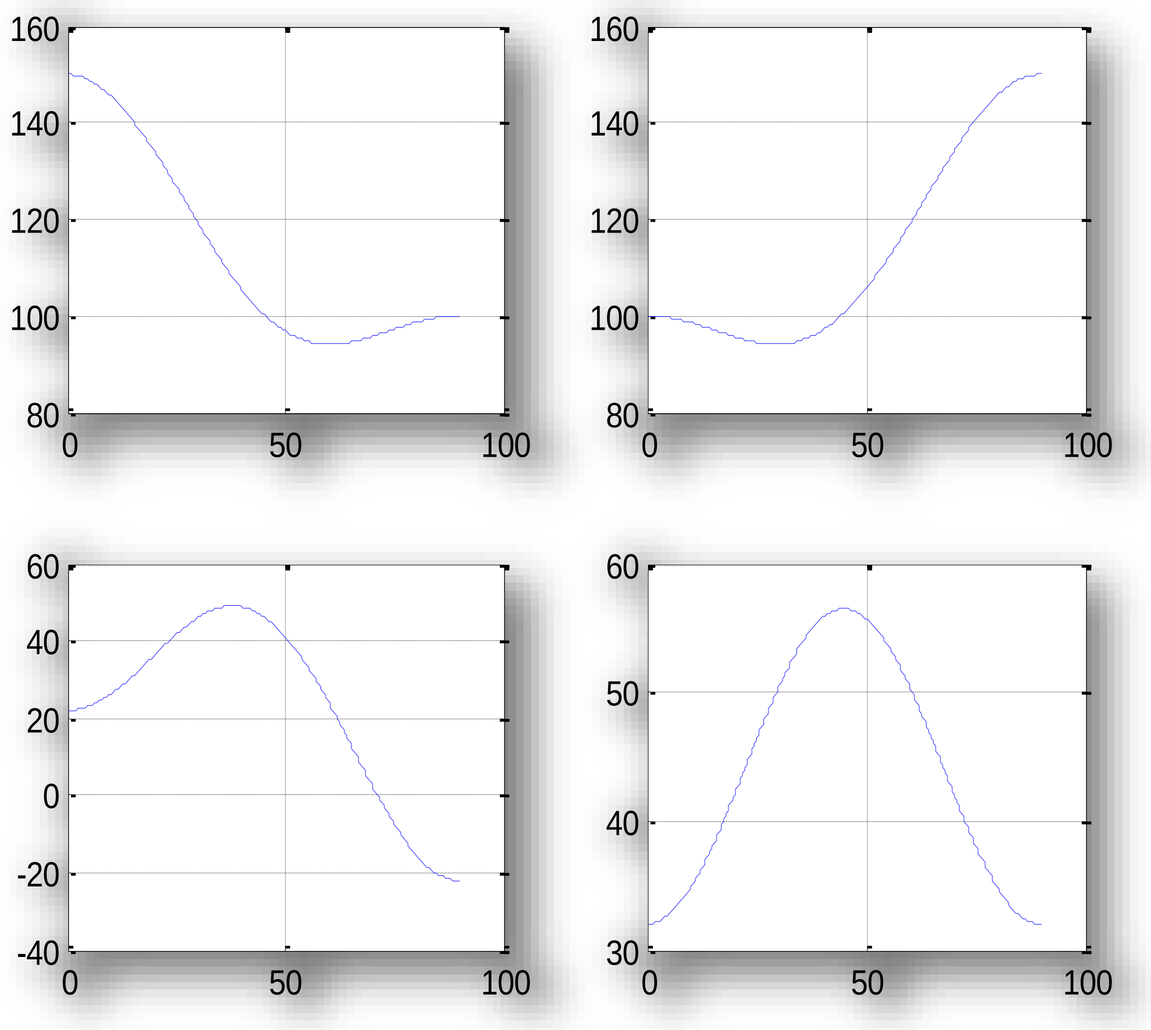


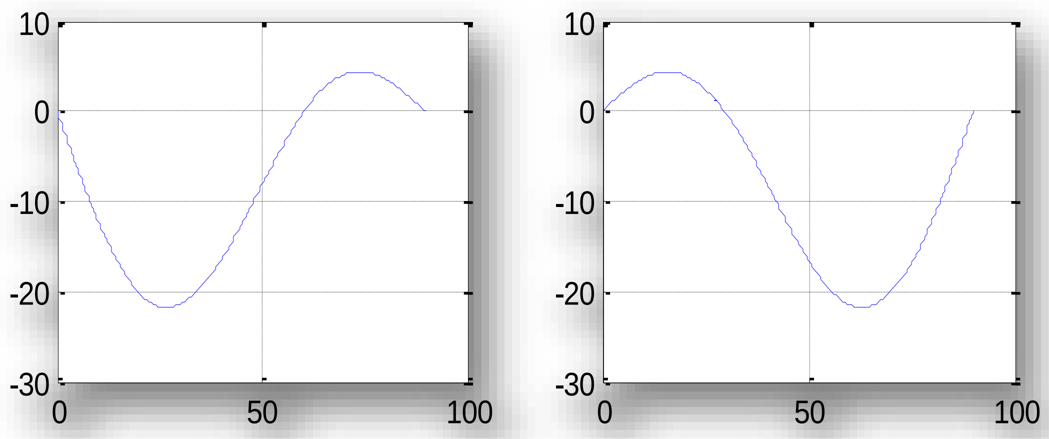
Fig -3- Curve of the first mutual influence coefficient1

USING A SINGLE PROGRAM:

```
clear
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
theta =0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
nu21 = nu12*e2/e1;
y1=m.^4*e1+n.^4*e2+2.*m.^2.*n.^2*(nu21*e1+2*g12);
y2=n.^4*e1+m.^4*e2+2.*m.^2.*n.^2*(nu21*e1+2*g12);
y3=m.^2.*n.^2*(e1+e2-(2*nu21*e1) )+ ((m.^2-n.^2))*g12;
y4=m.^2.*n.^2*(e1+e2-4*g12 )+ (m.^4+n.^4)*nu21*e1;
subplot( 221)
plot( theta,y1)
grid we ;
subplot( 222)
plot( theta,y2)
grid we ;
subplot( 223)
plot( theta,y3)
grid we ;
subplot( 224)
plot( theta,y4)
grid we ;
```




```
clear
clc
e1=150;
e2=100;
nu12=0.32;
g12=22;
theta =0:0.1:90;
m = cos ( theta*pi/180);
n = sin( theta *pi/180);
nu21 = nu12*e2/e1;
y1=-m.*n.*((m.^2*e1-n.^2*e2)-(m.^2-n.^2)*(nu21*e1+2*g12));
y2=-m.*n.*((n.^2*e1-m.^2*e2 )+ (m.^2-n.^2)*(nu21*e1+2*g12));
subplot( 221)
plot( theta,y1)
grid we ;
subplot( 222)
plot( theta,y2)
grid we ;
```



GENERAL CONCLUSION :

According to the results obtained and the graphs which illustrate the different variations of the parameters of a unidirectional fold we can deduce some points:

1- the orientation of the fibers in a unidirectional fold is very important to obtain a more rigid structure

2- the orientation of the fibers in a unidirectional fold generates a shear which is presented by the coefficients of mutual influence but this can be a disadvantage

3- the use of a calculation code is very effective for calculating any parameter or behavioral law of a composite

4-the precision of the results due to the precision of the fiber parameters is matrix

CHAPTER VIII
EXAMPLES OF COMPOSITE MATERIALS ANALYSIS

Homework 01/ Composite (Micromechanical of Lamina)

Problem 01

Calculate the following for a metal-matrix composite of aluminum reinforced with 60 vol % SiC fibers:

(a) The modulus of elasticity under isostrain conditions E_1.

(b) The modulus of elasticity under isostress conditions E_2.

Note:

The modulus of elasticity for aluminum is 69 x 10^3 MPa

The modulus of elasticity for SiC is 430 x 10^3 MPa

Problem 02

For Glass/Epoxy Unidirectional lamina, given the following material properties:

Fiber volume fraction V_f = 40%

E_f = 85 GPa ; $\nu_f = 0.20$; G_f = 35.5 GPa

E_m = 3.4 GPa ; $\nu_m = 0.20$; G_m = 1.3 GPa

Find the following:

1- matrix volume fraction (V_m),

2- longitudinal elastic modulus (E_1),

3- transverse elastic modulus (E_2),

4- major Poisson's ratio(ν_{12}),

5- in-plane shear modulus(G_{12}).

Problem 03

Plot Poisson's ratio as a function of reinforcing fiber content $\nu(V_f)$ for an SiC fiber-reinforced Si3N4 composite system loaded parallel to the fiber direction and SiC contents between 50 and 75 vol %.

Note:

Poisson's ratio for SiC is $\nu_m = 0.19$

Poisson's ratio for Si3N4 ν_f is = 0.24

Problem 04

Compare the calculated value of the isostrain modulus of a tungsten fiber (50 vol%) /copper composite with the value of 260 x 103 MPa which is given in the literature (for a parallel loading of continuous filaments).

Note:

The modulus of tungsten is 407 x 10^3 MPa

The modulus of copper is 115 x 10^3 MPa.

Homework 02/ Composite (Stiffness and Compliance Matrices)

Problem 01

Consider the graphite-reinforced polymer composite material [1]:

$E1$ = 155.0 GPa, $E2 = E3$ = 12.10 GPa

$v23$ = 0.458, $v12 = v13$ = 0.248

$G23$ = 3.20 GPa, $G12 = G13$ = 4.40 GPa

1- Determine the reduced compliance and stiffness matrices.

2- Check that the two matrices obtained in (a) are indeed inverses of each other by multiplying them together to get the identity matrix.

Problem 02

Consider a 60-mm cube made of graphite-reinforced polymer composite material that is subjected to a tensile force of 100 kN perpendicular to the fiber direction, directed along the 2-direction. The cube is free to expand or contract.

1- Determine the changes in the 60-mm dimensions of the cube.

The material constants for graphite-reinforced polymer composite material are given as follows [1].

Problem 03

Consider a layer of graphite-reinforced composite material 200mm long, 100mm wide, and 0.200mm thick. The layer is subjected to an inplane tensile force of 4 kN in the fiber direction which is perpendicular to the 100-mm width.

1- Determine the transverse strain $\varepsilon 3$.

Assume the layer to be in a state of plane stress and use the elastic constants given in problem [1].

Problem 04

Consider a 40-mm cube made of glass-reinforced polymer composite material that is subjected to a compressive force of 150 kN perpendicular to the fiber direction, directed along the 3-direction. The cube is free to expand or contract.

1- Determine the changes in the 40-mm dimensions of the cube.

The material constants for glass-reinforced polymer composite material are given as follows:

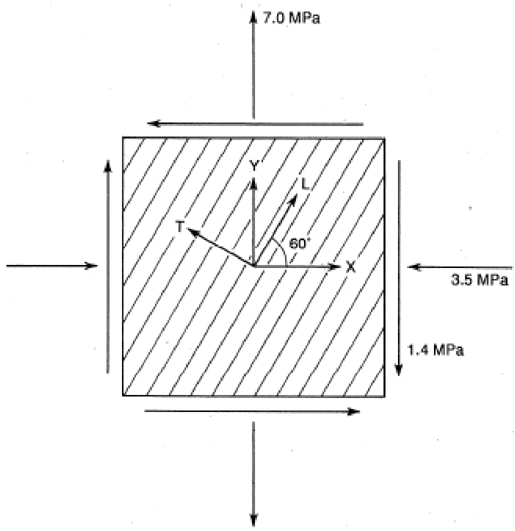
E_1 = 50.0 GPa, $E_2 = E_3$ = 15.20 GPa

v_{23} = 0.428, $v_{12} = v_{13}$ = 0.254

G_{23} = 3.28 GPa, $G_{12} = G_{13}$ = 4.70 GPa

Homework 03/ Composite (stress and strain in arbitrary axes

Problem 01



Elastic constants:
$E_L = 14\ GPa$
$E_T = 3.5\ GPa$
$G_{LT} = 4.2\ GPa$
$\nu_{LT} = 0.4$
$\nu_{TL} = 0.1$

Stresses:
$\sigma_x = -3.5\ MPa$
$\sigma_y = 7.0\ MPa$
$\tau_{xy} = -1.4\ MPa$

1. Compute the stresses in orthotropy axes (1,2).

2. Compute the strains in natural axes (1,2).

3. Find the strains in the lamina θ=60° axes *(x,y)*.

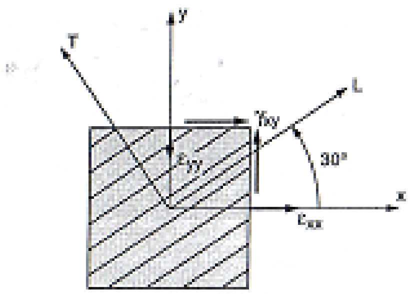
Problem 02

Unidirectional layer is contained in one of its planes (x, y) to the state following deformations:

ε_{xx}= 1 %

ε_{yy}= - 0.5 %

γ_{xy}= 2 %



The fiber direction at an angle of **30°** with the x direction

The elastic constant of the composite material are:

*E*1 = 40 GPa, *E*2 = 10 GPa

*v*12 = 0.32 *G*12 = 4.50 GPa

Considering that the layer is in a state of plane stress, determine:

1- Reduced stiffness matrices in principal (local) axis (1,2).

2- Stiffness matrices in arbitrary axis (global) (x,y).

3. Stresses σ_{xx}, σ_{yy}, σ_{xy} in the axes (x, y) system.

4. Stresses in the principal axes (1.2) of the layer (and schematically in figure).

Homework 04/ Composite (Ultimate Strengths of a Unidirectional Lamina)

Problem 01

Consider a glass/epoxy lamina with a 70% fiber volume fraction, the properties for glass and epoxy are:

$E_f = 85\ GPa$	$E_m = 3.4\ GPa$	$E_1 = 60.52\ GPa$	$G_f = 35.42\ GPa$,
$\nu_f = 0.20$.	$\nu_m = 0.30$.	$E_2 = 10.37$ GPa.	$G_m = 1.308\ GPa$,
$(\sigma_f)_{ult} = 1550\ MPa$	$(\sigma_m)_{ult} = 72\ MPa$	$\nu_{12} = 0.23$	$G_{12} = 4.014\ GPa$.
$(\tau_f)_{ult} = 35\ MPa$.	$(\tau_m)_{ult} = 34\ MPa$.		$(\tau_{12})_{mult} = 34\ MPa$.

Find:

1. Ultimate failure strain of the fiber $(\varepsilon_f)_{ult}$
2. The ultimate tensile strain of the matrix $(\varepsilon_m)_{ult}$
3. Longitudinal tensile strength $(\sigma_1^T)_{ult}$
4. Minimum fiber volume fractions $(V_f)_{minimum}$
5. Critical fiber volume fractions $(V_f)_{critical}$
6. The fiber diameter to fiber spacing ratio $\frac{d}{s}$
7. Ultimate transverse tensile strain (use 2 method) $(\varepsilon_2^T)_{ult}$
8. Longitudinal compressive strength (use 3 mode) $(\sigma_1^C)_{ult}$
9. Ultimate transverse tensile strength $(\sigma_2^T)_{ult}$
10. Ultimate compressive strain of the matrix (ε_m^C)
11. Ultimate transverse compressive strain(ε_2^C)
12. Ultimate transverse compressive strength $(\sigma_2^C)_{ult}$
13. Ultimate shearing strain of the matrix $(\gamma_{12})_{ult}$
14. Ultimate shear strength $(\tau_{12})_{ult}$

REFERENCES

References

[1]. « **Mécanique des matériaux et structures composites** », Jean Marie Berthelot, Edition Le mans Novembre 2010

[2]. « **Analysis of Composite Structures** », Christian Decolon, Edition in 2000 by Hermes Science Publications, Paris

[3]. « **Mécanique des Matériaux composites** », J. Molimard, EMSE 2004, Version 2, Septembre 2004

[4]. « **Composite Structures: Theory and Practice** », Peter Grant and Carl Q. Rousseau, editors, ASTM Stock Number: STP 1383.

[5]. « **Mechanics of composites materiels** », Laszlo P.Kollar & George S. Springer, Cambridge University Press 2003.

[6]. « **Mechanics of Composite Materials With MATLAB** », George Z. Voyiadjis Peter I. Kattan, Springer-Verlag Berlin Heidelberg 2005 Printed in The Netherlands