Series TD 2 Constrained and unconstrained optimization

Exercise N° 1

Consider the following objective function:

$$f(x_1, x_1) = 20x_1 + 26x_2 + 4x_1x_2 - 4{x_1}^2 - 3{x_2}^2$$

- Find the values of the design variables x_1, x_1 which maximize the objective function $f(x_1, x_1)$.

Exercise N° 2

Determine the value x^* for which the following objective function is maximum.

$$f(x) = \frac{1}{10\sqrt{2\pi}} e^{-\left(\frac{1}{2}\right)\left[\frac{(x-100)}{10}\right]^2}$$

Exercise N° 3

Identify the nature of the stationary points to each of the following objective functions:

(1) $f = 2 - x^2 - y^2 + 4xy$

(2) $f = 2 + x^2 - y^2$

(3) $f = xy$

(4) $f = x^3 - 3xy^2$

Exercise N° 4

We define the following optimization problem as:

$$Minimize\ f = 9 - 8x_1 - 6x_2 - 4x_3 + 2{x_1}^2 + 2{x_2}^2 + {x_3}^2 + 2x_1x_2 + 2x_1x_3$$

$$subjected\ to:$$

$$x_1 + x_2 + 2x_3 = 3$$

- Solve the above optimization problem using :
  - **(a)** The direct variable substitution method.
  - **(b)** The Lagrange multipliers method.

Exercise N° 5

Consider the optimization problem defined by:

$$Minimize\ f(\boldsymbol{X}) = \frac{1}{2}({x_1}^2 + {x_2}^2 + {x_3}^2)$$

$$Subjected\ to:$$

$$\begin{cases} g_1(\boldsymbol{X}) = x_1 - x_2 = 0 \\ g_2(\boldsymbol{X}) = x_1 + x_2 + x_3 - 1 = 0 \end{cases}$$

- Solve the above optimization problem using :
  - **(a)** The direct variable substitution method.
  - **(b)** The Lagrange multipliers method.

Exercise N° 6

Determine the values of the variables x, y, z that minimize the following objective function:

$$f(x, y, z) = \frac{6xyz}{x + 2y + 2z}$$

For which the variables x, y, z are restricted by the following relation:

$$xyz = 16$$

Exercice N° 7

We formulate an optimization problem as:

$$Minimize\ f = (x_1 - 2)^2 + (x_2 - 1)^2$$

$$Subjected\ to:$$

$$\begin{cases} 2 \geq x_1 + x_2 \\ x_2 \geq {x_1}^2 \end{cases}$$

Use the optimality condition to find the local minimum point among the following points:

$$\boldsymbol{X}_1 = \begin{Bmatrix} 1.5 \\ 0.5 \end{Bmatrix}, \quad \boldsymbol{X}_2 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}, \quad \boldsymbol{X}_3 = \begin{Bmatrix} 2 \\ 0 \end{Bmatrix}$$

Exercise N° 8

We define the optimization problem as:

$$Minimize\ f(x_1, x_2) = 2x_1 + \beta x_2$$

$$Subjected\ to:$$

$$\begin{cases} g_1(x_1, x_2) = {x_1}^2 + {x_2}^2 - 5 \leq 0 \\ g_2(x_1, x_2) = x_1 - x_2 - 2 \leq 0 \end{cases}$$

1. Use the optimality condition and determine the value(s) of β for which the point ${x_1}^* = 1, {x_2}^* = 2$ is optimal to the given problem.