Series TD 1 : Introduction

Exercise N° 1

Consider the following matrices:

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 2 & -4 \\ 2 & 4 & -2 \\ -4 & -2 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{bmatrix}$$

Using the method of eigenvalues, for each of the matrices, determine whether it is positive definite, negative definite or indefinite.

Exercise N° 2

Consider the following matrices:

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 2 & -4 \\ 2 & 4 & -2 \\ -4 & -2 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{bmatrix}$$

Using the method of principal minors, for each of the matrices, determine whether it is positive definite, negative definite or indefinite.

Exercise N° 3

For each of the following objective functions, calculate its Hessian matrix and indicate whether it is positive definite, negative definite or indefinite.

- **(a)** $f(x_1, x_2) = {x_1}^2 - {x_2}^2$
- **(b)** $f(x_1, x_2) = 4x_1x_2$
- **(c)** $f(x_1, x_2) = {x_1}^2 + 2{x_2}^2$
- **(d)** $f(x_1, x_2) = -{x_1}^2 + 4x_1x_2 + 4{x_2}^2$
- **(e)** $f(x_1, x_2, x_3) = -{x_1}^2 + 4x_1x_2 - 9{x_2}^2 + 2x_1x_3 + 8x_2x_3 - 4{x_3}^2$

Exercise N° 4

Study the convexity, concavity or none for each of the functions below:

- **(a)** $f(x) = -2x^2 + 8x + 4$
- **(b)** $f(x) = x^2 + 10x + 1$
- **(c)** $f(x_1, x_2) = {x_1}^2 - {x_2}^2$
- **(d)** $f(x_1, x_2) = -{x_1}^2 + 4x_1x_2$
- **(e)** $f(x) = e^{-x}, \quad x > 0$
- **(f)** $f(x) = \sqrt{x}, \quad x > 0$
- **(g)** $f(x_1, x_2) = x_1x_2$
- **(h)** $f(x_1, x_2) = (x_1 - 1)^2 + 10(x_2 - 2)^2$

Exercise N° 5

Match each of the following objective function with the corresponding optimal characteristics.

(a) $f(x_1, x_2) = 4x_1 - 3x_2 + 2$	Local maximum at the Point (1, 2).
(b) $f(x_1, x_2) = (2x_1 - 2)^2 + (x_2 - 2)^2$	Saddle Point at the origin.
(c) $f(x_1, x_2) = -(x_1 - 1)^2 - (x_2 - 2)^2$	No minimum point.
(d) $f(x_1, x_2) = x_1x_2$	Inflection Point at the origin.
(e) $f(x) = x^3$	Local minimum at the point (1, 2).