CHAPTER: MAGNETOSTATICS

- Magnetic Force and Lorentz law
    - Magnetic Force on a Current-Carrying Wire
    - Motion of Charged Particles in a Magnetic Field
- Biot-Savart Law
- Ampere’s Law

Introduction: In electrostatics, electric fields constant in time are produced by stationary charges.
In magnetostatics magnetic fields constant in time are produced by steady currents.
We have seen that a charged object produces an electric field $\vec{E}$ at all points in space.
In a similar manner, a bar magnet is a source of a magnetic field $\vec{B}$.

	Electric Field	**Magnetic Field**
Nature	Created around electric charge	Created around moving electric charge (Current-carrying conductor) and magnets
Units	▪ Newton per coulomb ▪ volts per meter	Gauss or Tesla $1 \text{ tesla} = 1\text{ T} = 1\text{ N/A}\cdot\text{m}$ (Volt . second) per $(\text{meter})^2$
Force	▪ Proportional to the electric charge ▪ Direction along E lines	Proportional to charge and speed of electric charge Direction: by the right hand rule
Work	Electric force does work in displacing a charged particle	No work because F $\perp$ displacement (point of application)
Movement In B field	Perpendicular to the magnetic field	Perpendicular to the electric field
Pole	Monopole or Dipole	Dipole

The Force on a Charge Moving in a Magnetic Field

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$\begin{aligned}\vec{\mathbf{v}} \times \vec{\mathbf{B}} &= (v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}) \\ &= (v_yB_z - v_zB_y)\hat{\mathbf{i}} + (v_zB_x - v_xB_z)\hat{\mathbf{j}} \\ &\quad + (v_xB_y - v_yB_x)\hat{\mathbf{k}}\end{aligned}$$

$$\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}},\ \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

$\vec{\mathbf{F}}_B$: The magnetic force

q: a particle of charge

v: The particle's speed

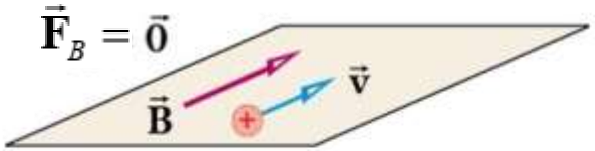
$\vec{\mathbf{B}}$: The magnetic field, the ***gauss*** is non-SI unit

- The electric force is along the direction of the electric field, the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, the magnetic force does no work when a particle is displaced.
- **magnetic force cannot speed up or slow down a charged particle.**

The magnitude of $\vec{\mathbf{F}}_B$ is given by

$$F_B = |q|vB\sin\theta$$

$\theta \rightarrow$ angle between $\bar{v}$ and $\bar{B}$



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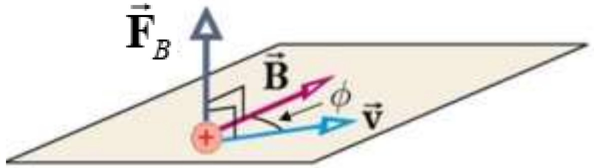

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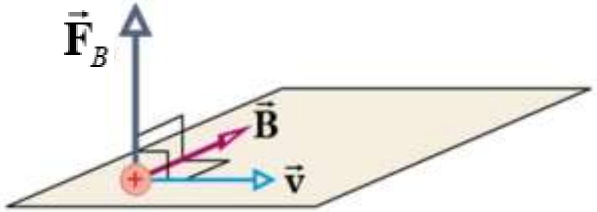

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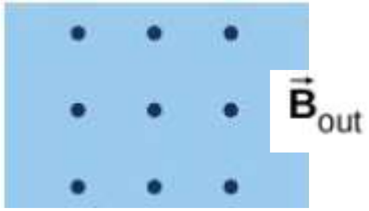
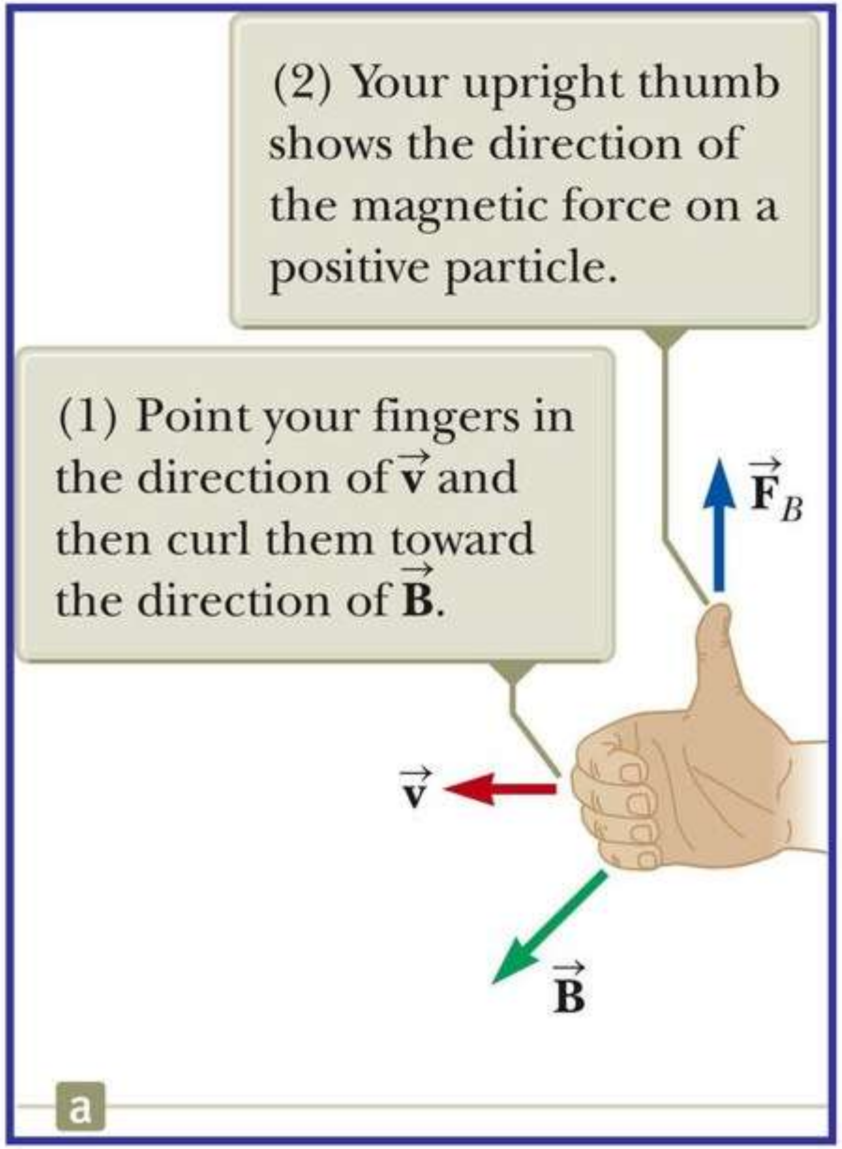
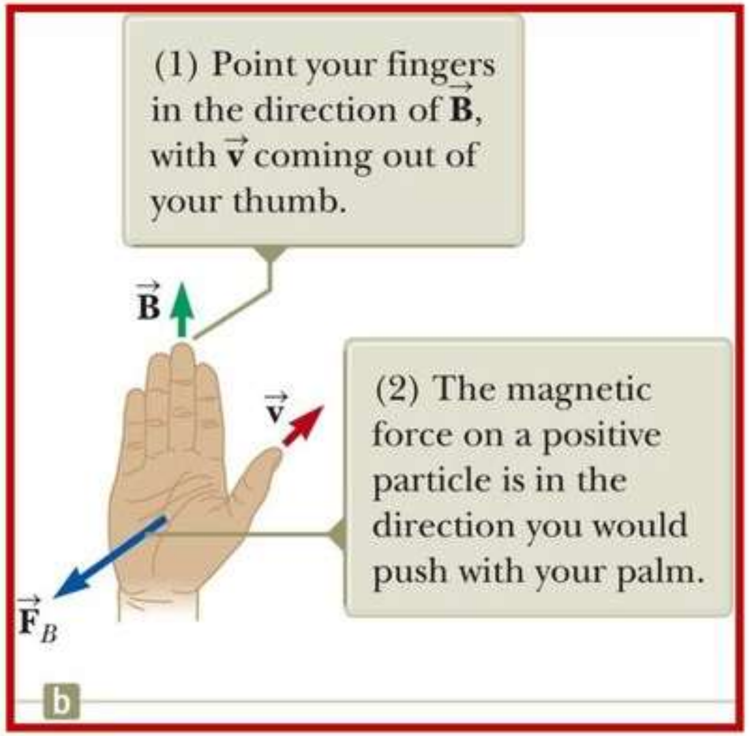


Right-hand rule determine the direction of magnetic force.
So the magnetic force is always perpendicular to **v** and **B**.

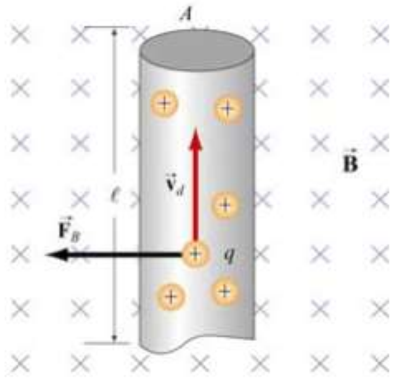
- This rule is based on the right-hand rule for the cross product.
- Your thumb is in the direction of the force if **q** is positive.
- The force is in the opposite direction of your thumb if **q** is negative.



- The force on a positive charge extends outward from the palm.
- The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand.
- The force on a negative charge is in the opposite direction.



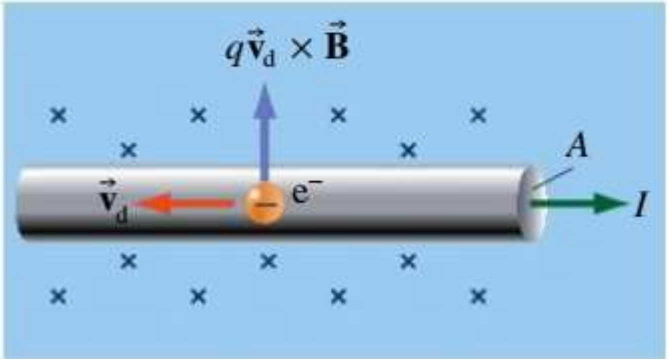
Magnetic Force on a Current-Carrying Wire



Magnetic force on a conducting wire

Consider a conductor of length 'ℓ', carrying a steady current 'I', kept inside a uniform magnetic field ***B*** at right angle to it.
If 'A' is the cross-sectional area of the conductor and, 'n' = number of charges per unit volume) and, e = charge of electron,
v_d = drift velocity of electron ,
the Total force on conductor= Total No. of electrons × force on each electron

$$F_B = (nA\,\ell)ev_d B \qquad I = nAev_d$$



$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$ For a straight conductor in a uniform magnetic field

$$\vec{F}_B = NI\vec{\ell} \times \vec{B}$$ For N parallel conductors flowing through the same current

$$F_B = I\ell B \sin\theta$$ The magnitude of the force is given by the familiar equation

$\theta \rightarrow$ angle between $\overline{\ell}$ and $\overline{B}$

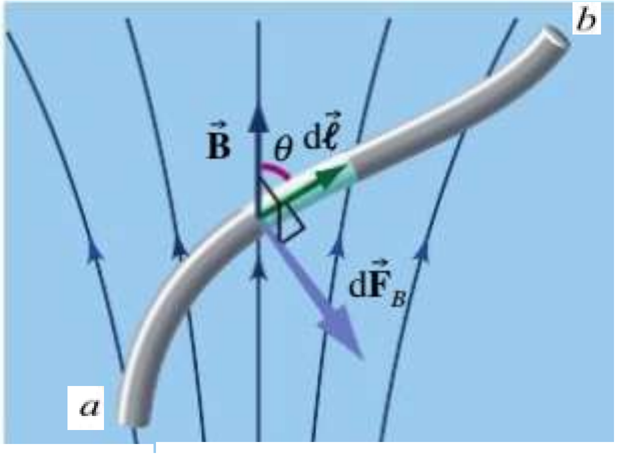


Figure A curved wire carrying a current I.

$$d\vec{F}_B = I\,d\vec{\ell} \times \vec{B}$$

$$\vec{F}_B = \int d\vec{F}_B$$

$$= I\int_a^b d\vec{\ell} \times \vec{B}$$

For a wire of arbitrary shape, the magnetic force can be obtained by summing over the forces acting on the small segments that make up the wire

$$F_{B_x} = \int dF_{B_x} \qquad F_{B_y} = \int dF_{B_y} \qquad F_{B_z} = \int dF_{B_z}$$

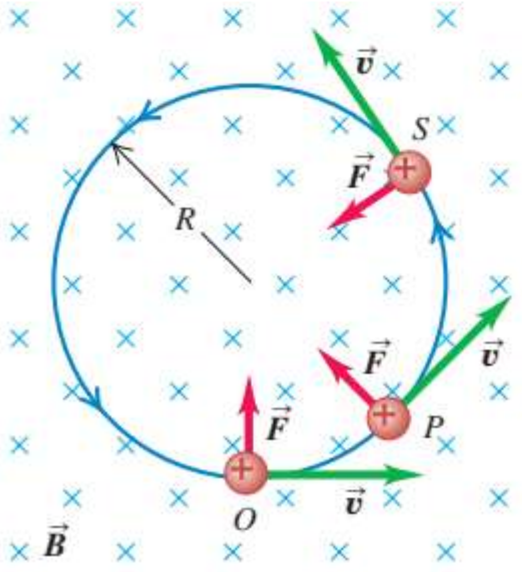
Lorentz force law

The force on a moving particle arising from combined electric and magnetic fields is

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

Motion of Charged Particles in a Magnetic Field

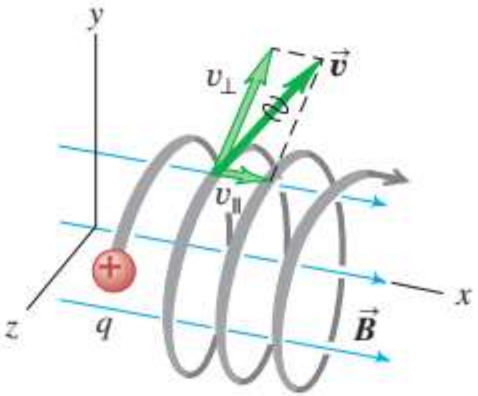
Figure shows a simple example. A particle with positive charge q is at point O, moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ directed into the plane of the figure. The vectors $\vec{v}$ and $\vec{B}$ are perpendicular, so the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ has magnitude $F = qvB$ and a direction as shown in the figure. The force is *always* perpendicular to $\vec{v}$, so it cannot change the *magnitude* of the velocity, only its direction. To put it differently, the magnetic force never has a component parallel to the particle's motion, so the magnetic force can never do *work* on the particle. This is true even if the magnetic field is not uniform.



A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.

Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

particle moving in a uniform magnetic field $\vec{B}$. The magnetic field does no work on the particle, so its speed and kinetic energy remain constant.



This particle's motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.

If the direction of the initial velocity is *not* perpendicular to the field, the velocity *component* parallel to the field is constant because there is no force parallel to the field. Then the particle moves in a helix

MAGNETIC FIELD LINES

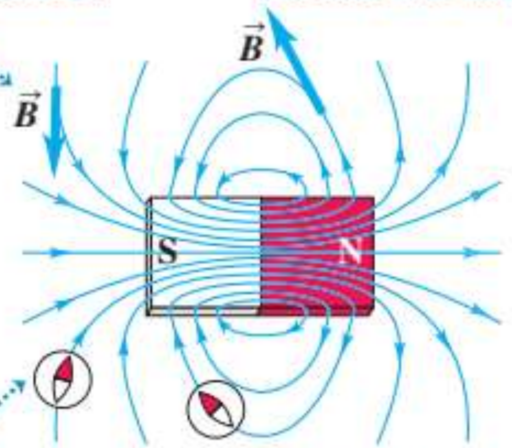
(1) Magnetic field can be represented by magnetic field lines, the direction of field lines representing the directions of the magnetic field.

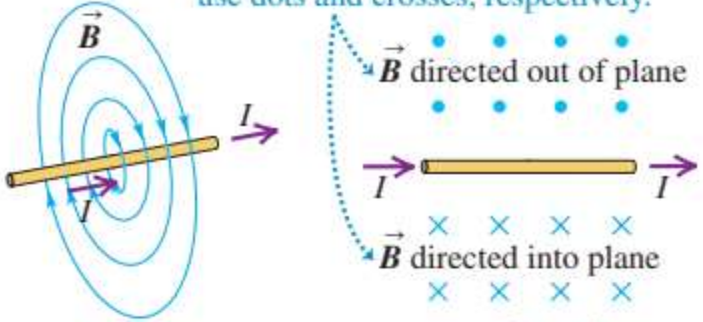
(2) However, the magnetic field lines do not represent the direction of force on a moving charge inside the magnetic field. This is because the magnetic force acting on a moving charge inside a magnetic field is at right angles to the direction of the magnetic field.

(3) The tangent at any point on the magnetic field lines represent the direction of the magnetic field. Consequently, two magnetic field lines do not intersect. If they intersect, then at the point of intersection, they will have two tangents, indicating two directions, which is not possible for a vector quantity.

(4) The magnetic field lines linked with a closed surface are continuous closed curves.

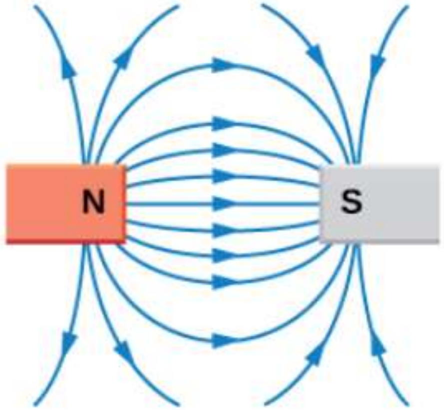
(5) The magnetic field lines are concentrated in a region of stronger field and spread out in a region of weaker field. In a region of uniform magnetic field, the magnetic field lines are parallel to each other and equally spaced.



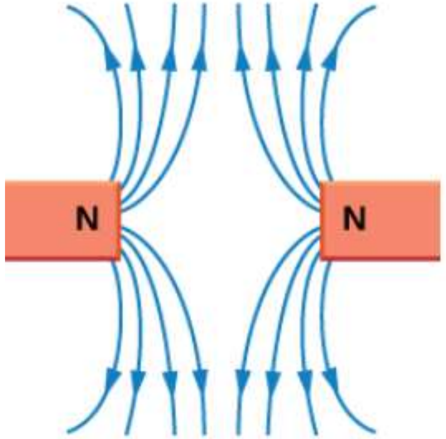


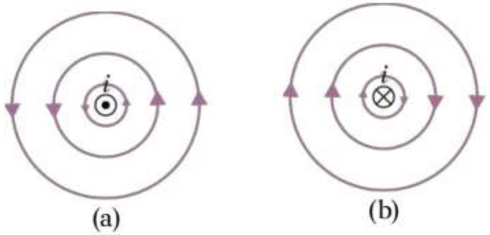
Perspective view

Wire in plane of paper



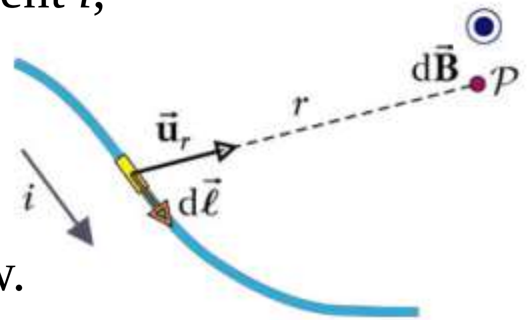
Magnetic field lines between like and unlike poles





❑ Biot-Savart Law:

When charges move in a conducting wire and produce a current i, the magnetic field at any point P due to the current can be calculated by adding up the magnetic field contributions $\mathrm{d}\vec{\mathbf{B}}$ from small segments of the wire $\mathrm{d}\vec{\ell}$



$$\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi}\frac{\mathrm{d}\vec{\ell}\times\vec{\mathbf{u}}_r}{r^2}$$

These segments be pointing in the direction of the current flow. The infinitesimal current source can then be written as $i\,\mathrm{d}\vec{\ell}$

$$\vec{\mathbf{u}}_r = \frac{\vec{\mathbf{r}}}{r} = \frac{\Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}$$

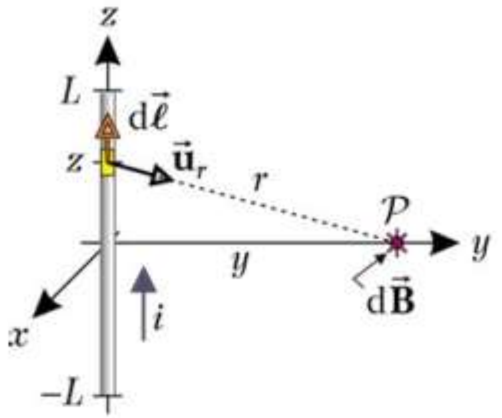
$$\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A}$$

where μ_0 is a constant called the *permeability of free space*:

$$\vec{\mathbf{B}} = \int_{\text{wire}} d\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi}\int_{\text{wire}}\frac{\mathrm{d}\vec{\ell}\times\vec{\mathbf{u}}_r}{r^2}$$

the magnetic field at the point P requires integrating over the current source

- **Magnetic Field due to a Finite Straight Wire**



$$\mathrm{d}\vec{\ell} = \mathrm{d}z\,\vec{k}$$

Consider a differential element , carrying current *i* in the *z*-direction.

$$\vec{\mathbf{u}}_r = \frac{\vec{\mathbf{r}}}{r} = \frac{y\,\vec{j} - z\,\vec{k}}{\sqrt{y^2+z^2}}\,.$$

Is the corresponding unit vector

$$\mathrm{d}\vec{\ell}\times\vec{\mathbf{u}}_r = \frac{\mathrm{d}z\;y\,(\vec{k}\times\vec{j}) - \mathrm{d}z\;z\,(\vec{k}\times\vec{k})}{\sqrt{y^2+z^2}} = \frac{-\vec{i}y\,\mathrm{d}z}{\sqrt{y^2+z^2}}$$

Is the cross product

The differential contribution to the magnetic field is obtained as

$$d\vec{\mathbf{B}} = \frac{\mu_0 i}{4\pi}\frac{(-\vec{\imath})y\,\mathrm{d}z}{\sqrt{y^2+z^2}}\left(\frac{1}{\sqrt{y^2+z^2}}\right)^2 = \frac{\mu_0 i}{4\pi}\frac{(-\vec{\imath})y\,\mathrm{d}z}{(y^2+z^2)^{3/2}}.$$

Integrating from $-L$ to L :

$$\vec{\mathbf{B}} = \int_{-L}^{L}\frac{\mu_0 i}{4\pi}\frac{(-\vec{\imath})y\,\mathrm{d}z}{(y^2+z^2)^{3/2}} = -\frac{\mu_0 i y}{4\pi}\vec{\imath}\int_{-L}^{L}\frac{\mathrm{d}z}{(y^2+z^2)^{3/2}}.$$

we obtain

$$\vec{\mathbf{B}} = -\frac{\mu_0 i y}{4\pi}\vec{\imath}\left[\frac{z}{y^2\sqrt{y^2+z^2}}\right]\Bigg|_{-L}^{L} = -\frac{\mu_0 i}{4\pi y}\left[\frac{L}{\sqrt{y^2+L^2}}-\frac{(-L)}{\sqrt{y^2+L^2}}\right]\vec{\imath}$$

$$\vec{\mathbf{B}} = -\frac{\mu_0 i}{2\pi y}\frac{L}{\sqrt{y^2+L^2}}\vec{\imath}.$$

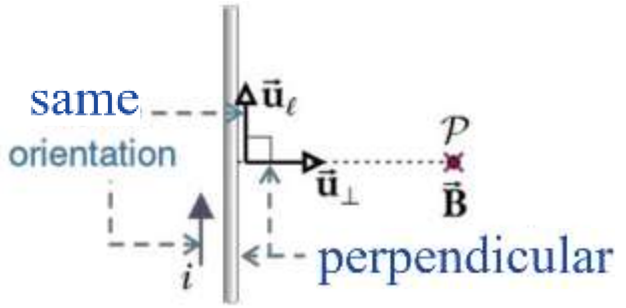
by choosing y = r >> L The magnetic field becomes :

$$B \approx \frac{\mu_0 i}{2\pi r}\frac{L}{\sqrt{r^2}} = \frac{\mu_0 i L}{2\pi r^2}. \qquad (r \gg L)$$

For infinite length:

$$B \approx \frac{\mu_0 i L}{2\pi y L} = \frac{\mu_0 i}{2\pi y} \qquad y = r \ll L$$

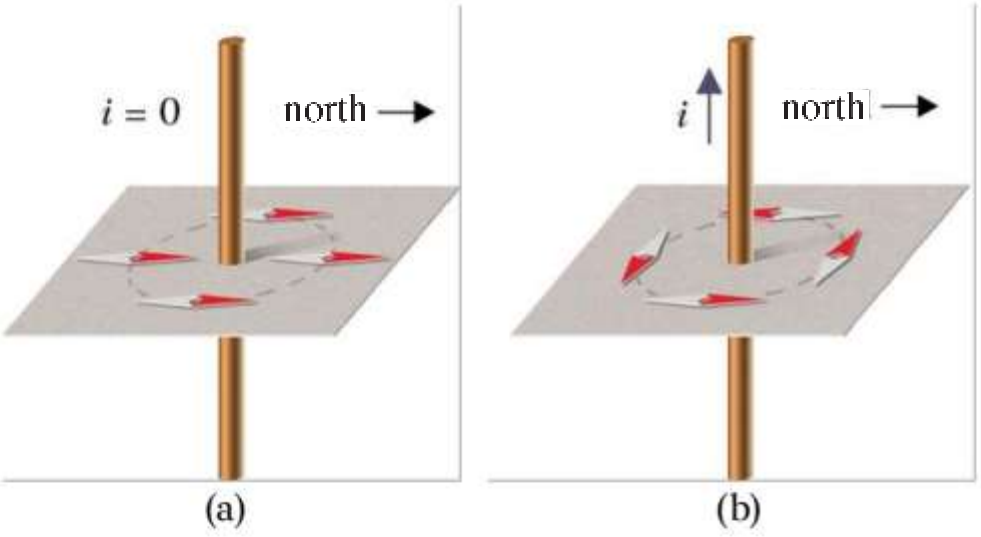
$$\boxed{\vec{\mathbf{B}} = \frac{\mu_0 i}{2\pi r}\vec{\mathbf{u}}_\ell \times \vec{\mathbf{u}}_\perp}$$



Unit vectors for the magnetic field of an infinite wire

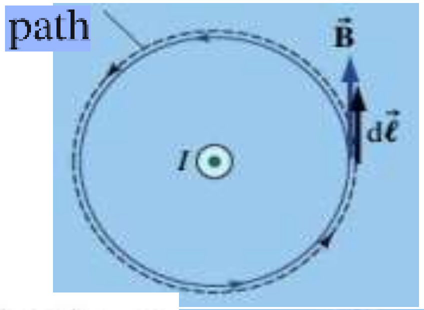
❑ Ampere’s Law:

By placing compass needles near a wire. As shown in Figure a, all compass needles point in the same direction in the absence of current. However, when $i \neq 0$, the needles will be deflected along the tangential direction of the circular path.

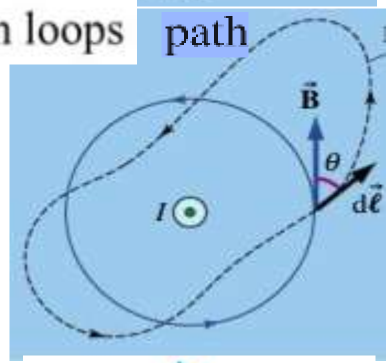


Deflection of compass needles near a current-carrying wire

Let us now divide a circular path of radius r into a large number of small length vectors $d\vec{\ell}$,
by choosing a closed path,
or an “Amperian loop” that follows one particular magnetic field line.
We Obtain:



Amperian loops



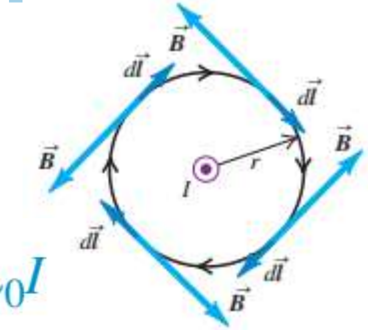
$$\cos\theta = 1 \qquad \vec{\mathbf{B}}\cdot d\vec{\ell} = B\,d\ell$$

$$\oint \vec{\mathbf{B}}\cdot d\vec{\ell} = B\oint d\ell = B(2\pi r)$$

$$= \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r) = \mu_0 I$$

The generalization to any closed loop of arbitrary shape that involves many magnetic field lines is known as Ampere’s law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$



$$\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$$

Ampere’s law in magnetism is analogous to Gauss’s law in electrostatics.
In order to apply them, the system must possess certain symmetry.