Chapter III: Electrokinetic

Course 1

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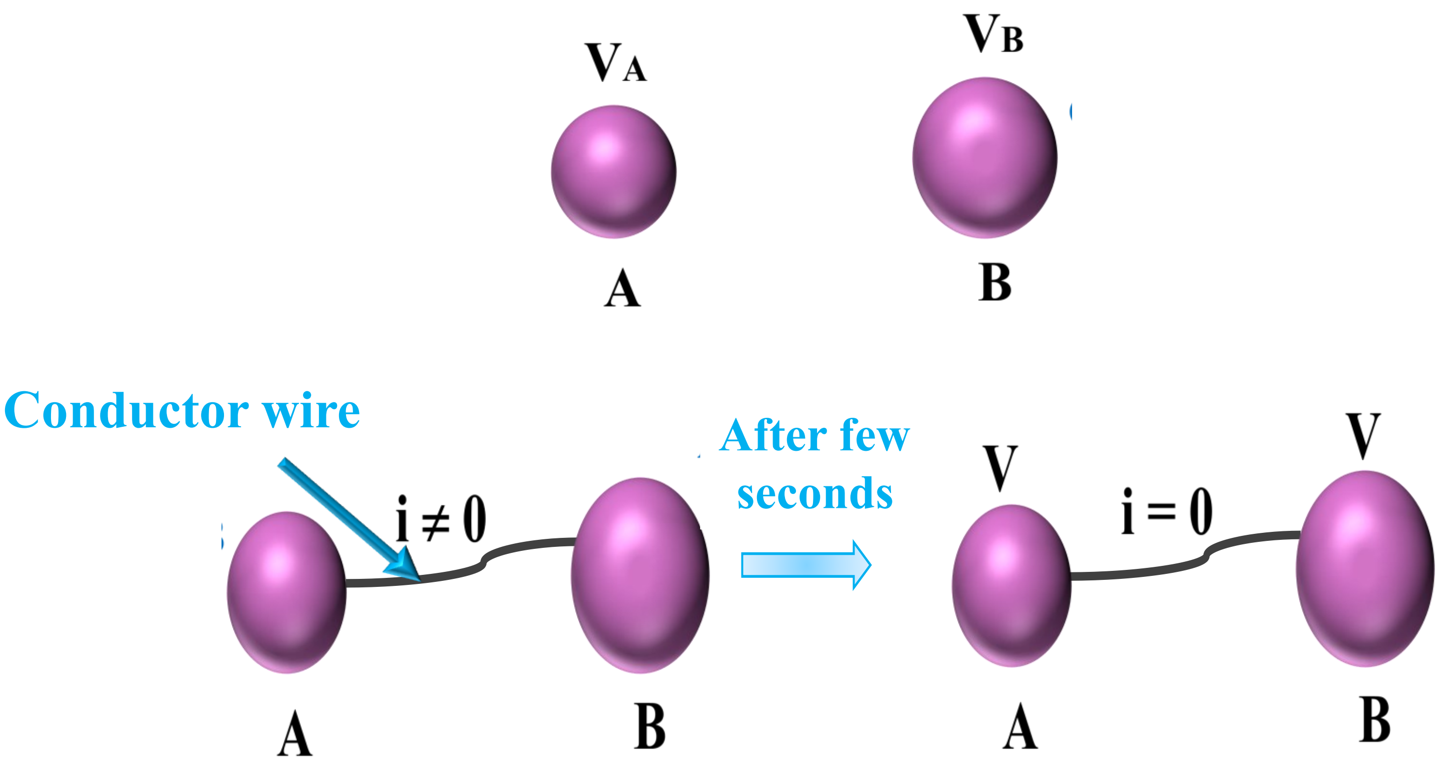
Introduction

Electrokinetic studies electric charges in motion within conductors, giving rise to electric current. Unlike electrostatics, where charges are stationary, here we explore the movement of charges in metallic conductors, covering fundamental laws like Ohm's law and Kirchhoff's laws, and concepts of voltage, current, and resistance.

Electric current

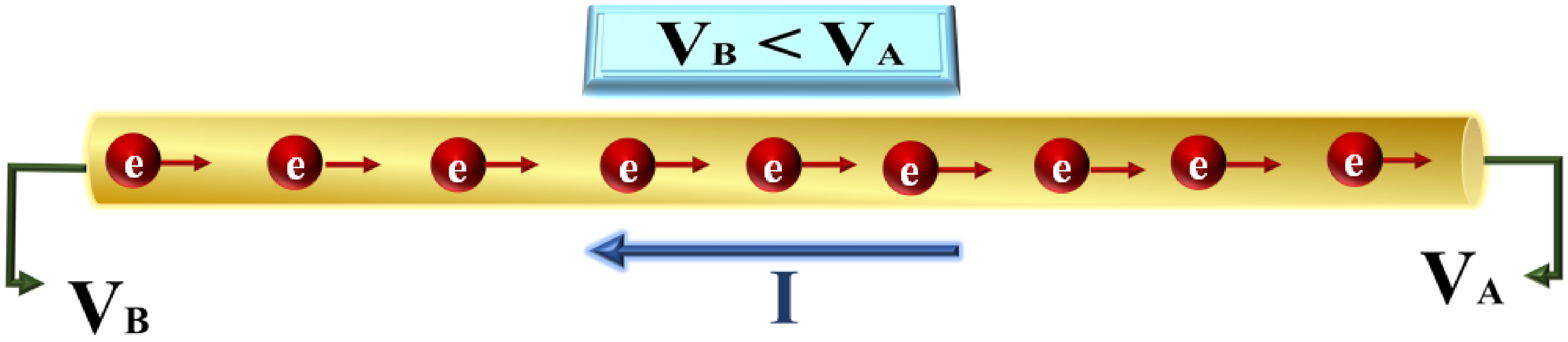
Let A and B be two charged conductors with potentials V_A and V_B, where $V_A > V_B$. When the two conductors are connected by a conducting wire, they form a single conductor.



Definition:

Electric current is a collective motion of charge carriers. In metals, it is the movement of electrons that causes electric current.

Electric current appears in a conductor when a potential difference is established between its terminals.



By convention, the direction of current is considered opposite to the movement of electrons (e). Therefore, electric current I flows in the direction of decreasing potentials

Electric current intensity (current):

The electrical current I is the rate at which charge flows, it is the quantity of electricity (charge) that crosses a section S of the conductor per unit of time.

$I = \frac{dQ}{dt}$, with dQ representing the electric charge crossing section S in a time interval dt.

Case of direct current, or DC,

Direct current is an electric current in which the motion of charges is nearly continuous within a conductor, flowing consistently in one direction. Its intensity remains constant over time.

$I = \frac{\Delta Q}{\Delta t}$, with ΔQ representing the electric charge crossing section S in a time interval dt.

In the International System of Units (SI), electric current is measured in Amperes (**A**)

Example

Consider a charge moving through a cross-section of a wire where the charge is modeled as $Q(t) = Q_M\left(1 - e^{-t/\tau}\right)$. Here, Q_M is the charge after a long period of time, as time approaches infinity, with units of coulombs, and τ is a time constant with units of seconds (see Figure A). What is the current through the wire?

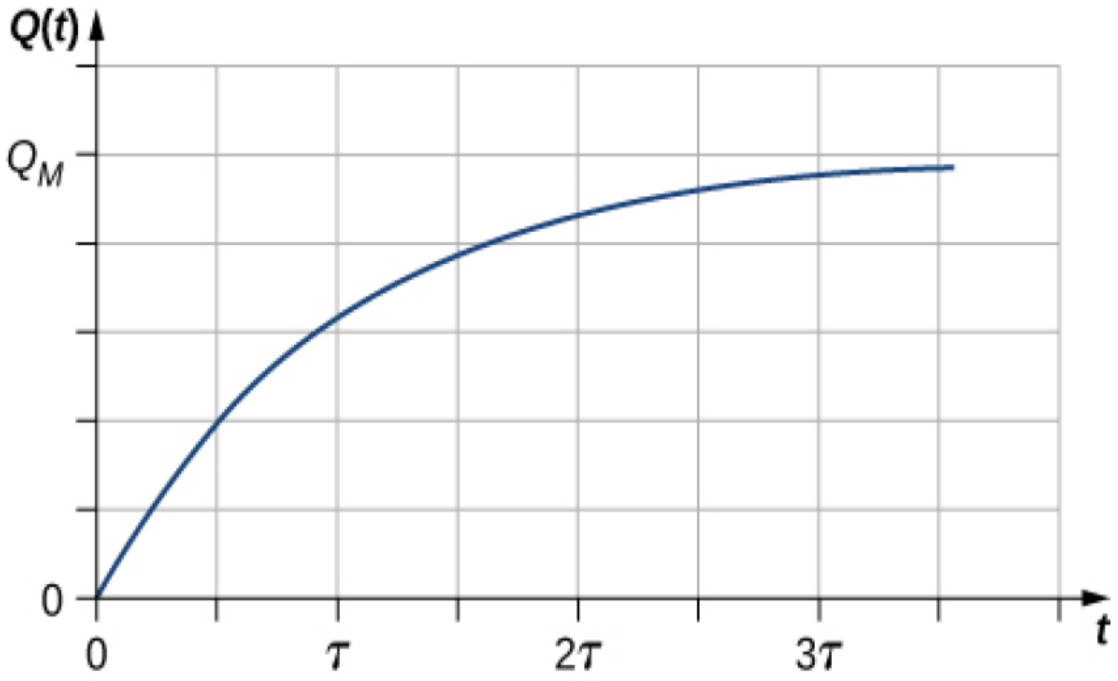


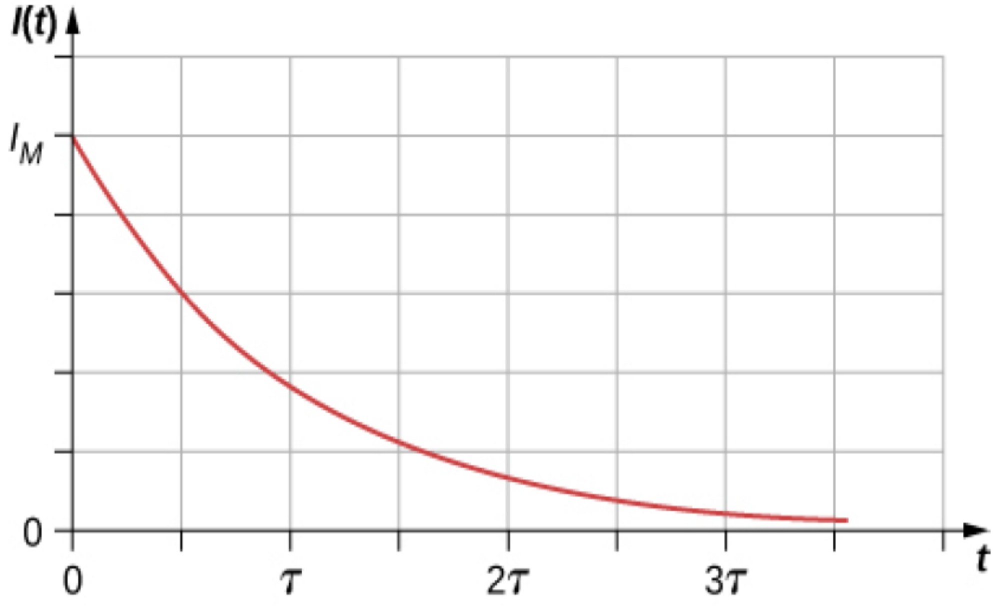
Figure A: A graph of the charge moving through a cross-section of a wire over time

Solution

We know that $$I = \frac{dQ}{dt}$$

The derivative can be found using $\frac{d}{dx}e^u = e^u \frac{du}{dx}$.

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[Q_M\left(1 - e^{-t/\tau}\right)\right] = \frac{Q_M}{\tau}e^{-t/\tau}.$$



Example:

The main purpose of a battery in a car or truck is to run the electric starter motor, which starts the engine. The operation of starting the vehicle requires a large current to be supplied by the battery. Once the engine starts, a device called an alternator takes over supplying the electric power required for running the vehicle and for charging the battery.

a- What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine?

b- How long does it take 1.00 C of charge to flow from the battery?

Solution

a- Entering the given values for charge and time into the definition of current gives

$$I = \frac{\Delta Q}{\Delta t} = \frac{720\ \mathrm{C}}{4.00\ \mathrm{s}} = 180\ \mathrm{C/s} = 180\ \mathrm{A}.$$

b- Solving the relationship for time and entering the known values for charge and current gives

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\ \mathrm{C}}{180\ \mathrm{C/s}} = 5.56 \times 10^{-3}\mathrm{s} = 5.56\ \mathrm{ms}.$$

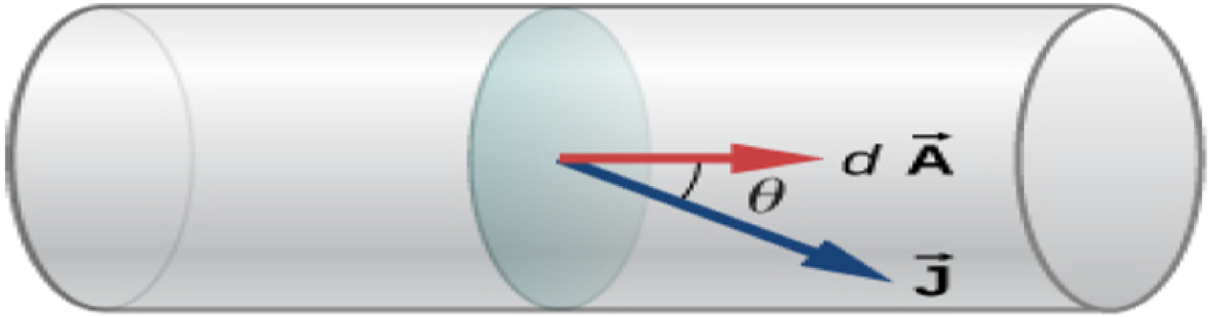
The substantial current required by starter motors demonstrates the swift movement of a considerable charge within a brief period. These starter motors necessitate high currents to overcome the engine's inertia, enabling the engine to start promptly. This significant current is essential to deliver the substantial energy required for engine ignition.

Current density

The **current density** is the flow of charge through an infinitesimal area, divided by the area. The current density must take into account the local magnitude and direction of the charge flow, which varies from point to point.

$$J = \frac{\mathrm{d}I}{dA} \quad \Rightarrow \quad I = \iint_{\text{Area}} \vec{J}.\mathrm{d}\vec{A}$$

Current density $\vec{J}$ describes the amount of charge per unit time and area. It provides a description of local current. The vector $\vec{J}$ describes the density of electric charge flux through a given surface, as shown in Figure III.3.



Other expression of the current density $\vec{J}$ is defined as: $\vec{J} = \rho \vec{v}$

with: ρ is the charge density, $\rho = n\,e$, where n is the number of charges per unit volume and e is the charge of the electron ($e=1,6\ 10^{-19}\ C$).

$\vec{v}$ is the average velocity of all the free electrons in the conductor is called **the drift velocity** of free electrons of the conductor.

Its unit in the International System is ampere per square meter ($\boldsymbol{A.m^{-2}}$).

Example 1:

The current supplied to a lamp with a 100-W light bulb is 0.87 amps. The lamp is wired using a copper wire with diameter 2.588 mm.

- Find the magnitude of the current density.

Solution:

Calculate the current density:

$$J = \frac{I}{A} = \frac{0.87\text{ A}}{5.26 \times 10^{-6}\text{m}^2} = 1.65 \times 10^5 \frac{\text{A}}{\text{m}^2}.$$

The current density in a conducting wire depends on the current through the conducting wire and the cross-sectional area of the wire. For a given current, as the diameter of the wire increases, the charge density decreases.

Example 2:

Calculate the drift velocity of electrons in a copper wire with a diameter of 2.053 mm carrying a 20 A current, given that there is one free electron per copper atom. The density of copper is and the atomic mass of copper is 63.54 g/mol.

Solution:

First, we calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, the number of free electrons is the same as the number of copper atoms per m^3. We can now find n as follows:

$$n = \frac{1\ e^-}{\text{atom}} \times \frac{6.02 \times 10^{23}\,\text{atoms}}{\text{mol}} \times \frac{1\ \text{mol}}{63.54\ \text{g}} \times \frac{1000\ \text{g}}{\text{kg}} \times \frac{8.80 \times 10^{3}\,\text{kg}}{1\ \text{m}^3}$$
$$= 8.34 \times 10^{28}\,e^-/\text{m}^3.$$

The cross-sectional area of the wire is:

$$A = \pi r^2 = \pi\left(\frac{2.05 \times 10^{-3}\,\text{m}}{2}\right)^2 = 3.30 \times 10^{-6}\,\text{m}^2$$

Then the drift velocity:

$$v_d = \frac{I}{nqA} = \frac{20.00\ \text{A}}{(8.34 \times 10^{28}/\text{m}^3)(1.60 \times 10^{-19}\,\text{C})(3.30 \times 10^{-6}\,\text{m}^2)} = 4.54 \times 10^{-4}\ \text{m/s}.$$

Relationship between the electric field and the electric current density

In most conductors, there exists a proportionality between the current density and the local electrostatic field, as defined by Ohm's law. Therefore, we can express it as:

$$\vec{J} = \sigma \vec{E}$$

The proportionality coefficient **σ** is called the conductivity of the medium, which depends solely on the material's properties. It is expressed in ($\Omega^{-1} m^{-1}$).

Conductivity is an intrinsic property of a material. Another intrinsic property of a material is the **resistivity**, or electrical resistivity. The resistivity of a material is a measure of how strongly a material opposes the flow of electrical current. The symbol for resistivity is the lowercase Greek letter rho, **ρ** , and resistivity is the reciprocal of electrical conductivity: $\rho = \dfrac{1}{\sigma}$

The unit of resistivity in SI units is the ohm-meter ($\Omega\, m$)

We can define the resistivity in terms of the electrical field and the current density,

$$\vec{J} = \frac{1}{\rho}\vec{E}$$

Example

Material	Conductivity, σ $(\Omega \cdot m)^{-1}$	Resistivity, ρ $(\Omega \cdot m)$
Conductors		
Silver	6.29×10^{7}	1.59×10^{-8}
Copper	5.95×10^{7}	1.68×10^{-8}
Gold	4.10×10^{7}	2.44×10^{-8}
Aluminum	3.77×10^{7}	2.65×10^{-8}
Tungsten	1.79×10^{7}	5.60×10^{-8}
Iron	1.03×10^{7}	9.71×10^{-8}
Platinum	0.94×10^{7}	10.60×10^{-8}
Steel	0.50×10^{7}	20.00×10^{-8}
Lead	0.45×10^{7}	22.00×10^{-8}

From the two relations of current density $\vec{J}$, we can write the relationship between the average velocity of electrons and the electric field:

$$\vec{v} = \frac{\sigma}{ne}\vec{E}$$

Where the ratio $\frac{\sigma}{ne} = \mu$, μ is the mobility of charges, expressed in ($m^2/V.\ s$).

Ohm law on a macroscopic scale

The experiment showed that the voltage across a metallic conductor, at constant temperature, is proportional to the current passing through it. This proportionality coefficient is called resistance, denoted by R.

$$R = \frac{V_{AB}}{I} = \frac{V_A - V_B}{I}$$

Resistance is the opposition encountered by electric current in a material due to collisions between electrons and atoms.

It's measured in ohms (Ω) and depends on factors like material type and temperature.

American National
Standards Institute (ANSI)

(a)

International Electrotechnical
Commission (IEC)

(b)

Ohm law on a microscopic scale

On a microscopic scale, Ohm's law relates the current density (J) to the electric field (*E*) and conductivity (σ) of a material through the equation:

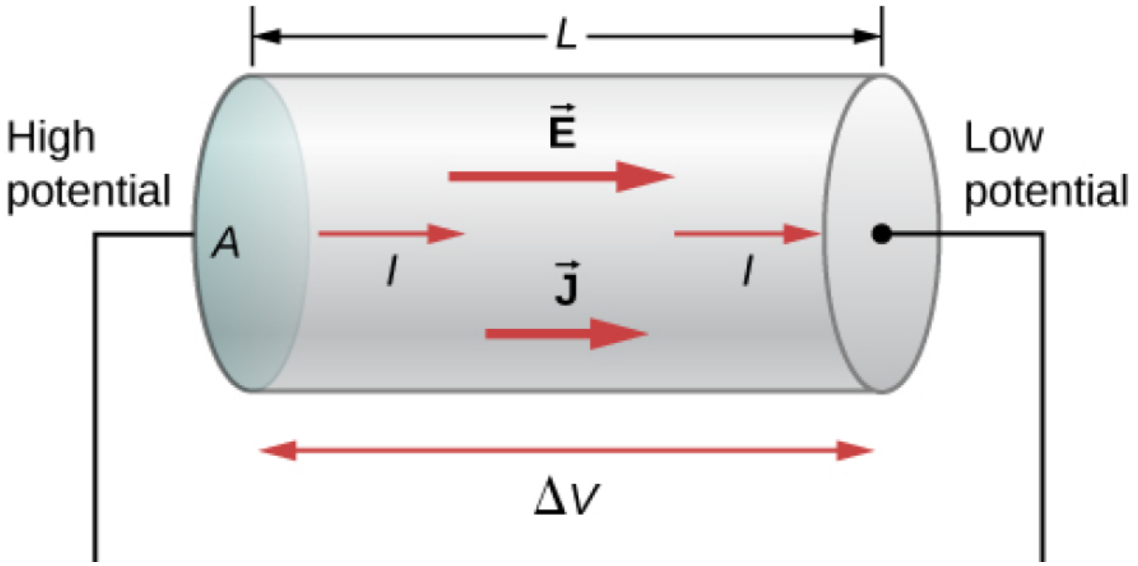
$$\vec{J} = \sigma \vec{E}$$

The resistance of a cylindrical segment

$$J = \sigma E \Rightarrow \frac{I}{S} = \sigma \frac{V_{AB}}{L} \Rightarrow \frac{V_{AB}}{I} = \frac{L}{S\sigma}$$

$$\Rightarrow R = \frac{L}{S\sigma}$$

$$\Rightarrow R = \rho \frac{L}{S}$$



Example:

Calculate the current density, resistance, and electrical field of a 5-m length of copper wire with a diameter of 2.053 mm carrying a current of I=10 mA.

the resistivity of copper $\rho = 1.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}$

Solution

First, we calculate the current density:

$$J = \frac{I}{A} = \frac{10 \times 10^{-3}\,\mathrm{A}}{3.31 \times 10^{-6}\,\mathrm{m}^2} = 3.02 \times 10^{3}\,\frac{\mathrm{A}}{\mathrm{m}^2}.$$

The resistance of the wire is

$$R = \rho\frac{L}{A} = \left(1.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}\right)\frac{5.00\ \mathrm{m}}{3.31 \times 10^{-6}\,\mathrm{m}^2} = 0.025\,\Omega\,.$$

Finally, we can find the electrical field:

$$E = \rho J = 1.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}\left(3.02 \times 10^{3}\,\frac{\mathrm{A}}{\mathrm{m}^2}\right) = 5.07 \times 10^{-5}\,\frac{\mathrm{V}}{\mathrm{m}}.$$

The Joule effect, also known as Joule heating or resistive heating

When electrons (electric current) flow through a conducting material, they cause the atoms to vibrate, resulting in an increase in the temperature of the conductor. This phenomenon is known as the Joule effect. Therefore, the flow of current (I) through an electrical conductor leads to energy loss, which manifests as heating.

This phenomenon is named after **James Prescott Joule**, who first observed and studied it in the 19th century.

Power in Electric Circuit

Power in electric circuits refers to the rate at which energy is transferred or converted within the circuit. It can be calculated using various formulas depending on the type of circuit and components involved.

The electric power gained or lost by any device has the form

$$P = IV.$$

Joule's Law

The power dissipated by a resistor I

Joule's Law, also known as the Joule-Lenz Law, states that the heat produced in a conductor by the passage of an electric current is directly proportional to the resistance of the conductor, the square of the current, and the time for which the current flows. Mathematically, it can be expressed as:

$$W = R \times I^2 \times t$$

Thus, the dissipated energy depends on three factors:

- ✓ The resistance (R) of the material.
- ✓ The intensity (I) of the current flowing through the material.
- ✓ The time (t) during which the current flows through the material.

Disadvantages of the Joule effect:

- The Joule effect sometimes explains why electronic components "burn out."
- It can even account for the triggering of certain fires: when a short circuit occurs, the current intensity increases sharply, leading to even higher heat generation, sometimes enough to initiate a fire.
- The Joule effect also represents energy loss when transporting electricity.

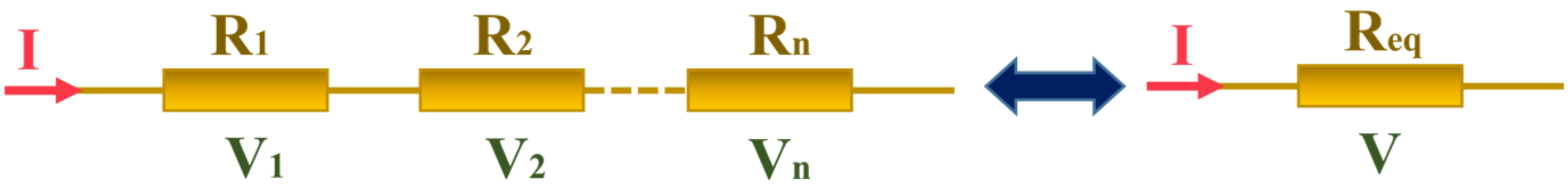
Advantages of the Joule effect:

- This phenomenon forms the basis for the operation of fuses. They trip due to heat and thus protect electrical circuits.
- The Joule effect can also be advantageous when seeking heating. It is thus exploited in electric radiators.

Resistor Configurations

1. Resistances in series:

- In this case, all resistances Ri are traversed by the same electric current I.



$$V = V_1 + V_2 + V_3 + \cdots\cdots + V_n \quad \Rightarrow \quad R_{eq} I = R_1 I + R_2 I + R_3 I + \cdots\cdots + R_n I$$

Thus, the equivalent resistance is obtained in the case of series grouping:

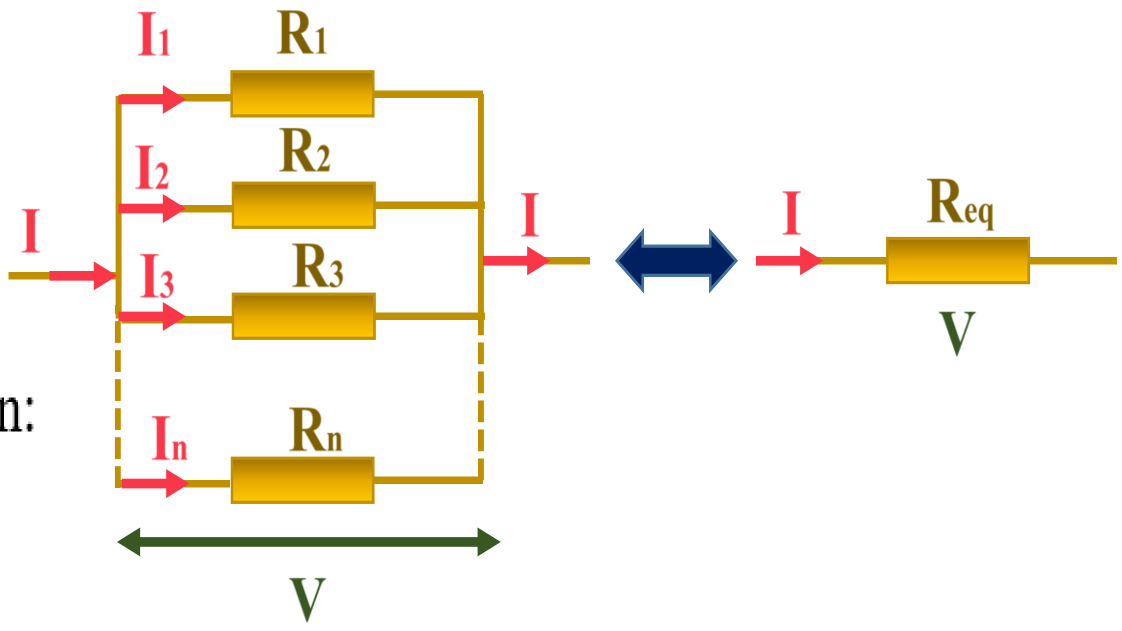
$$R_{eq} = R_1 + R_2 + R_3 + \cdots\cdots + R_n \quad \Rightarrow \quad R_{eq} = \sum_{1}^{n} \mathrm{R_i}$$

2. Resistances in Parallel:

In this case, the electric current I is shared among all branches of the resistances Ri.

$$I = I_1 + I_2 + I_3 + \cdots\cdots + I_n$$

with: $I = \dfrac{V}{R_{eq}}; I_1 = \dfrac{V}{R_1}; I_2 = \dfrac{V}{R_2}; \ldots\ldots..; I_n = \dfrac{V}{R_n}$



When we substitute into the previous expression, we obtain:

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots + \frac{V}{R_n}$$

So, we obtain the equivalent resistance in the case of parallel grouping:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \Rightarrow \quad \frac{1}{R_{eq}} = \sum_{1}^{n} \frac{1}{R_i}$$