Chapter II
Conductors in electrostatic equilibrium

Course 4

Dr IACHACHENE FARIDA FHC-UMBB-2023-2024

II.6.4 Energy Stored in a Capacitor

Electric energy of a capacitor refers to the stored energy within a capacitor due to the accumulation of electric charge on its plates when it is connected to a voltage source. This energy is stored in the electric field between the capacitor's plates and can be released when the capacitor is connected to a circuit. The energy stored in a capacitor is given by the formula:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

In this case, $V = \Delta V$ it refers to the potential difference (also known as voltage) between the two plates of the capacitor.

II.6.5 Association of Capacitors

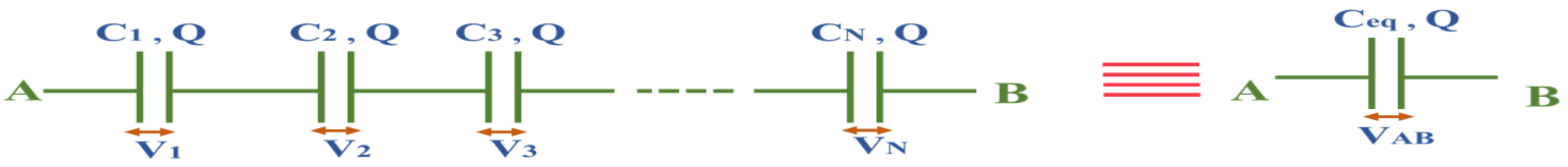
In practice, systems are found consisting of a set of capacitors connected in series or in parallel which can be substituted by a single capacitor and vice versa. The objective of grouping capacitors:

- To have a capacitor capable of withstanding high voltage differences.
- Or to have a capacitor with very large capacitance.

The characteristics of the equivalent capacitor can be determined as follows:

a- Series Combination

Let's consider N capacitors connected in series with capacitances C_1, C_2,... C_N, respectively. These can be replaced by an equivalent capacitor with capacitance C_{eq}. In series combination, all capacitors carry the same charge Q.



The potential difference (voltage) between A and B is given by: $V_{AB}=V_1+ V_2+ V_3+ .....+V_N$

All capacitors carry the same charge Q, which results in:

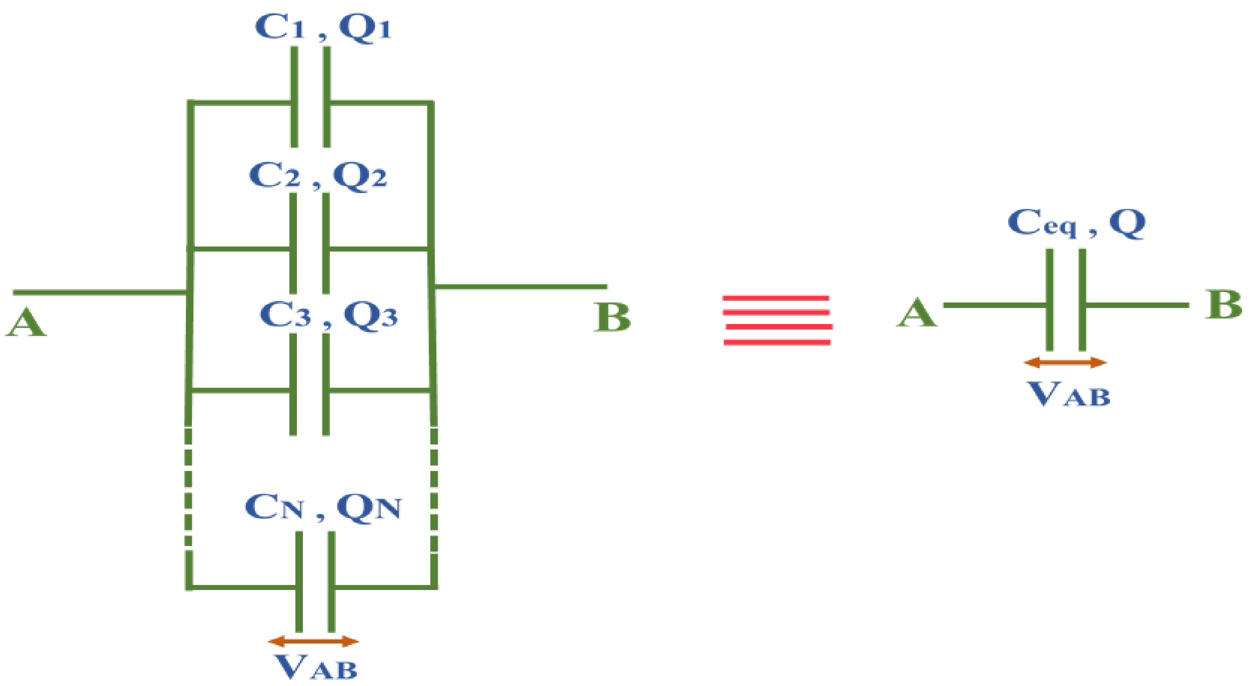
$$\frac{Q}{C_{eq}}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}+.......+\frac{Q}{C_N} \Rightarrow \frac{Q}{C_{eq}}=Q\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+.......+\frac{1}{C_N}\right)$$

$$\Rightarrow \frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+.......+\frac{1}{C_N}$$

Then, the equivalent capacitance C_{eq} of a system composed of N capacitors in series, each with capacitance C_i, is given by: $$\frac{1}{C_{eq}}=\sum_{i=1}^{N}\frac{1}{C_i}$$

b- Parallel Combination

When capacitors are connected in parallel, the voltage across each of them is equal.



In this case, the equivalent capacitor will have the charge Q.

$$Q = Q_1 + Q_2 + Q_3 + \ldots\ldots + Q_N$$

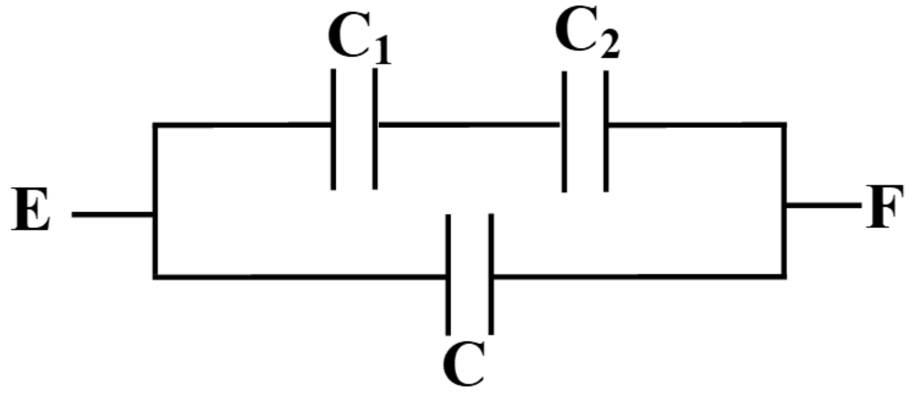
$$C_{eq}\, V_{AB} = V_{AB}C_1 + V_{AB}C_2 + V_{AB}C_3 + \ldots\ldots + V_{AB}C_N \Rightarrow C_{eq} = C_1 + C_2 + C_3 + \ldots\ldots + C_N$$

Then, the equivalent capacitance Ceq of a system composed of N capacitors in parallel, each with capacitance Ci, is given by: $C_{eq} = \sum_{i=1}^{N} C_i$

Example:

Consider the arrangement of capacitors in the diagram below.

Given that $C_1 = 6\ \mu F$, $C_2 = 3\ \mu F$, and $C = 2\ \mu F$.



1- Calculate the equivalent capacitor between E and F.

2- We apply a voltage between E and F of VEF = 1000 volts. Find the charge carried by the equivalent capacitor, deduce the charge carried by capacitors C, C_1, and C_2.

Solution :

1- Calculate C_{eq} :

C_1 in series with C_2 : $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow C' = 2\,\mu c$

C' in parallel with C $\rightarrow C_{eq} = C + C' = 4\,\mu c$

2- Find the charge carried by the equivalent capacitor, deduce the charge carried by capacitors C, C_1, and C_2

- ✓ C_{eq} Deducted from C in parallel with C' $\Rightarrow$ (C_{eq} ,C and C' have same voltage)

$$\Rightarrow Q = C\,V_{EF} = 2\;10^{-3}\;c$$

- ✓ C_1 in series with C_2 $\Rightarrow$ (C' , C_1 and C_2 have same charge)

$$\Rightarrow \quad Q_1 = Q_2 = Q' = C'\,V_{EF} = 2\;10^{-3}\;c$$