

مقياس احتمال (2)

جامعة بومرداس

حل المسئلة (4)

التصريف ① : $A = \{a, b, d, f\}$ $B = \{b, d, e\}$

$A \cup B = \{a, b, d, e\}$; $\overline{A \cap B} = \{c\} = A^c \cap B^c$

$\overline{B} = \{a, c\}$; $(\overline{A \cap B}) = \Omega - (A \cap B) = \{a, c, e\}$

$\overline{A} \cap B = \{e\}$; $A \cap B = \{b, d\}$

$\overline{A} = \{c, e\}$; $B \cap A = A \cap B = \{b, d\}$; $B \setminus A = B - A = \{x : x \in B \text{ و } x \notin A\}$
 $B \setminus A = \{e\}$

$A \cup \overline{B} = \{a, b, c, d\}$

$\overline{B} \setminus \overline{A} = B^c \setminus A^c = \overline{B} - \overline{A} = \{x : x \notin B \text{ و } x \notin \overline{A}\} = \{x \notin B \text{ و } x \in A\}$
 $= A \setminus B = \{a\}$

$(\overline{A \cup B}) = \{x : x \notin (A \cup B) \text{ و } x \in \Omega\} = \{c\}$

التصريف ② : $P(A) = 0,3$ $P(B) = 0,7$, $P(A \cap B) = 0,2$

(1) احتمال وقوع حدثين على الأقل :
 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0,3 + 0,7 - 0,2 = \boxed{0,8}$

(2) عدم وقوع الحدث A :
 $P(\overline{A}) = 1 - P(A) = 1 - 0,3 = \boxed{0,7}$

(3) وقوع الحدث A وعدم وقوع B :
 $P(A \cap \overline{B}) = P(A \cap B^c) = P(A - B) = P(A) - P(A \cap B) = 0,3 - 0,2 = \boxed{0,1}$

$P(A \cup B) = \frac{3}{4}$

$P(\overline{A}) = \frac{2}{3}$

$P(A \cap B) = \frac{1}{4}$

$P(A) = ?$

(1)

$$* P(A) = ?$$

$$P(A) + P(\bar{A}) = P(\Omega) = 1 \Rightarrow P(A) = 1 - P(\bar{A}) = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

$$P(B) = ?$$

$$* P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

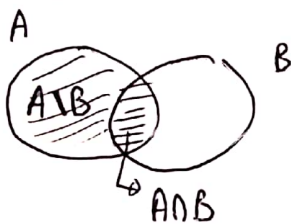
$$P(B) = P(A \cup B) - P(A) + P(A \cap B) \\ = \frac{3}{4} - \frac{1}{3} + \frac{1}{4} = \boxed{\frac{2}{3}}$$

$$* P(A \cap \bar{B}) = P(A \setminus B) = P(A - B) = P(A) - P(A \cap B) \\ = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

التصريح ④ :

$$* P(A \setminus B) = P(A) - P(A \cap B)$$

يمكن تجزئة الحدث A إلى العديتين المتناقبتين $A \setminus B$ و $A \cap B$
 $A = (A \setminus B) \cup (A \cap B) \Rightarrow P(A) = P(A \setminus B) + P(A \cap B)$



$$P(A \setminus B) = P(A) - P(A \cap B) \dots \text{①}$$

$$* P(A \cup B) \stackrel{?}{=} P(A) + P(B) - P(A \cap B)$$

لاحظ أن $(A \cup B)$ يمكن تجزئته إلى العديتين $(A \setminus B)$ و B أو $(B \setminus A)$ و A أي أن:

$$A \cup B = (A \setminus B) \cup B$$

$$P(A \cup B) = P(A \setminus B) + P(B) \quad (\text{بعون ① من قبل})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$* P(A \cup B \cup C) \stackrel{?}{=} P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

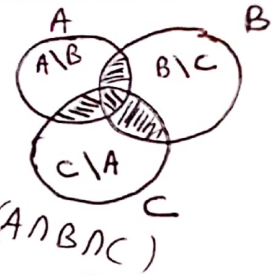
لدينا:

$$A \cup B \cup C = (A \setminus B) \cup (B \setminus C) \cup (C \setminus A) \cup (A \cap B \cap C)$$

$$P(A \cup B \cup C) = P(A \setminus B) \cup (B \setminus C) \cup (C \setminus A) \cup (A \cap B \cap C)$$

(ع)

$$= P(A) - P(A \cap B) + P(B) - P(B \cap C) + P(C) - P(C \cap A) + P(A \cap B \cap C)$$



$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

التصريح 5 :

$$D = \{ A \bar{B} \bar{C}, \bar{A} B \bar{C}, \bar{A} \bar{B} C \}$$

$$E = \{ A B \bar{C}, A \bar{B} C, \bar{A} B C \}$$

$$F = \{ A B \bar{C}, A \bar{B} C, \bar{A} B C, A \bar{B} \bar{C}, \bar{A} B \bar{C}, \bar{A} \bar{B} C, \bar{A} \bar{B} \bar{C} \}$$

$$G = \{ A \bar{B} \bar{C}, \bar{A} B \bar{C}, \bar{A} \bar{B} C, \bar{A} \bar{B} \bar{C} \}$$

$$H = \{ A \bar{B} \bar{C}, \bar{A} B \bar{C}, \bar{A} \bar{B} C, A B \bar{C}, A \bar{B} C, \bar{A} B C, A B C \}$$

التصريح 6 : $P(A_1) = P_1 = 0,7$; $P(A_2) = P_2 = 0,5$; $P(A_3) = P_3 = 0,4$

1) أهمية الهدف طلقة واحدة فقط : $P(A) = ?$

$$\begin{aligned} P(A) &= P(A_1 \cap \bar{A}_2 \cap \bar{A}_3) + P(\bar{A}_1 \cap A_2 \cap \bar{A}_3) + P(\bar{A}_1 \cap \bar{A}_2 \cap A_3) \\ &= P(A_1) \cdot P(\bar{A}_2) \cdot P(\bar{A}_3) + P(\bar{A}_1) \cdot P(A_2) \cdot P(\bar{A}_3) + P(\bar{A}_1) \cdot P(\bar{A}_2) \cdot P(A_3) \\ &= (0,4) \cdot (0,5) \cdot (0,3) + (0,6) \cdot (0,5) \cdot (0,3) + (0,6) \cdot (0,5) \cdot (0,7) = 0,36 \end{aligned}$$

2) أهمية الهدف طلقتين فقط : $P(B)$

$$\begin{aligned} P(B) &= P(A_1 \cap A_2 \cap \bar{A}_3) + P(A_1 \cap \bar{A}_2 \cap A_3) + P(\bar{A}_1 \cap A_2 \cap A_3) \\ &= P(A_1) \cdot P(A_2) \cdot P(\bar{A}_3) + P(A_1) \cdot P(\bar{A}_2) \cdot P(A_3) + P(\bar{A}_1) \cdot P(A_2) \cdot P(A_3) \\ &= (0,4) \cdot (0,5) \cdot (0,3) + (0,4) \cdot (0,5) \cdot (0,7) + (0,6) \cdot (0,5) \cdot (0,7) = 0,41 \end{aligned}$$

3) أهمية الهدف ثلاث طلقات : $P(C) = ?$

$$P(C) = P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3) = (0,4) \cdot (0,5) \cdot (0,7) = 0,14$$

$$D = A \cup B \cup C$$

4) أهمية الهدف طلقة واحدة على الأقل $P(D)$

$$P(D) = P(A) + P(B) + P(C) = 0,36 + 0,41 + 0,14 = 0,91$$

(3)

التصريف ⑦

$$P(A) = 0,5$$

$$P(A \cap B) = 0,25$$

$$P(B) = 0,33$$

$$* P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0,25}{0,33} = \boxed{0,75}$$

$$* P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0,25}{0,5} = \boxed{0,5}$$

$$* P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{1 - P(B)} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = 0,5 + 0,33 - 0,25 = \boxed{0,58}$$

$$\Rightarrow P(\bar{A}/\bar{B}) = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - 0,58}{1 - 0,33} = \frac{0,42}{0,67} = \boxed{0,62}$$

$$* P(\bar{B}/\bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = \frac{P(\overline{B \cup A})}{1 - P(A)} = \frac{1 - P(B \cup A)}{1 - P(A)} = \frac{0,42}{0,5} = \boxed{0,84}$$