Chapter II
Conductors in electrostatic equilibrium

Course 2

Dr IACHACHENE FARIDA   FHC-UMBB-2023-2024

II.5 Electrization of a conductor:

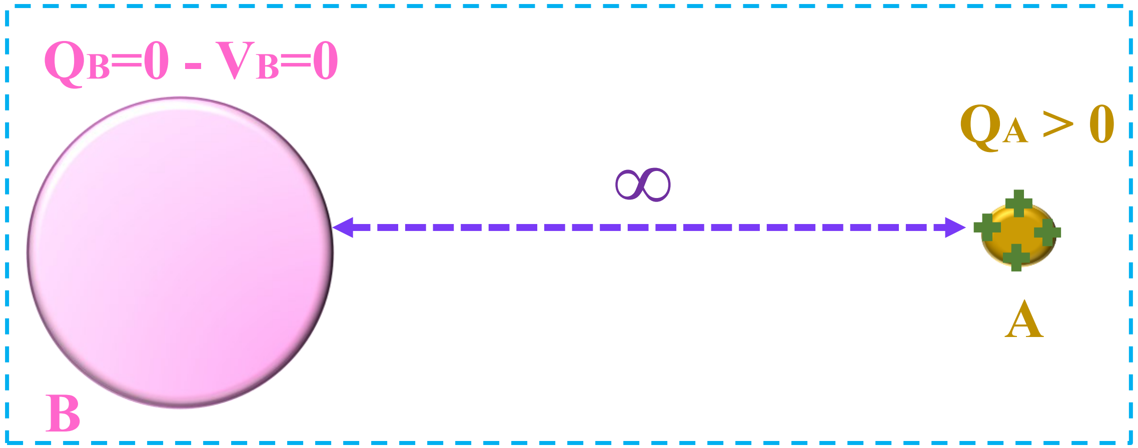
Phenomenon of influence between charged conductors

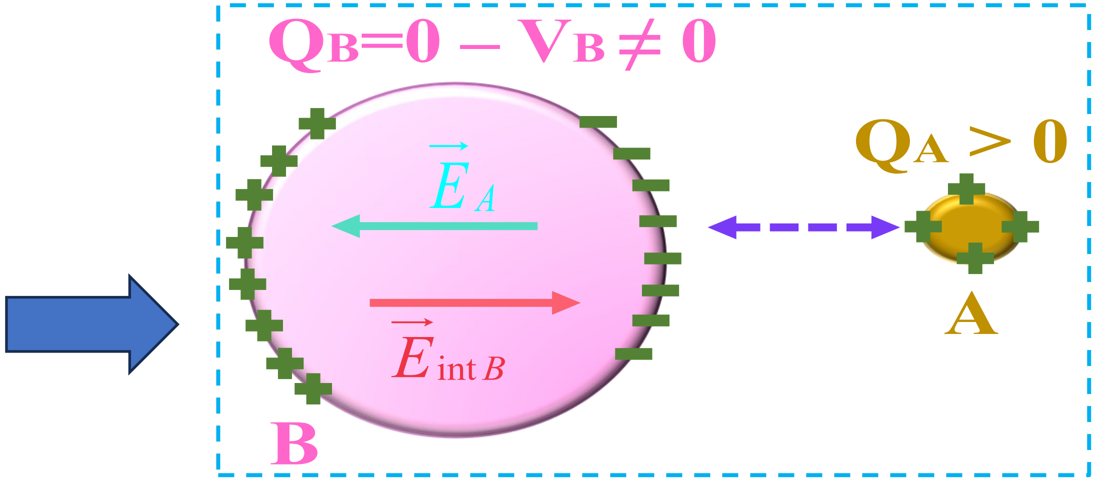
1. Partial Influence

Electrostatic influence occurs when a charged conductor A is brought near another conductor B, whether it is charged or not.

A - Case of conductor B uncharged (neutral) $Q_B=0$

Let B be a neutral conductor ($Q_B=0$). If we bring a positively charged conductor A near conductor B, conductor B will be influenced by the presence of conductor A in its vicinity (See Figures below). Negative charges (-) appear on the surface of conductor B facing conductor A, and positive charges (+) appear on the other side of conductor B.





Interpretation:

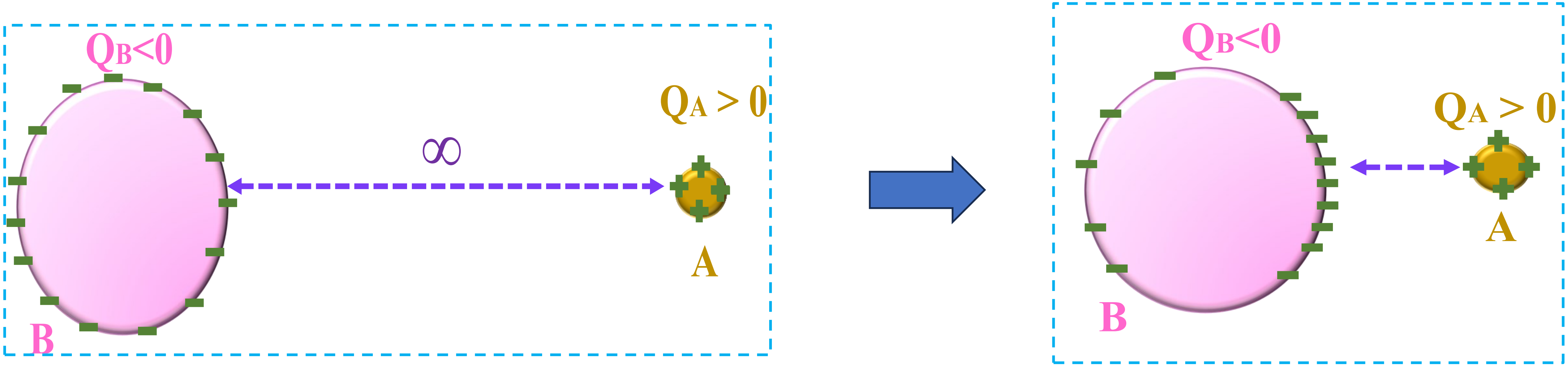
Consider a neutral conductor B, and let's bring a positively charged conductor A near it. Conductor A creates an electric field $\vec{E}_A$ A in its vicinity. The free electrons of conductor B will move under the influence of the electric field $\vec{E}_A$ in the opposite direction of $\vec{E}_A$ ($\vec{F} = -e\vec{E}_A$). These electrons will accumulate on the side facing A, while positive charges will appear on the other side. This transfer of charge stops when $\|\vec{E}_A\| = \|\vec{E}_{int\,B}\|$, which means when the field inside conductor B cancels out. At this point, the conductor is in electrostatic equilibrium.

Note:

The phenomenon of influence does not change the total charge of a conductor B; its charge remains constant, equal to its initial value. It only modifies the distribution of this charge on the surface, thus changing its potential V_B. In the previous example, the charge of B remains constant ($Q_B=0$), but its potential changes ($V_B \neq 0$).

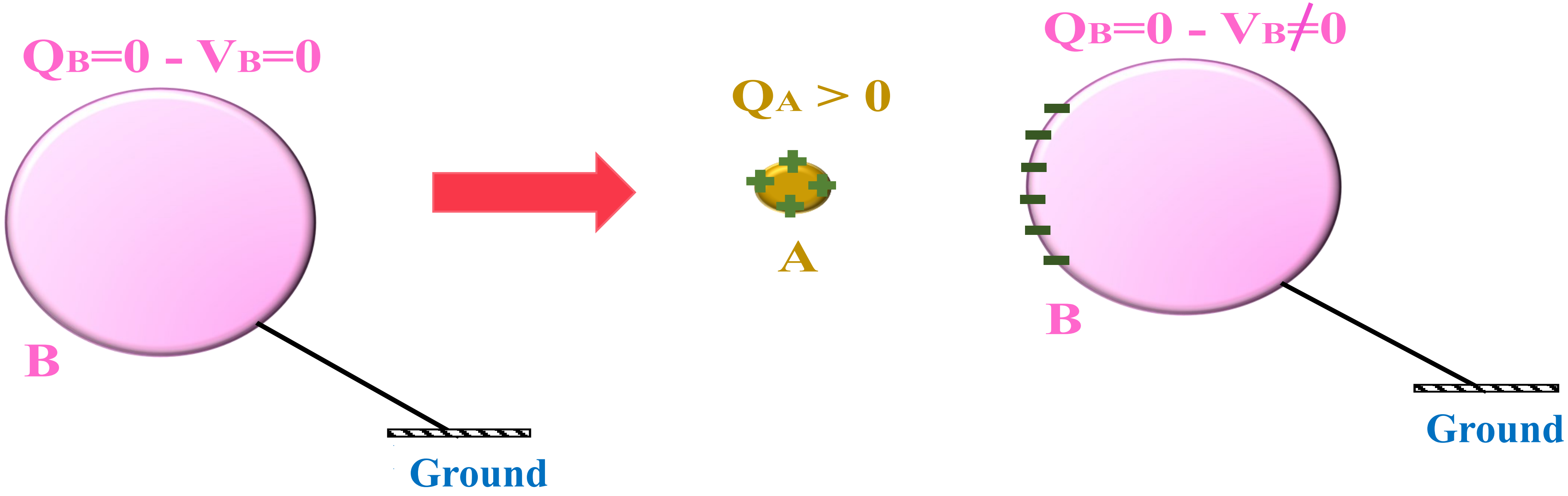
B - Case of conductor B negatively charged ($Q_B < 0$)

In this case, the distribution of charge on conductor B will be altered (see figures below).



C - Case of a conductor maintained at a constant potential:

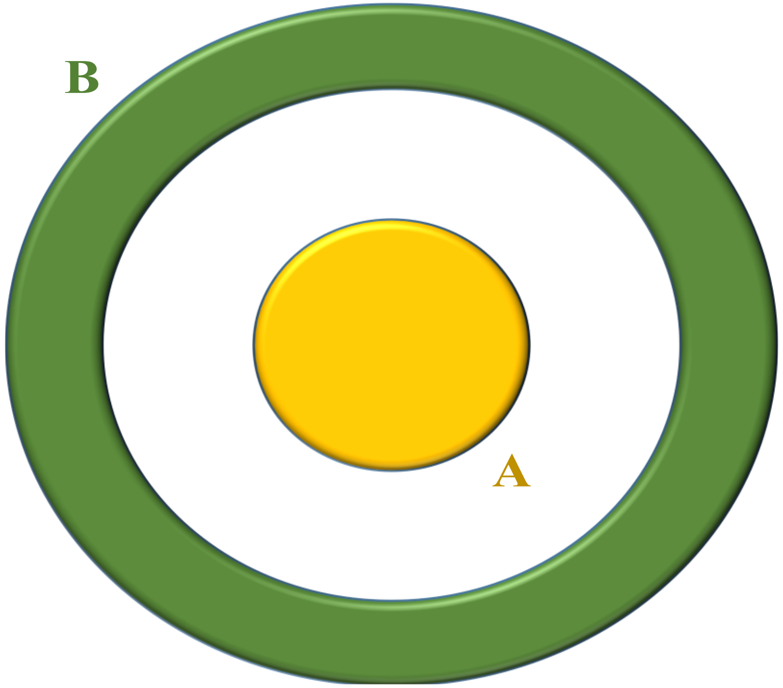
If conductor B is maintained at a constant potential V_B (connected to a generator or to ground), in this case, when we bring conductor A close to B, the charge of B will be modified, but its potential remains constant (Q_B changes while V_B remains constant).



2 Total Influence

We speak of total influence when all field lines originating from A terminate on B. This is achieved when B completely surrounds A. Application of the theorem of corresponding elements shows that the charge appearing on the inner surface of B is equal and opposite to the charge of conductor A ($Q_{Bint}=-Q_A$).

Example: Let conductor A be positively charged $Q_A > 0$, if $Q_B=0$, see figure below.



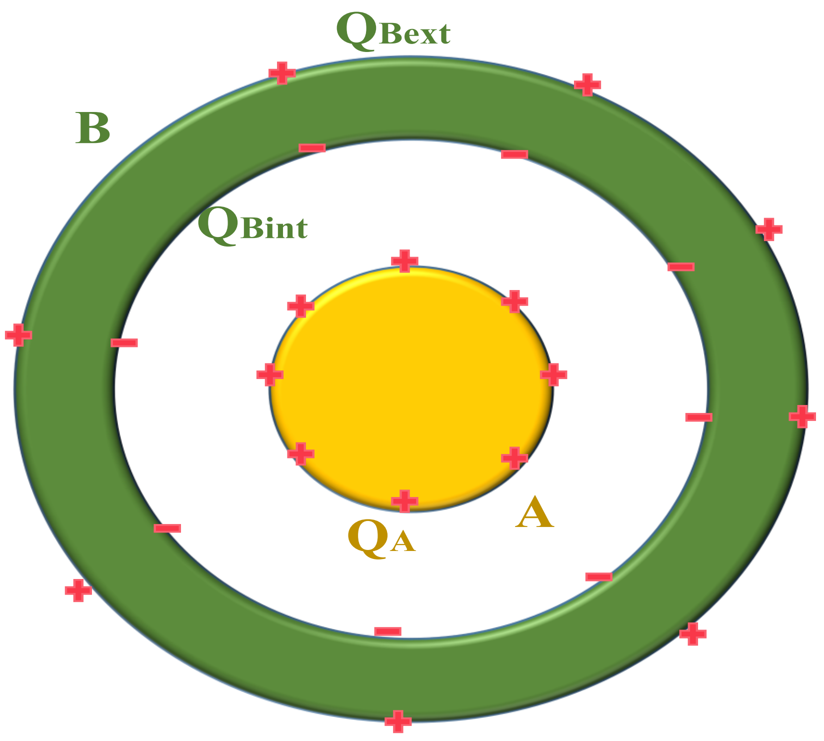
By total influence: $Q_{Bint} = -Q_A$

The total charge of B is conserved, the phenomenon of influence does not change the charge of a conductor; it is the distribution of the charge that is modified. Therefore:

$$Q_B = Q_{Bext} + Q_{Bint}$$

$$\Rightarrow Q_{Bext} = Q_B - Q_{Bint}$$

$$\Rightarrow Q_{Bext} = -Q_{Bint}$$



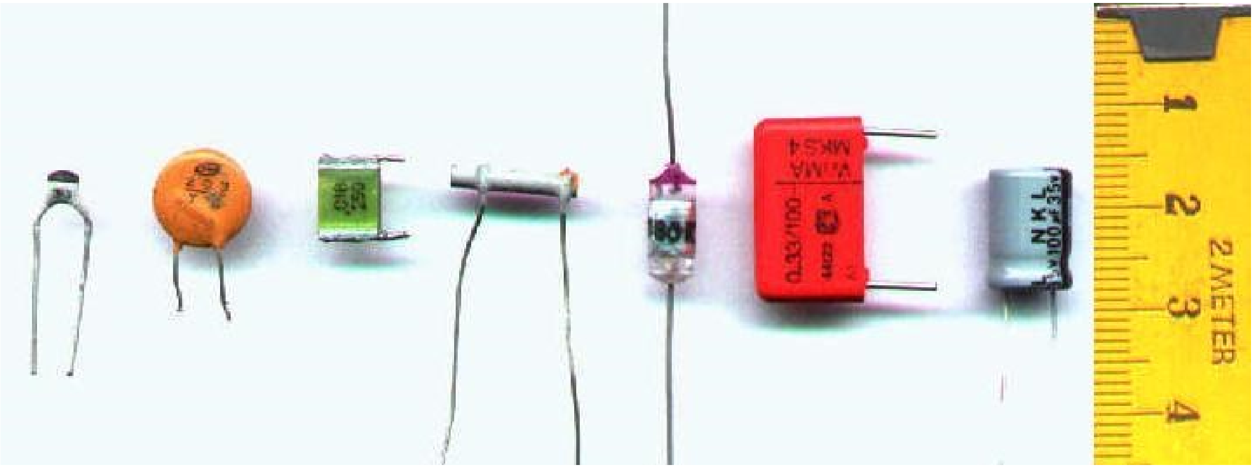
So, the distribution of charge on conductor B is:

$$\begin{cases} Q_{Bint} = -Q_A \\ Q_{Bext} = +Q_A \end{cases}$$

6. Capacitor

1. Definition

A capacitor is a device used to store electrical energy. It consists of two conductors in total influence.



2. The capacitance of a capacitor

In electronic circuits, capacitors are represented by:

If Q is the charge carried by the plates of the capacitors and V is the potential difference (voltage) between the two plates, experiments have shown that the ratio Q/V remains constant. This constant is called the capacitance of a capacitor, denoted by C.

The capacitance value C of a capacitor, which is a positive quantity, depends on:

- The geometry of the conductors forming the capacitor, including their dimensions and shape.
- The distance between the two plates (or electrodes).
- The nature of the material (dielectric) separating the two plates, characterized by ε (ε_0 in the case of vacuum). The plates are separated by an insulator which serves to increase the capacitance of the capacitor.

3. Steps to determine the expression of a capacitor's capacitance:

a. Determine the expression of the electric field inside the capacitor, i.e., between the two plates.

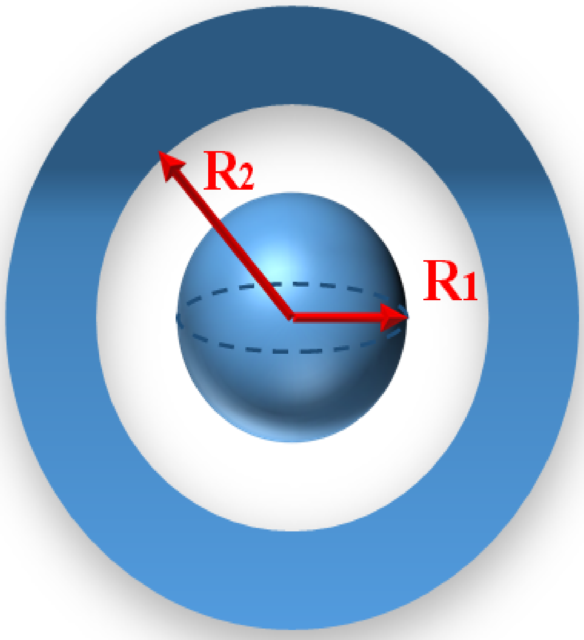
b. Find the expression of the voltage difference (ΔV) between the two plates.

c. Substitute the expressions for Q and ΔV into the capacitance formula, and simplify if necessary. which represents the expression of the capacitance C.

Example:

a/ Spherical capacitor

The spherical capacitor consists of two concentric conductive spheres with radii R_1 and R_2 ($R_1 < R_2$) separated by an insulator. See the figure below.



If the first electrode (sphere of radius R_1) is charged with $Q>0$, then the second electrode will carry a charge $Q<0$.

- Determine the expression for the capacitance of the spherical capacitor:

• Expression of the electric field inside the capacitor:

By applying Gauss's theorem, the Gaussian surface is a sphere of radius r concentric with the two spheres.

$$\Phi = \oiint_{S_G} \vec{E}.d\vec{S} = \frac{Q_{int\,er}}{\varepsilon_0}$$

$$\Rightarrow \oiint_{S_G} \vec{E}.d\vec{S} = E(r) \oiint_{S_G} dS = E(r)4\pi r^2 \quad and \quad Q_{int\,er} = Q$$

$$E(r)4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \Rightarrow E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

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$$E(r)4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \Rightarrow E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

- Expression of the potential difference between the two electrodes ΔV:

Since the electric field $\vec{E} = E\,\vec{u}_r$ is radial and $\vec{E} = -\overrightarrow{grad}V$, then we can write:

$$E = -\frac{dV}{dr} \Rightarrow dV = -Edr$$

$$\int_{V_-}^{V_+} dV = -\frac{Q}{4\pi\varepsilon_0}\int_{R_2}^{R_1}\frac{dr}{r^2} \Rightarrow V_+ - V_- = \frac{Q_B}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

then:

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

- Deduce the expression of the capacitance C:

$$C = \frac{Q}{\Delta V} \Longrightarrow C = \frac{4\pi\varepsilon_0 (R_2 R_1)}{R_2 - R_1}$$

It is clear that the capacitance of a capacitor depends on the medium. ε_0, of its dimension $(R_2 R_1)$ and the distance between the two electrodes $(R_2 - R_1)$.