Chapter II
Conductors in electrostatic equilibrium

Course 1

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Conductors in electrostatic equilibrium

II.1 Introduction

Conductors enable the movement of electric charges throughout their structure. A conductor is considered to be in electrostatic equilibrium when its charges are stationary, showing that they are not in motion. In this state, the conductor exhibits various properties relevant to its behavior in electrical systems.

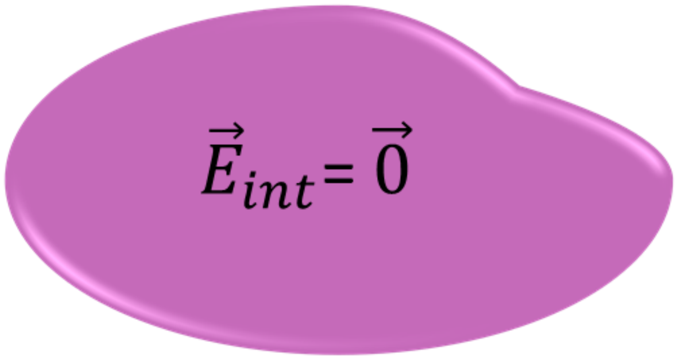
.II.2 Properties of a conductor in electrostatic equilibrium

Let there be in vacuum a conductor charged in electrostatic equilibrium by a charge Q.

a- The electrostatic field inside a conductor in electrostatic equilibrium

The electric field $\vec{E}$ is null at every point within the conductor in electrostatic equilibrium. This is because in electrostatic equilibrium, the charges within the conductor are stationary, and therefore not subject to any force ($\vec{F} = \vec{0}$). We know that que

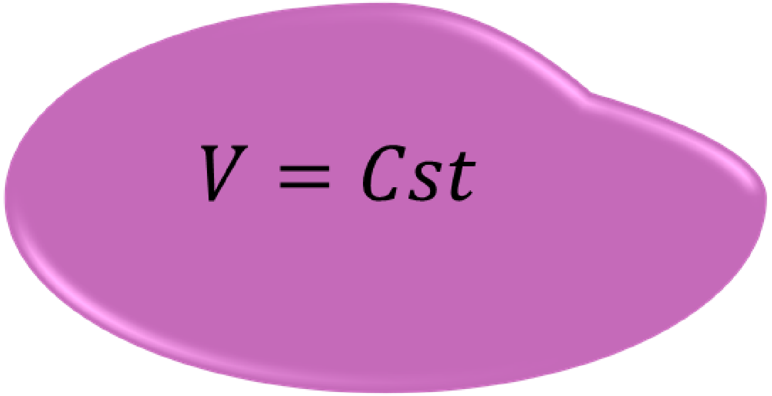
$\vec{F} = q\vec{E}_{int}$, which implies that.



Electric field inside a conductor in electrostatic equilibrium

b- The electrostatic potential inside a conductor in electrostatic equilibrium

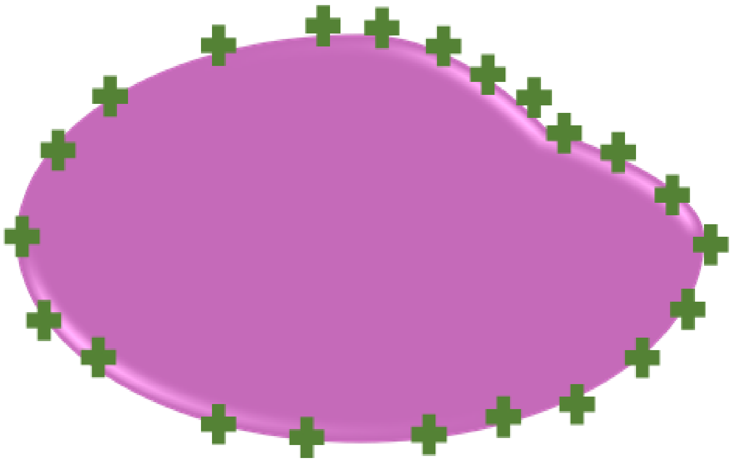
At electrostatic equilibrium, the conductor constitutes an equipotential volume, meaning its potential V is uniform (with the same value throughout its volume and surface). This is because: $\vec{E} = -\overrightarrow{grad}V \Rightarrow \vec{0} = -\overrightarrow{grad}V \Rightarrow V = Cte$



The conductor forms an equipotential volume

c- The distribution of charge inside a conductor in electrostatic equilibrium:

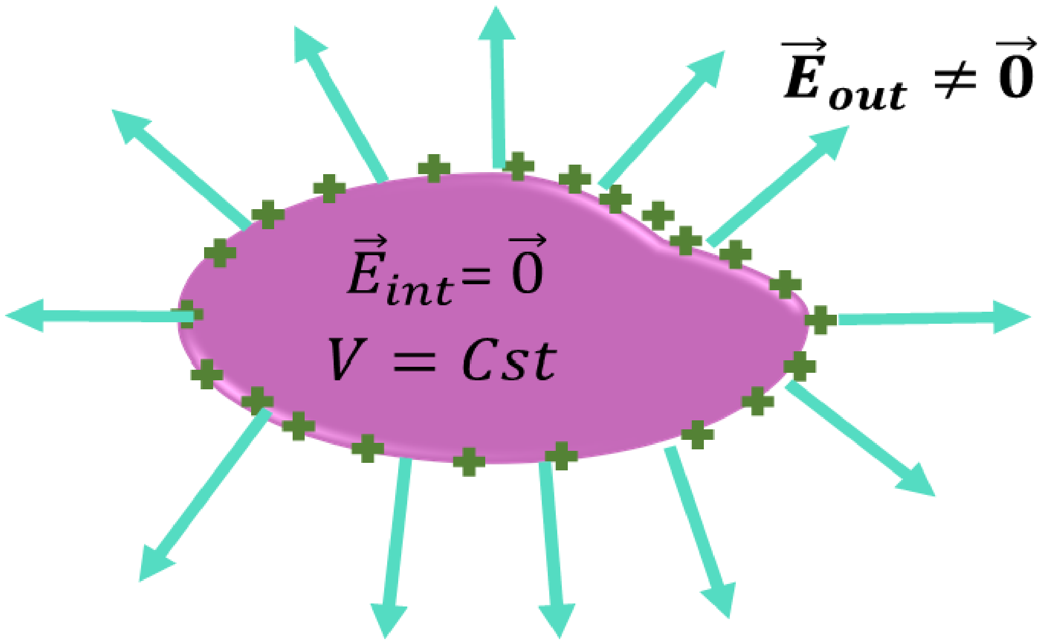
Within a conductor in electrostatic equilibrium, the electric charge is zero inside, and its charge is entirely distributed on its surface.



Surface distribution

d- The electric field lines of a conductor in equilibrium

The field lines are perpendicular to the surface of a charged conductor in electrostatic equilibrium because the conductor forms an equipotential surface (See Chapter I).



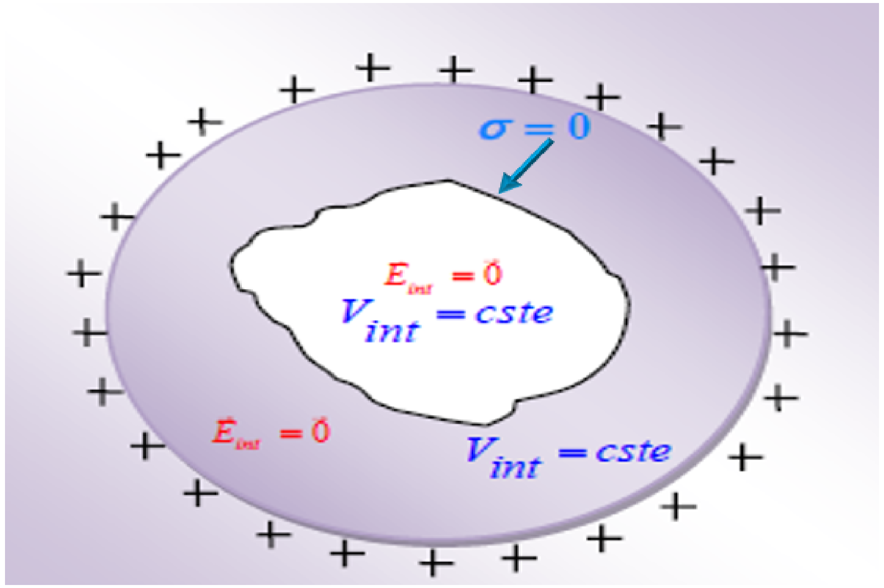
e- The electric field lines of a conductor in equilibrium

A hollow conductor, with its cavity devoid of charges, possesses the following properties when in electrostatic equilibrium:

- The electric field is null within the conductor's bulk and within the cavity.

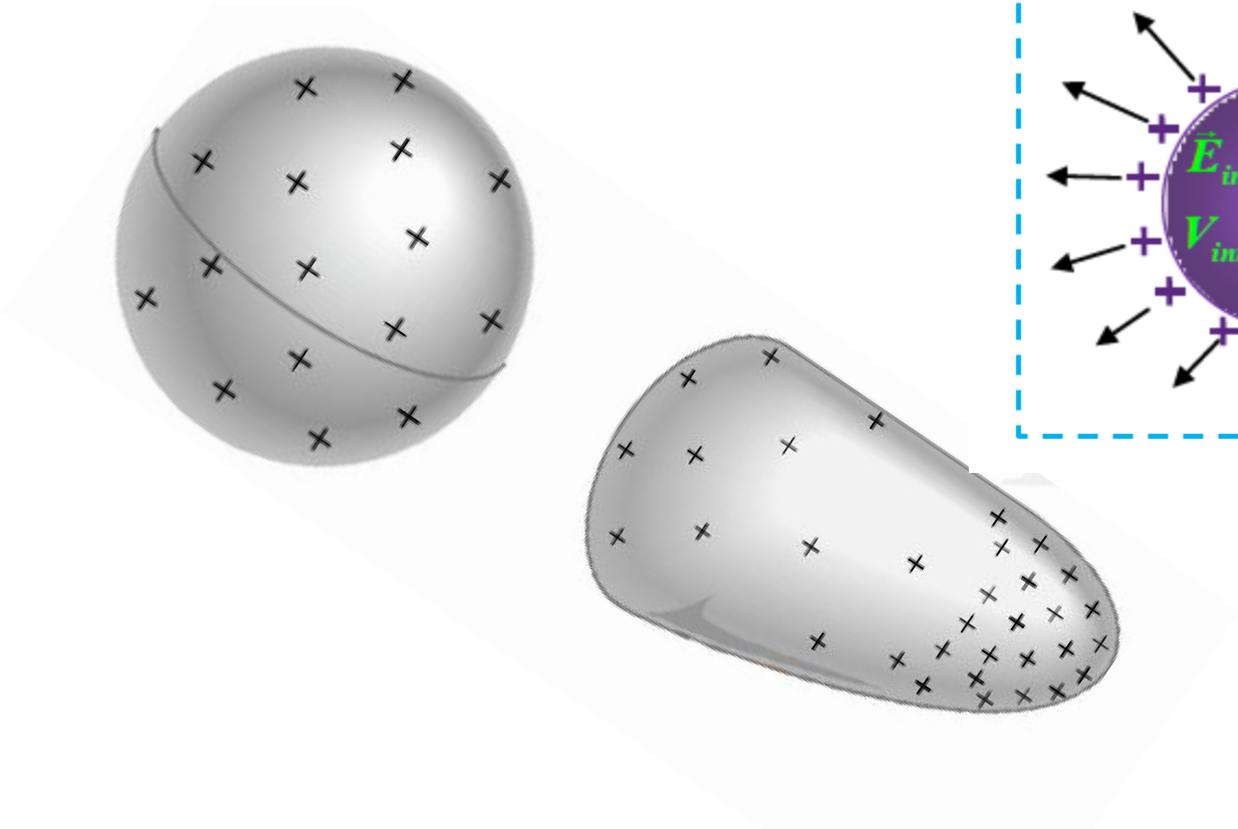
- The potential remains constant at every point within the volume enclosed by the conductor's outer surface.

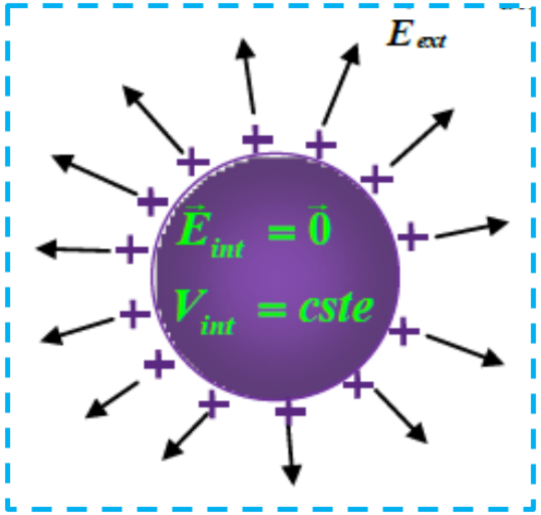
- Charges are solely distributed on the outer surface.

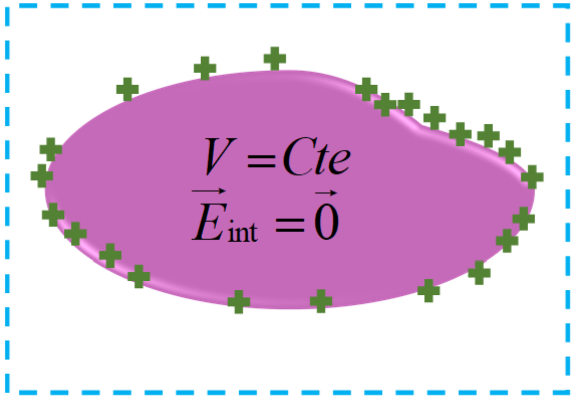


a- Point-effectiveness:

In a conductor in electrostatic equilibrium, electric charges tend to accumulate on pointed surfaces (i.e., those with small radii of curvature). Indeed, the accumulation of charges depends on the geometric shape of the conductor.

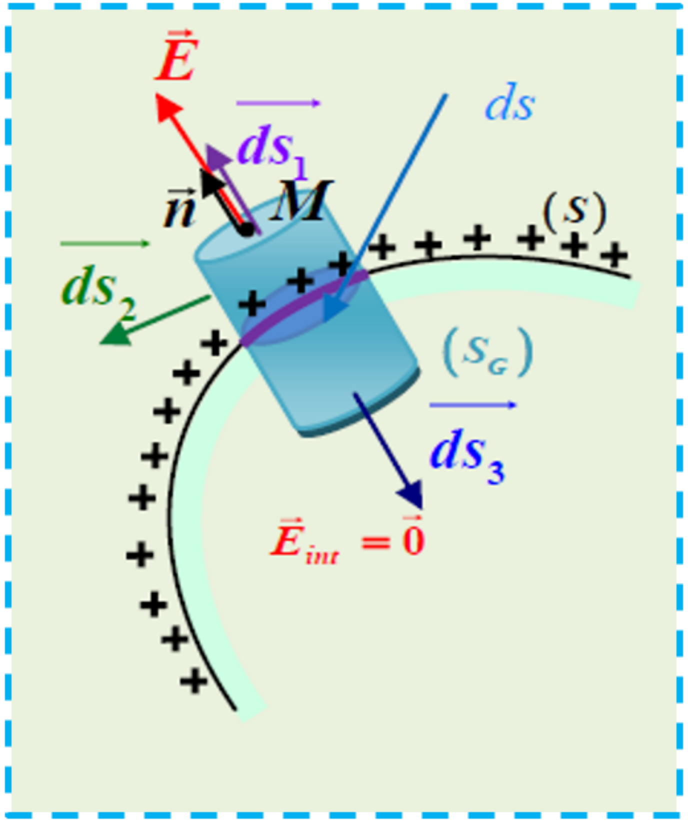






f- Electric field near a conductor in electrostatic equilibrium:

To calculate the electric field in the immediate vicinity of a charged conductor in electrostatic equilibrium, we use Gauss's theorem. For this purpose, let's consider a surface element dS carrying an elementary charge $dq = \sigma ds$. The Gauss surface S_G is a cylinder with a height h and radius r (See the figure below). Let's calculate the flux of the electric field through ***SG***:



$$\Phi_{S_G} = \oiint_{S_G} \vec{E}.d\vec{s} = \frac{Q_{net}}{\varepsilon_0}$$

Le flux total à travers la surface de Gauss (**A closed cylinder S_G**)

$$\Phi_{S_G} = \Phi_1 + \Phi_2 + \Phi_3$$

$$\oiint_{S_G} \vec{E}.\vec{ds} = \iint_{S_1} \vec{E}.\vec{ds}_1 + \iint_{S_2} \vec{E}.\vec{ds}_2 + \iint_{S_3} \vec{E}_{int}.\vec{ds}_3$$

$$\vec{E} \,//\, \overrightarrow{dS_1} \Rightarrow \vec{E}.\overrightarrow{dS_1} = E.dS_1$$

$$\vec{E} \perp \overrightarrow{dS_2} \Rightarrow \vec{E}.\overrightarrow{dS_2} = 0$$

$$\vec{E}_{int} = 0 \Rightarrow \vec{E}_{int}.\overrightarrow{dS_3} = 0$$

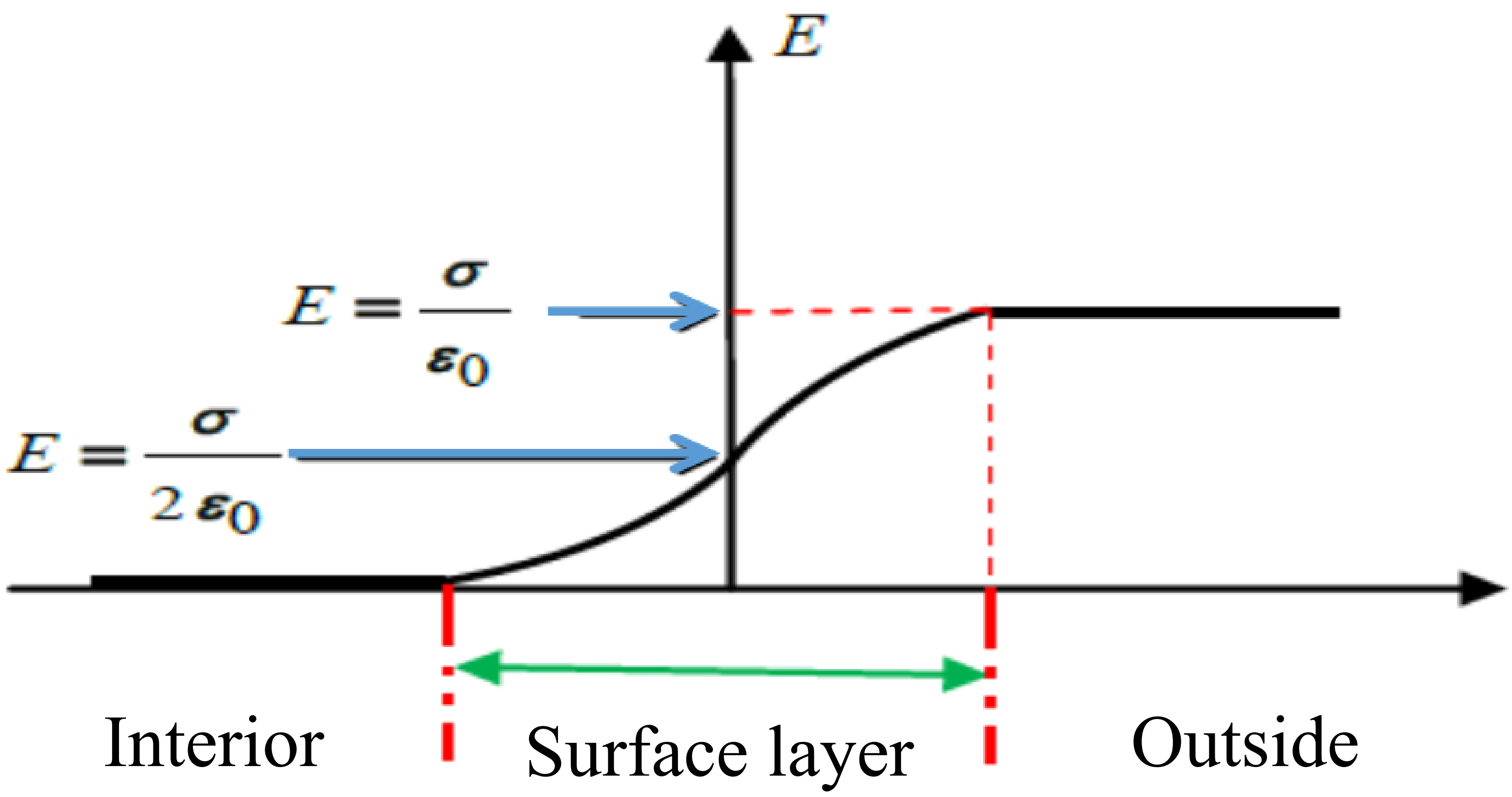
We find: $\Phi = \iint_{S_1} \vec{E}\,.d\vec{s}_1 = E.S_1$ et $Q_{int} = \sigma S_1$

Then: $E.S_1 = \dfrac{\sigma S_1}{\varepsilon_0} \Rightarrow E = \dfrac{\sigma}{\varepsilon_0}$

So, the expression for the electric field in the immediate vicinity of a charged conductor in electrostatic equilibrium is given by:

$$\mathrm{E} = \frac{\sigma}{\varepsilon_0}, \quad \textit{It is Coulomb's theorem}.$$

- Variation of the electric field in the vicinity of the conductor:



a- Electrostatic pressure

The electrostatic pressure **P** exerted on the surface element ***dS*** charged with a surface charge density: it is a force per unit area, given by the equation:

$$P = \frac{dF}{dS}$$

The charge located on the surface dS experiences an electric force $dF = dqE$

with $dq = \sigma\, dS$ and $E = \frac{\sigma}{2\varepsilon_0}$ This results in:

$$dF = \frac{\sigma^2}{2\varepsilon_0} dS$$

Hence, the electrostatic pressure P exerted on the charged surface element dS is given by:

$$P = \frac{\sigma^2}{2\varepsilon_0}$$

II.3 Capacitance of a conductor

The experiment shows that there is a proportionality between the charge Q and the potential V of an isolated conductor in electrostatic equilibrium. In other words, the ratio between the charge carried by the conductor and its potential is constant.

$$\frac{Q}{V} = \frac{Q'}{V'} = \frac{Q''}{V''} = Cst$$

This constant is called the capacitance of an isolated conductor C. Its value depends on the geometry and dimensions of the conductor as well as the medium in which it is located. It characterizes the ability of the conductor, when brought to a given potential, to store charge.

It is given by the expression: $C = \frac{Q}{V}$

The unit of C in the International System is the farad F. In practice, submultiples of the farad are mostly used:

- Le microfarad $1\mu F = 10^{-6}$ F
- Le nanofarad 1nF $=10^{-9}$F
- Le picofarad 1pF $=10^{-12}$F

II.4 Electrostatic energy of an isolated conductor:

Let's say we have an isolated conductor charged with a charge Q and its potential is V .

The electrical potential energy of this conductor is given by:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Where W is expressed in joules, C in farads, V in volts, and Q in coulombs.