Chapter I
Electrostatics

Course 8

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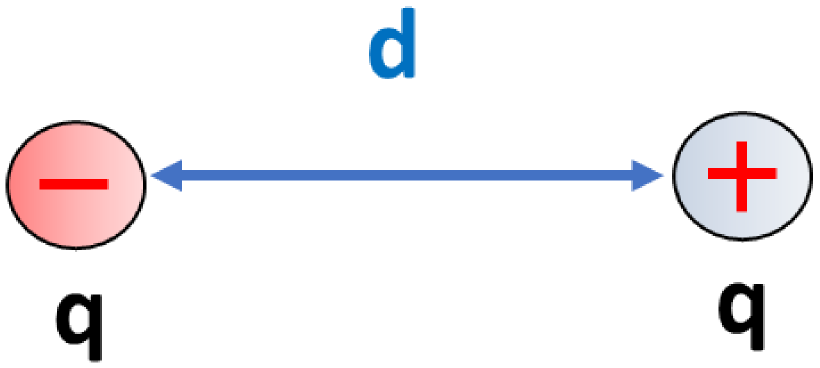
5. Electric dipoles

1. Definition:

An electric dipole consists of two particles with charges of equal magnitude q but opposite signs (+q and -q), separated by a small distance "d", as shown in Figure below.

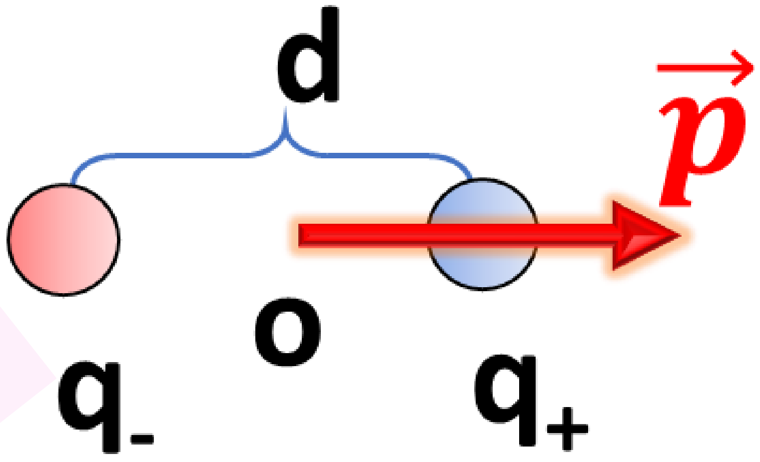


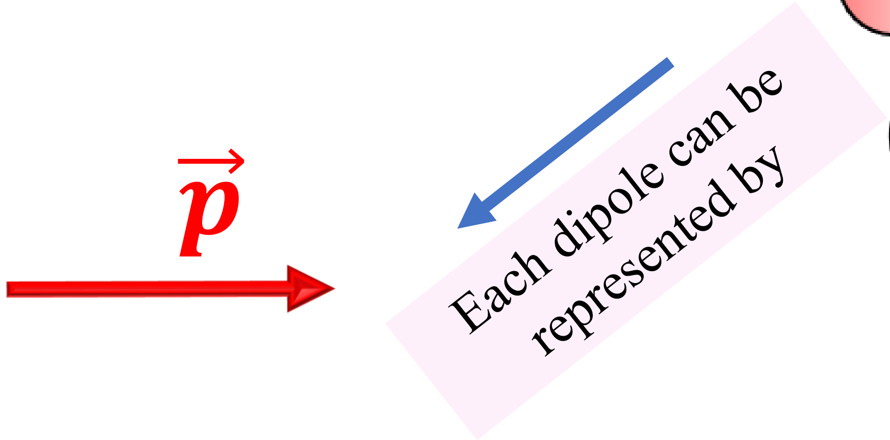
2. The electric dipole moment

Each dipole is characterized by its dipole moment $\vec{p}$ oriented from the negative charge to the positive charge, defined as:

$$\vec{p} = q\vec{d}$$

Its unit in the IS is coulomb meter (C·m)





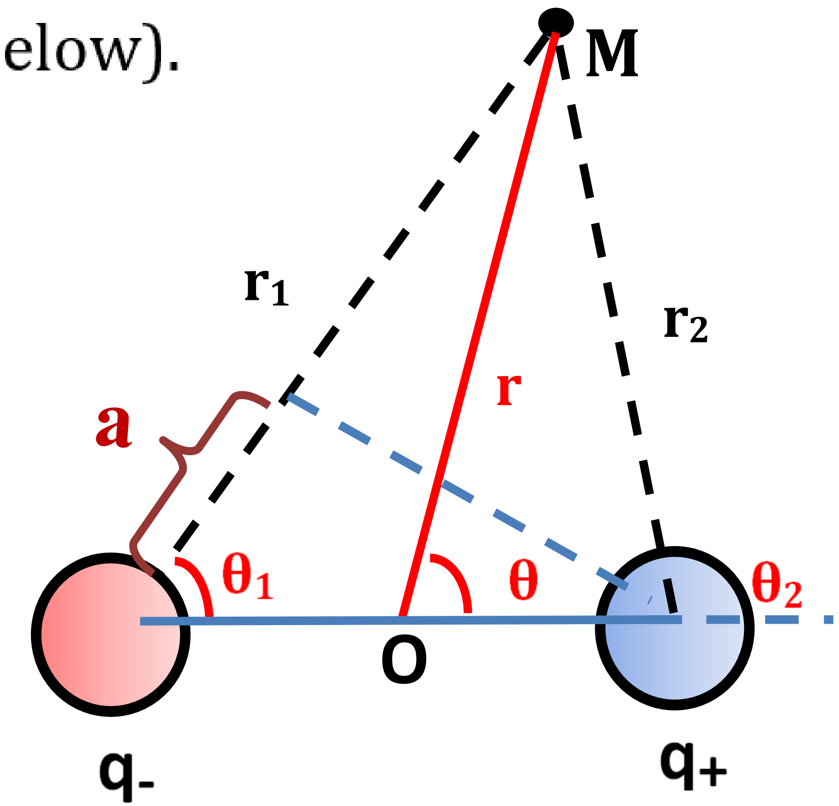
Example:

In nature, there are naturally popularized molecules, such as: **HCl, H_2O, SO_2.**

Explanation: In a bond, the electron pair may not be shared equally between the two atoms: one of the two atoms may have a greater attraction force on the electron cloud than the other. This capacity for atoms to attract the electron cloud is called electronegativity. This unequal sharing of the electron charge then transforms the atomic pair into a dipole. It is as if there were a partial electron transfer from the least electronegative atom to the most electronegative atom. This fictitious transfer is introduced by partial charges: the most electronegative atom, which attracts the electron pair, is assigned a partial negative charge, denoted $-\delta$, while the other atom is assigned a partial positive charge, denoted $+\delta$.

3. Electric potential created by a dipole

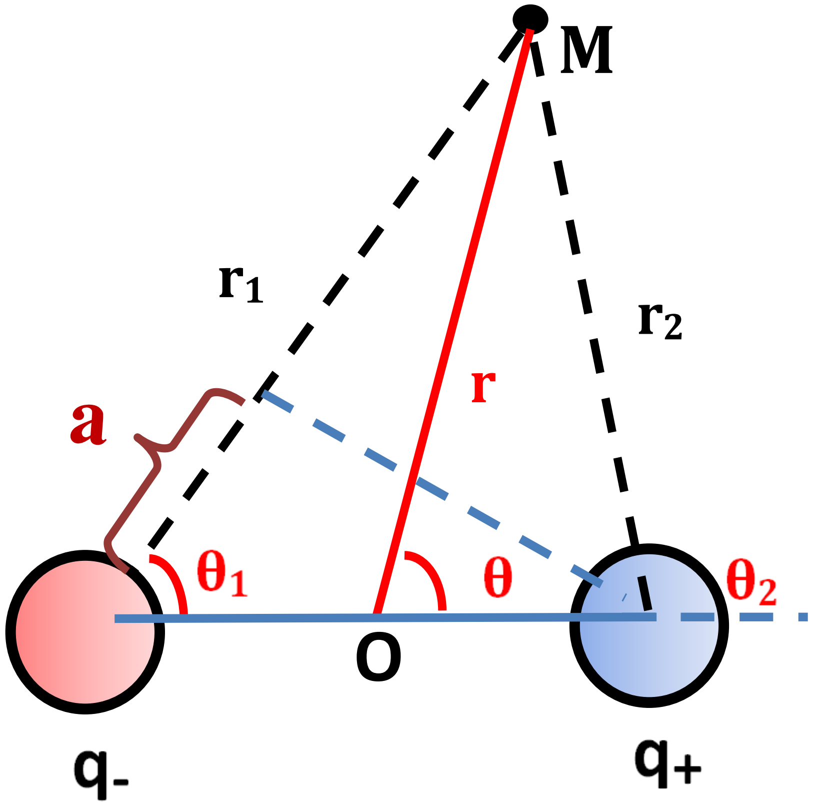
Let's consider a dipole consisting of two-point charges (+q and -q) separated by a distance (d). We will calculate the electrostatic potential created by this dipole at a point (M) (far away from the charges), located at a distance (r) from the midpoint (O) of the dipole (Figure below).



$$V(M) = V(q_+) + V(q_-) \Rightarrow V = k\left(\frac{q_+}{r_2}\right) + k\left(\frac{q_-}{r_1}\right)$$

$$\Rightarrow V = k\left(\frac{q}{r_2}\right) + k\left(\frac{-q}{r_1}\right)$$

$$\Rightarrow V = kq\left(\frac{r_1 - r_2}{r_1 r_2}\right) \qquad (*)$$



As follows: $r \gg d \Rightarrow \begin{cases} r_1 \approx r_2 \approx r \\ \theta_1 \approx \theta_2 \approx \theta \end{cases}$

So we can approximate $\begin{cases} a = r_1 - r_2 = d\cos\theta \\ and \\ r_1 r_2 = r^2 \end{cases}$

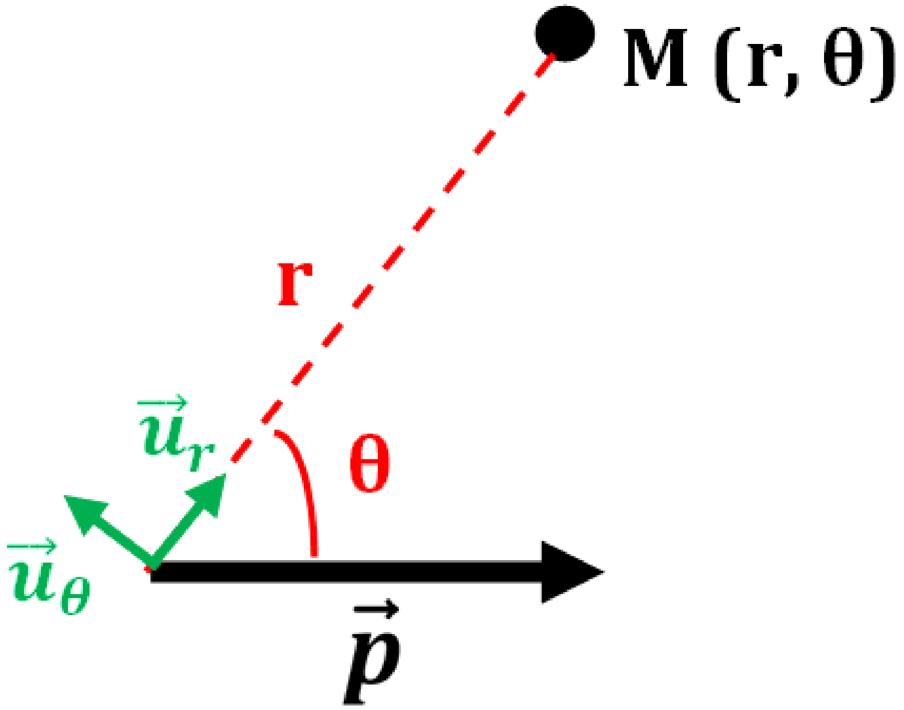
By substituting into equation (*), we obtain

dipole moment

$$(*) \Rightarrow V = k\frac{\overbrace{qa}^{P}\cos\theta}{r^2} \Rightarrow V = \frac{1}{4\pi\varepsilon_0}\left(\frac{P\cos\theta}{r^2}\right)$$

Hence, the electric potential created by an electric dipole at a point (**M**) detected by polar coordinates **(r, θ), M (r, θ)** , is defined as:

$$V(r, \theta) = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$



4. Field created by a dipole:

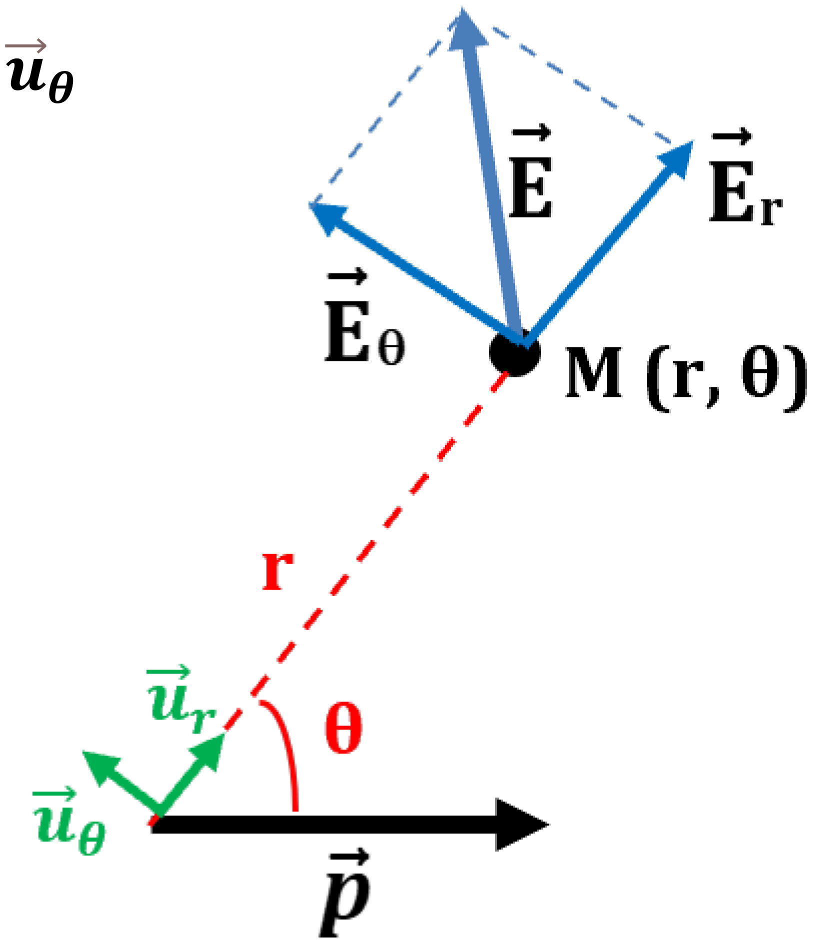
To calculate the electrostatic field, we now only need to use the following relationship: $\vec{E} = -\overrightarrow{grad}V(r,\theta)$ In cylindrical coordinates.

We obtain thus: $\vec{E} = -\frac{\partial V}{\partial r}\vec{u}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta - \cancel{\frac{\partial V}{\partial z}\vec{u}_z}$ 0

The components of the electric field in polar coordinates:

$$\vec{E}(r,\theta) = \begin{cases} E_r = \dfrac{1}{2\pi\varepsilon_0} p\left(\dfrac{\cos\theta}{r^3}\right) \\ E_\theta = \dfrac{1}{4\pi\varepsilon_0} p\left(\dfrac{\sin\theta}{r^3}\right) \end{cases}$$

$$\vec{E}(r, \theta) = \frac{1}{2\pi\varepsilon_0} p \left(\frac{\cos\theta}{r^3}\right)\vec{u}_r + \frac{1}{4\pi\varepsilon_0} p \left(\frac{\sin\theta}{r^3}\right)\vec{u}_\theta$$



5. Dipole placed in a uniform electric field

If we place a dipole, with electric moment $\vec{p}$, in a uniform external field $\vec{E}$ (the field at every point in space is constant in magnitude and direction), the charges that constitute it are subject to equal and opposite forces.

$$\overrightarrow{F_+} = -\overrightarrow{F_-}$$

The dipole then experiences the action of a torque $\vec{\tau}$ (عزم) moment of force (عزم القوه)

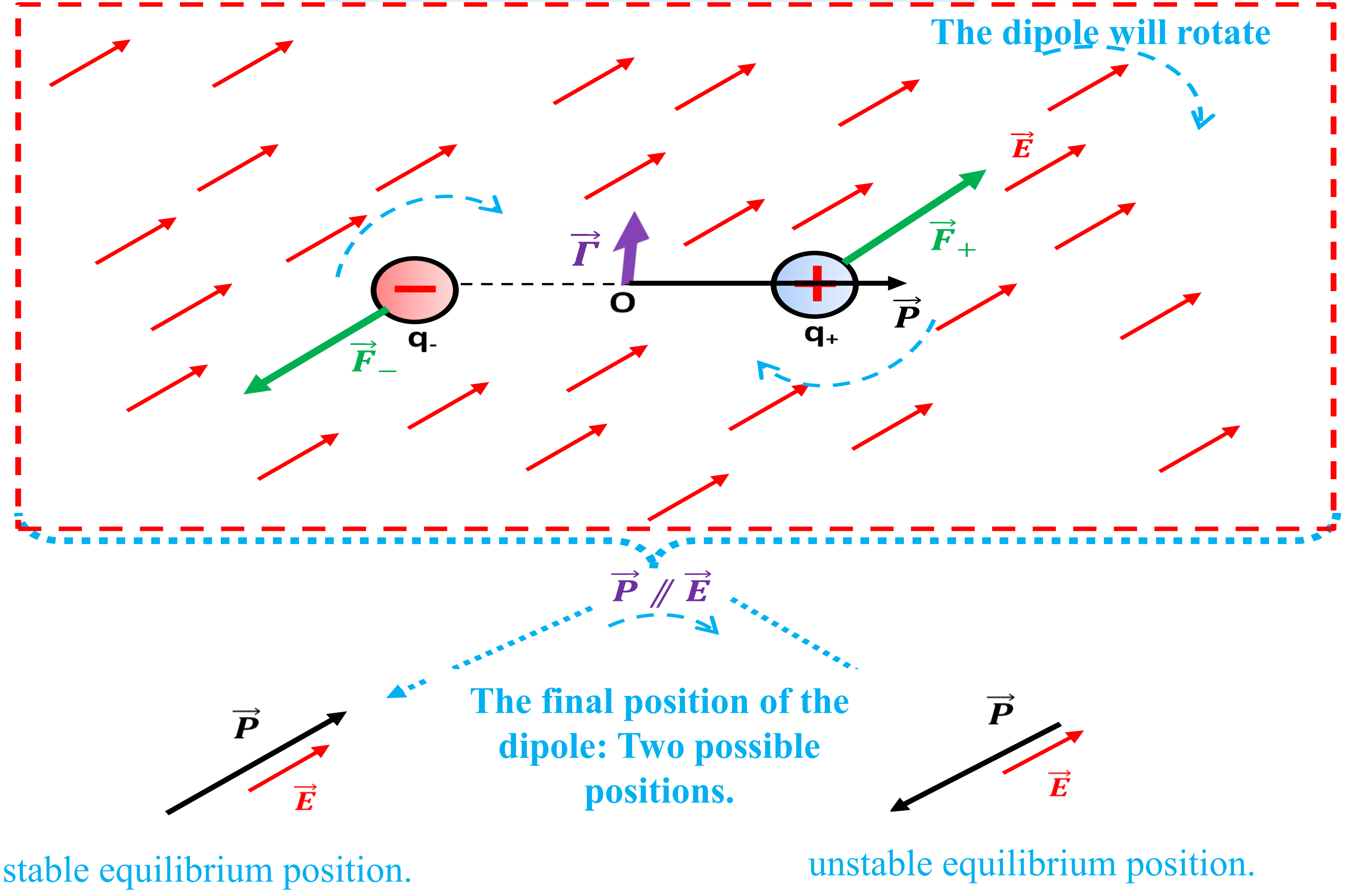
$$\vec{\tau} = \vec{p} \Lambda \vec{E} \qquad \vec{\tau} \perp Plane\,(\vec{p}, \vec{E})$$

In terms of magnitude: $\|\vec{\tau}\| = \|\vec{p}\| \|\vec{E}\| |sin\,\alpha| \Rightarrow \|\vec{\tau}\| = aqE|sin\,\alpha|$

This torque causes the dipole to rotate in order to align it parallel to the external field $\vec{E}$.

So, if we release a dipole in a space where the electric field $\vec{E}$ is uniform, the dipole tends, under the action of torque $\vec{\tau}$, to rotate to reach an equilibrium position (eventually $\vec{\tau} = \vec{0}$) in which $\vec{p}$ and $\vec{E}$ are collinear.

In this zone, the electrical field is uniform.



6. Potential energy of a dipole

The potential energy $\boldsymbol{E_p}$ of a dipole, placed in an electric field $\vec{\boldsymbol{E}}$, is defined by:

$$\boldsymbol{E_p = -\vec{p}.\vec{E}\ \ (dot\ product) \Rightarrow E_p = -pE\cos\alpha}$$

- This energy is minimal for: $\boldsymbol{\alpha = 0 \Longrightarrow E_p = -pE}$
- This energy is maximal for: $\boldsymbol{\alpha = \pi \Longrightarrow E_p = pE}$

7. Examples

Example 1

Consider a dipole formed by two electric charges (-q and +q) separated by a distance a. Knowing that: $q = 1.5$ nC and $a = 5$ mm.

1) - What is the magnitude of its dipole moment?

2) - Calculate its internal energy.

3) - This dipole is placed in a uniform electric field $E = 45$ kV/m.

Express in joules and then in electron-volts, the maximum potential energy resulting from the interaction of this dipole and the field. ($1\ eV = 1.6 \times 10^{-19}$ J).

Solution:

1) Calculate P :

$\vec{p} = q\vec{a} \Rightarrow \|\vec{p}\| = |q|\|\vec{a}\|$

$\Rightarrow P = qa$ AN: $P = 7.510^{-12} C.m$

2)- Internal energy :

The internal energy of two particles is:

$$U = k\frac{q_1 q_2}{r^2} \Rightarrow U = k\frac{(-q)(+q)}{a^2}$$

$$U = -k\frac{q^2}{r^2} \qquad NA: U = 2.710^{-4} J$$

3)- Calculate the maximum potential energy:

The energy is maximal for the unstable positions α = π.

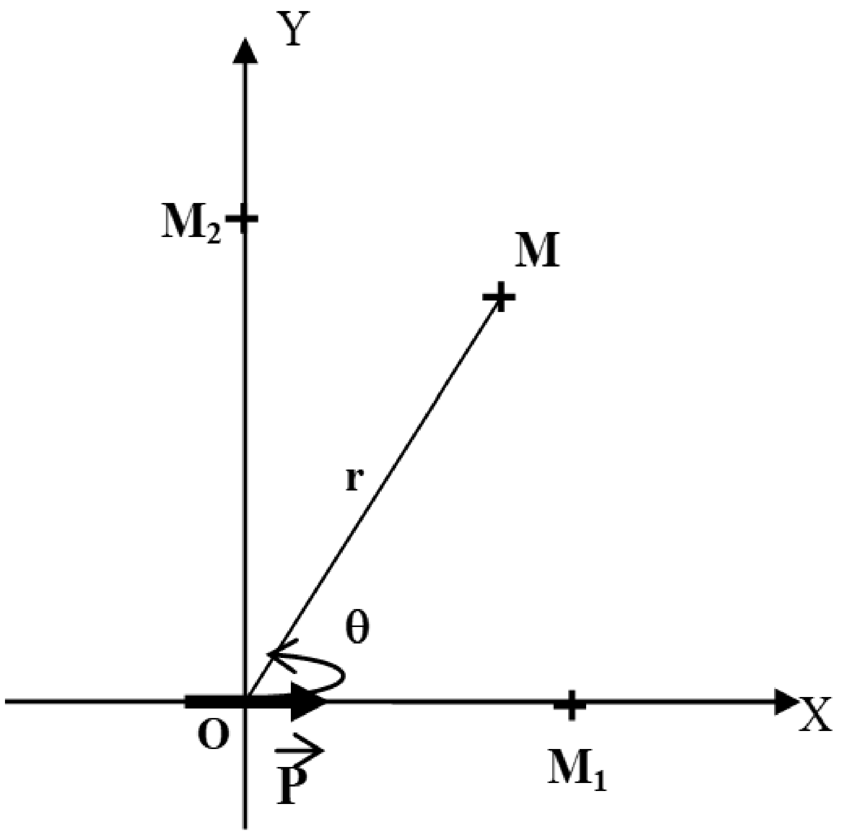
$$E_p = -pEcos\alpha = pE(cos\pi = -1)$$

$$NA: E_p = 3.37510^{-7} J = 2.11 \quad 10^{12} eV$$

Example 2

Let there be an electric dipole with a dipole moment $\vec{p}$, l he potential created by this dipole at point $M(r,\ \theta)$ is defined by:

$$V = kP\frac{\cos\theta}{r^2}$$



1- Determine the electric field at point M created by this dipole.

2- Deduce the value of the electric field and the electric potential created by this dipole at points **$M_1(a,0)$ et $M_2(0,b)$.**

Given: **$p=7.5\ 10^{-12}$ C.m, a= 2 mm, b=4mm.**

Solution:

Determine the electric field at point M created by this dipole.

We have:

$$\vec{E} = -\frac{\partial V}{\partial r}\vec{u}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta$$

So, by deriving the expression of V, we obtain the components of the field created by the electric dipole:

$$\vec{E}(r,\theta) = \begin{cases} E_r = \dfrac{1}{2\pi\varepsilon_0} p\left(\dfrac{\cos\theta}{r^3}\right) \\ E_\theta = \dfrac{1}{4\pi\varepsilon_0} p\left(\dfrac{\sin\theta}{r^3}\right) \end{cases}$$

2- Deduce the value of the electric field at M_1 and M_2.

We need to determine the position of each point in polar coordinates.

At point M1: r=2mm et θ=0

At point M2: r=4mm et θ=π/2

The potential:

$$V(M_1) = 910^9 7.510^{-12} \frac{cos\, 0}{(210^{-3})^2} = 16875V$$

$$V(M_2) = 910^9 7.510^{-12} \frac{cos(\pi/2)}{(410^{-3})^2} = 0V$$

Electrical field

$$\vec{E}(M_1) = \begin{cases} E_r = \dfrac{1}{2\pi\varepsilon_0} p\left(\dfrac{cos\, 0}{r^3}\right) = 1.687510^7 V/m \\ E_\theta = \dfrac{1}{4\pi\varepsilon_0} p\left(\dfrac{sin\, 0}{r^3}\right) = 0V/m \end{cases}$$

$$\vec{E}(M_2) = \begin{cases} E_r = \dfrac{1}{2\pi\varepsilon_0} p\left(\dfrac{\boldsymbol{cos}\, \pi/2}{r^3}\right) = 0V/m \\ E_\theta = \dfrac{1}{4\pi\varepsilon_0} p\left(\dfrac{\boldsymbol{sin}\, \pi/2}{r^3}\right) = 8.437510^6 V/m \end{cases}$$

Example 3

In a first approximation, a water molecule can be considered as formed by two H+ ions and one O^{2-} ion. The oxygen atom carrying the charge -2δ is linked to two hydrogen atoms, each carrying a charge of $+\delta$. δ is equal to 33% of the elementary charge; the angle between the two O-H bonds is denoted as α; the distance between an oxygen atom and a hydrogen atom is denoted as d.

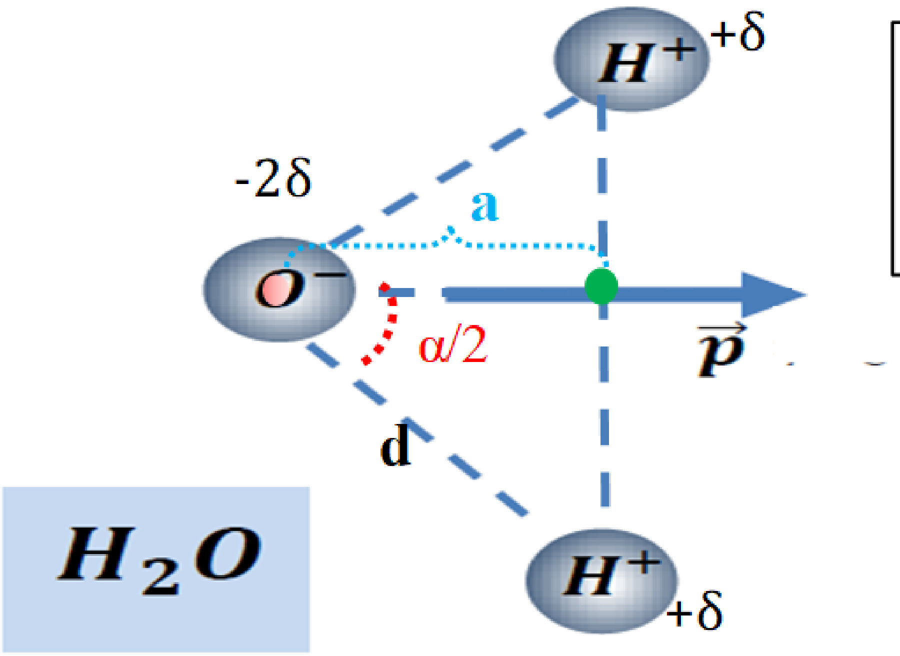
1) Draw a diagram of this molecule.

2) Determine the expression for the dipole moment of the water molecule and calculate its value.

$e = 1.60 \times 10-19$ C ; Distance O-H : $d = 0.952°$ A ($1°$ A $= 1 \times 10^{-10}$m); Angle α : $\alpha = 104°45'$ ($1'=1/60^{ième}$ de degré).

Solution

1) Diagram of the molecule H_2O :



-2δ Barycentre – (le centre des charges -)

+2δ Barycentre + (le centre des charges +)

2) Determine the expression for the dipole moment of the water molecule and calculate its value, in C m.

We know that: $\vec{p} = q\vec{a}$ avec $q = 2\delta = 2 * \underbrace{0.33\,e}_{\delta}$ et $a = d\cos\frac{\alpha}{2}$

$$p = 2\delta\, d\cos\frac{\alpha}{2}$$

$$p = 0.66\mathrm{e}\, d\cos\frac{\alpha}{2} \quad \mathrm{AN}: p = 6.137\,10^{-30}\ \mathrm{C.m}$$