Chapter I
Electrostatics

Course 6

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3. Continuous charge distribution

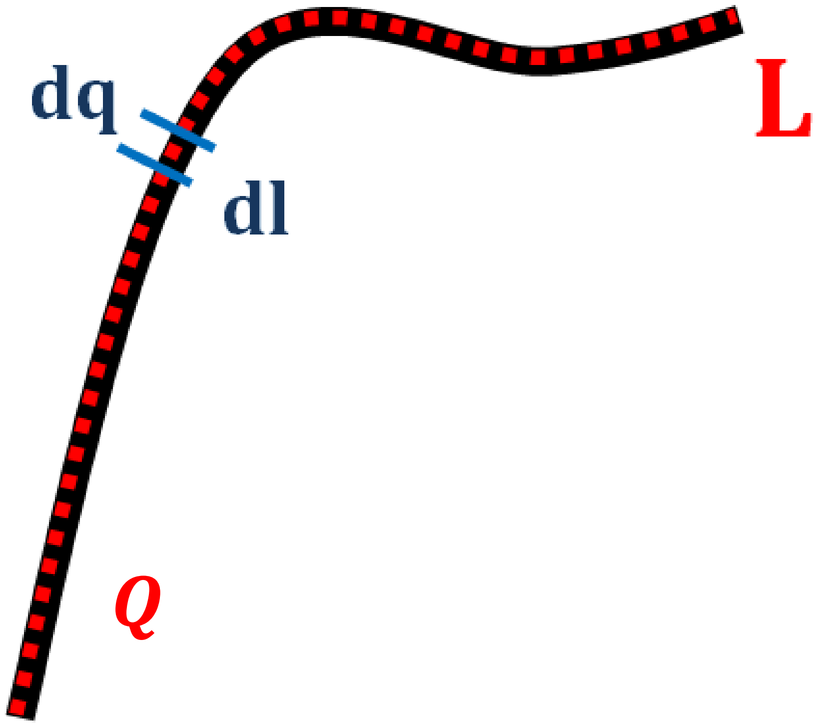
When charge Q is distributed continuously over the dimensions of a material (length, surface, or volume), it cannot be considered as a point charge. This concept necessitates the use of density parameters (linear, surface, or volume) to characterize the distribution of charge within the material.

What is continuous charge distribution?

In a continuous charge distribution, all the charges are closely bound together i.e. having very less space between them. But this closely bound system doesn't mean that the electric charge is uninterrupted. It clears that the distribution of separate charges is continuous, having a minor space between them.

a. Linear charge distribution:

The charge **Q** is distributed along a wire, in this case, we define the linear charge density, denoted by **λ**, given by the equation:



$$\boldsymbol{\lambda} = \frac{\boldsymbol{dq}}{\boldsymbol{dl}} = \frac{Elementary\ charge}{\text{Elementary length}}$$

Where: **dq** is the elementary charge carried by the elementary length **dl**.

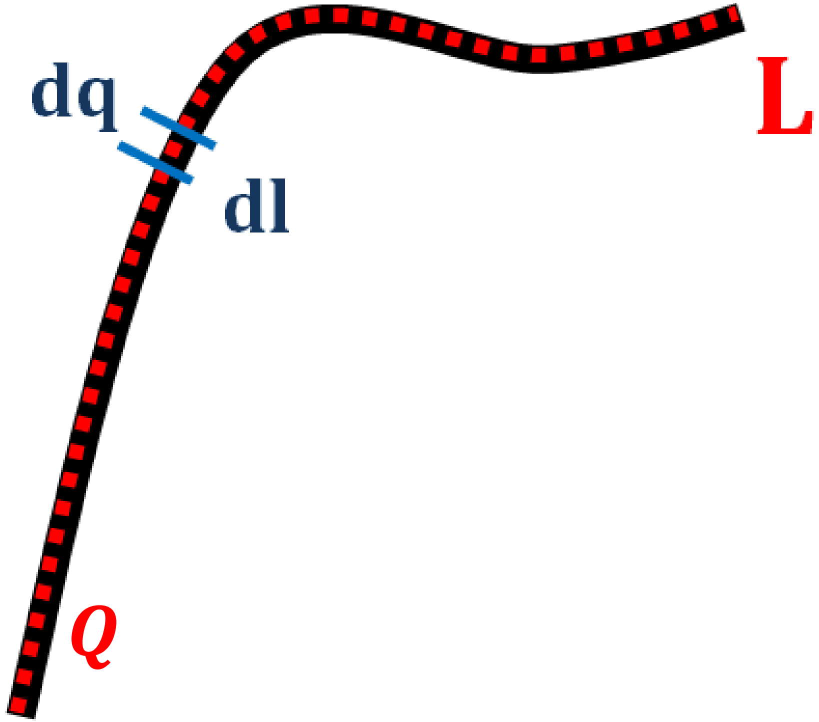
Its unit is Coulomb per meter (**C/m**).

The total charge carried by the wire of length **L** is:

$$dq = \lambda dl \Rightarrow Q = \int_0^L \lambda dl$$

If the charge **Q** is uniformly distributed

along the wire, then **λ** is constant.

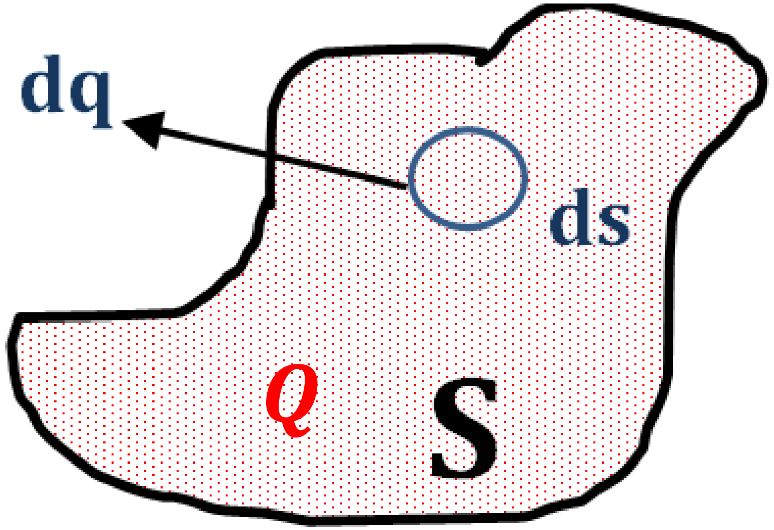


$$Q = \lambda \int_0^L dl \Rightarrow Q = \lambda L$$

b. Surface charge distribution

The charge ***Q*** is distributed over a surface **S**. In this case, we define the surface charge density, denoted by **σ**, given by the equation:

$$\boldsymbol{\sigma} = \frac{dq}{ds} = \frac{Ch\,arg\,e\,\text{élémentaire}}{Surface\text{élementaire}}$$



Where: **dq** is the elementary charge carried by the elementary surface ds.

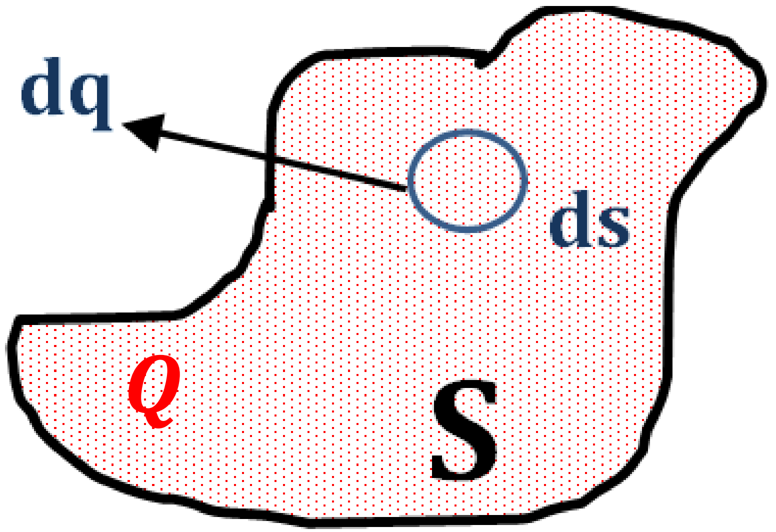
Its unit is Coulomb per square meter (C/m^2).

The total charge carried by the surface S is:

$$\boldsymbol{dq = \sigma ds \Rightarrow Q = \iint_S \sigma ds}$$

If the charge **Q** is uniformly distributed over the surface **S**, then **σ** is constant:

$$Q = \sigma \iint_S ds \Rightarrow Q = \sigma S$$



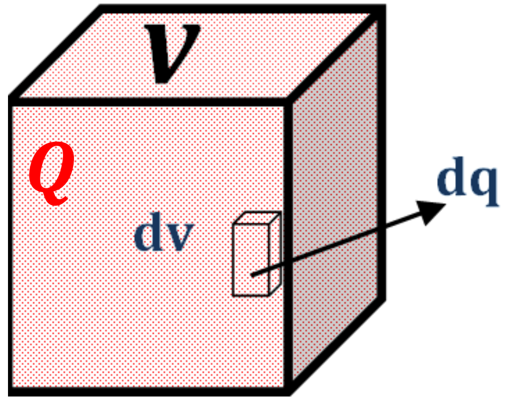
c. Volume charge distribution

The charge **Q** is distributed over a volume **V.** In this case, we define the volume charge density, denoted by ρ, given by the equation

$$\boldsymbol{\rho} = \frac{dq}{dv} = \frac{Ch\,arg\,e\,élémentaire}{volumeélementaire}$$

Where: **dq** is the elementary charge carried by the elementary volume **dv**.

Its unit is Coulomb per cubic meter (**C/m³**).

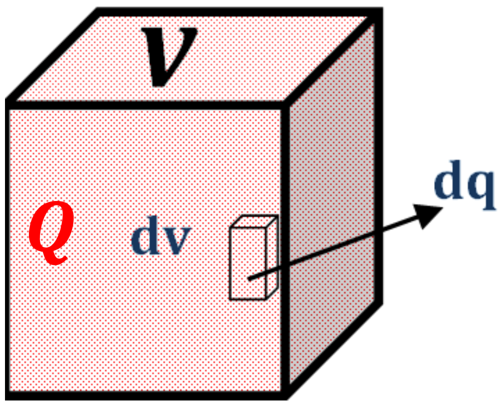


The total charge carried by the volume **v** is:

$$\boldsymbol{dq = \rho dv \Rightarrow Q = \iiint_{v} \rho dv}$$

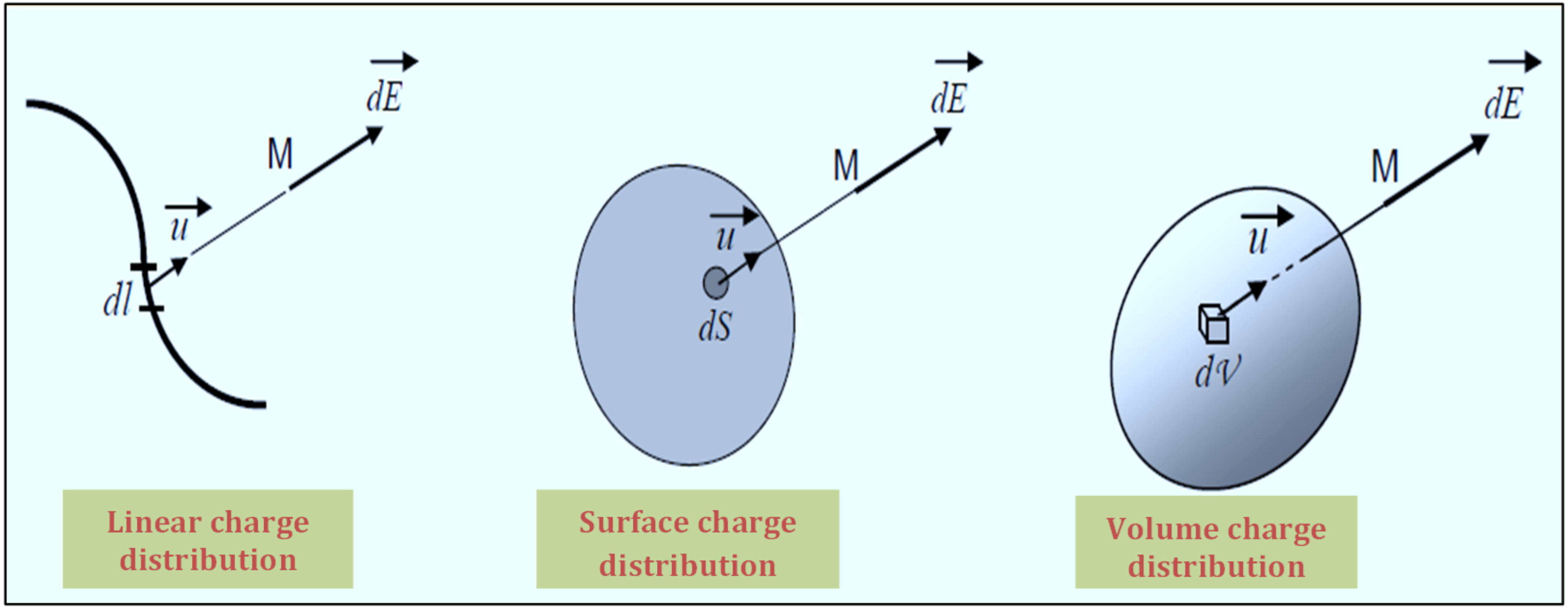
If the charge **Q** is uniformly distributed over the volume **v**, then **ρ** is constant:

$$Q = \rho \iiint_{v} dv \Rightarrow Q = \rho v$$



4. The electric field and potential created by a continuous distribution of charges

To calculate the electric field $\vec{\boldsymbol{E}}$ and the potential **V** at point **M** in space created by a continuous charge distribution, we start by expressing the elementary electric field $\boldsymbol{d}\vec{\boldsymbol{E}}$ and the elementary potential **dV** created by an elementary charge **dq** at point **M** in space. By integrating the obtained relations, we obtain the total electric field and potential created by this charge distribution (by the total charge **Q**) at point M


dE
M
u
dl
Linear charge distribution
dE
M
u
dS
Surface charge distribution
dE
M
u
dV
Volume charge distribution

- **Case of linear charge distribution:**

$$\vec{E}_M = \int d\vec{E}_M = k\int_L \frac{\lambda dl}{r^2}\vec{u}_r$$

$$V_M = k\int_L \frac{\lambda dl}{r}$$

- **Case of surface charge distribution:**

$$\vec{E}_M = \int d\vec{E}_M = k\iint_S \frac{\sigma ds}{r^2}\vec{u}_r$$

$$V_M = k\iint_S \frac{\sigma ds}{r}$$

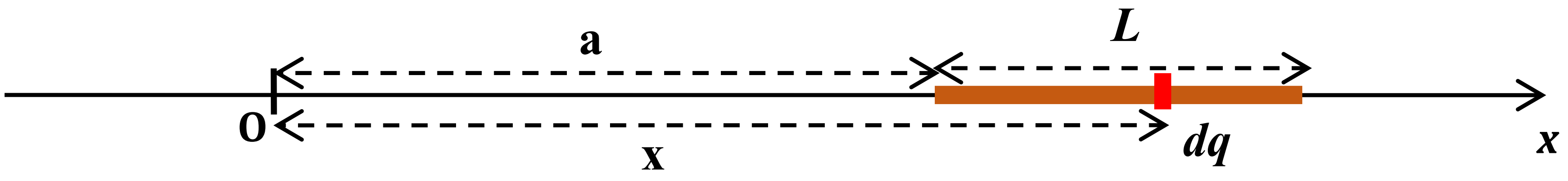
- **Case of volume charge distribution:**

$$\vec{E}_M = \int d\vec{E}_M = k\iiint_v \frac{\rho dv}{r^2}\vec{u}_r$$

$$V_M = k\iiint_v \frac{\rho dv}{r}$$

Example 1

A metal rod of length **L** carries a charge **Q** uniformly distributed with a linear charge density λ **>0** (see figure below).



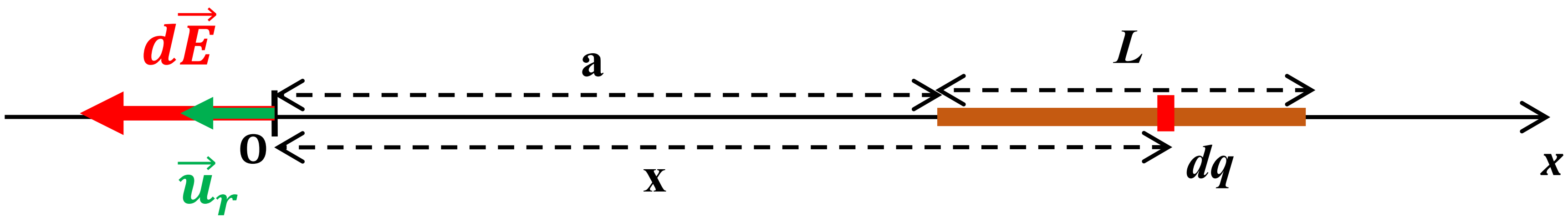
1. Find the net electric field $\vec{E}_o$ created by the rod at point **O** as a function of **L, a, and** λ **.**

2. Find the net potential V_o created by the rod at point O.

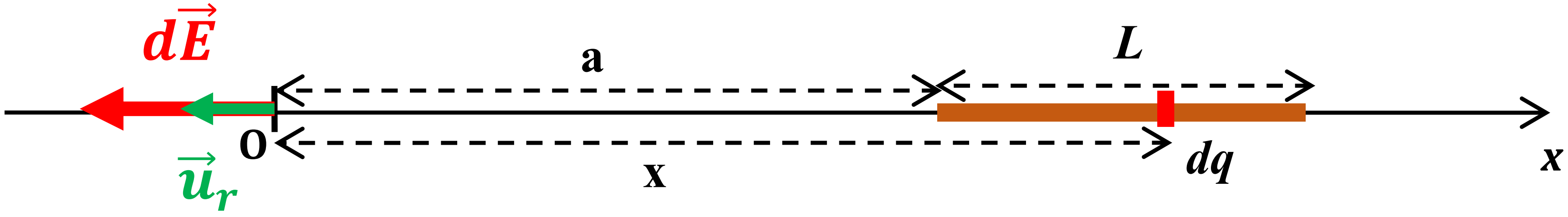
Solution

Firstly, you need to plot the elementary field vector $\vec{dE}$ at point O created by the elementary charge dq as well as the unit vector $\vec{u}_r$. This field emanates from dq towards **O** because the charge is positive.

The elementary field is given by the following relationship:



$$d\vec{E}_o = k\frac{dq}{r^2}\vec{u}_r$$



From the figure, we can deduce that: $\vec{u}_r = -\vec{\imath}$; $r = x, and$ $dq = \lambda dx$

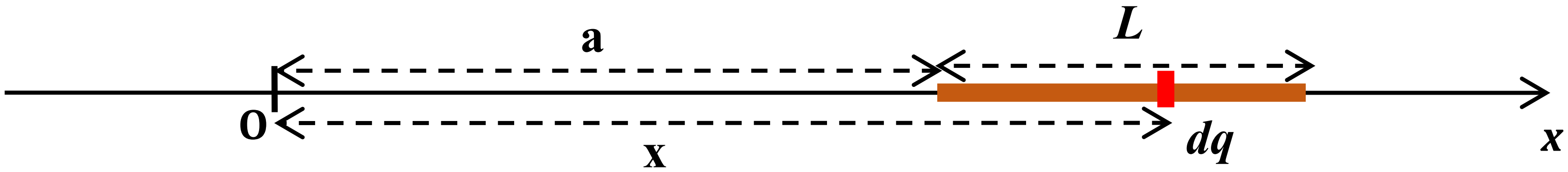
$$\Rightarrow \qquad d\vec{E}_o = -k\frac{\lambda dx}{x^2}\vec{\imath}$$

Finding The net field created by the rod at point O:

$$\vec{E}_o = \int d\vec{E}_o = -\underbrace{k\lambda}_{Cte}\int_a^{L+a}\frac{dx}{x^2}\vec{\imath} \Rightarrow \vec{E}_o = -k\lambda\left[-\frac{1}{x}\right]_a^{L+a}\vec{\imath}$$

$$\vec{E}_o = -\frac{k\lambda L}{a(L+a)}\vec{\imath}$$

2. Finding the net potential $\boldsymbol{V_o}$ created by the rod at point O.



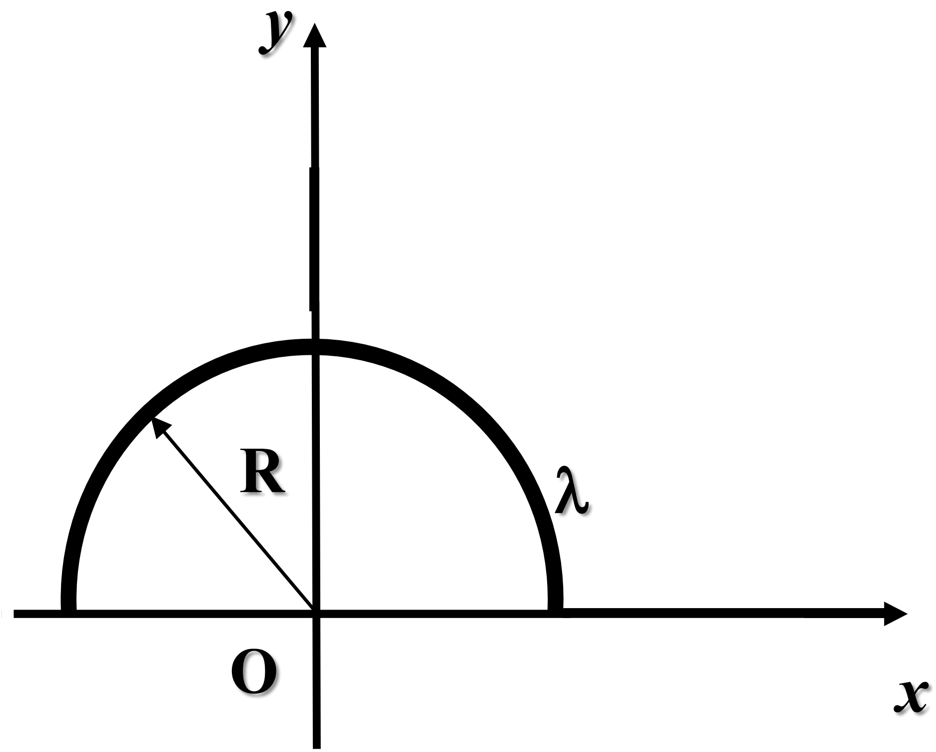
$$dV_o = k\frac{dq}{r} \Rightarrow dV_o = k\frac{\lambda dx}{x}$$

$$V_0 = \int dV_o = k\lambda\int_a^{L+a}\frac{dx}{x} \Rightarrow V_o = k\lambda[Lnx]_a^{L+a}$$

$$\boxed{V_o = k\lambda Ln\left(\frac{L+a}{a}\right)}$$

Example 2

Consider a linear charge distribution with density λ, uniformly spread over a wire in the shape of a semicircle with radius R, as shown in the figure below.



Find the expression for the electric field $\vec{E}_o$ created by the charge distribution λ at point O .

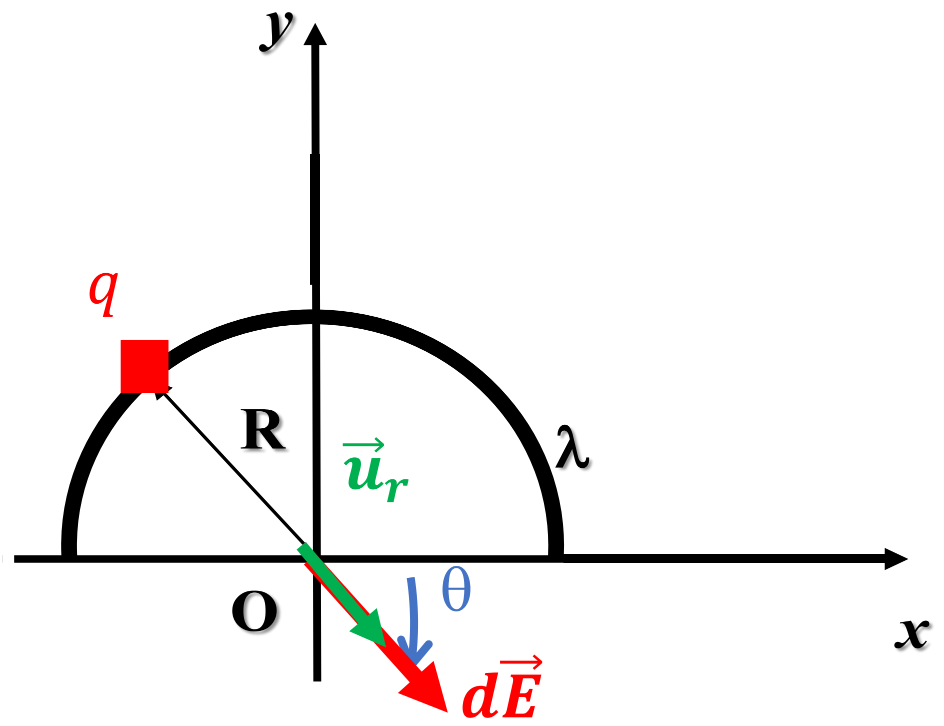
Solution

The elementary field created by an elementary charge is given by

$$d\vec{E}_2 = k\frac{dq}{R^2}\vec{u} \quad and \quad dq = \lambda dl,$$

The projection onto the axes:

$$\begin{cases} d\vec{E}_x = K\dfrac{\lambda dl}{R^2}cos\theta\vec{i} \\ d\vec{E}_y = -K\dfrac{\lambda dl}{R^2}sin\theta\vec{j} \end{cases} \quad and \ \ dl = Rd\theta$$



$$\vec{E} \quad \begin{cases} d\vec{E}_x = K\dfrac{\lambda R d\theta}{R^2}cos\theta\vec{\imath} \\ d\vec{E}_y = -K\dfrac{\lambda R d\theta}{R^2}sin\theta\vec{\jmath} \end{cases}$$

$$\vec{E} \quad \begin{cases} \vec{E}_x = K\dfrac{\lambda}{R}\displaystyle\int_0^{\pi} cos\theta d\theta\vec{\imath} = \dfrac{K\lambda}{R}[sin\,\pi - sin\,0]\vec{\imath} = 0\vec{\imath} \\ \vec{E}_y = -K\dfrac{\lambda}{R}\displaystyle\int_0^{\pi} sin\theta d\theta\,\vec{\jmath} = K\dfrac{\lambda}{R}[c\,os\,\pi - cos\,0]\vec{\jmath} = -\dfrac{2K\lambda}{R}\vec{\jmath} \end{cases}$$

$$\Rightarrow \vec{E} \quad = -\frac{2K\lambda}{R}\vec{\jmath}$$