Chapter I
Electrostatics

Course 5

Dr IACHACHENE FARIDA FHC-UMBB-2023-2024

Work and Energy

1. Electrostatic Potential Energy:

The particle's past

$V(\infty)=0$

$E_p(\infty)=0$

Position at ∞

q

Move q from ∞ to point M

Current position of the particle

Position M

$V(M)\neq 0$

$E_p(M)\neq 0$

$Zone\ \exists V \ \Longrightarrow \ \exists \vec{E}$

1. Electrostatic Potential Energy:

If we place an electric charge $\boldsymbol{q}$ at a point $\boldsymbol{M}$ where the electric potential is $\boldsymbol{V_M}$ (see Figure), it will have a potential energy $\boldsymbol{E_p}$ given by:

$$\boldsymbol{E_P = q\, V_M}$$

The unit of $\boldsymbol{E_p}$ is joule (J).

Note :

The electrostatic potential energy of a charged particle placed in an electrostatic field is equal to the work required to move that particle from its current position to infinity.

1. Work done by the electric force

When we place an electric charge $\boldsymbol{q}$ at a point in space where an electric field $\vec{\boldsymbol{E}}$ exists, it is then subjected to the action of an electric force:

$$\vec{F} = q.\vec{E}$$

The work done by this force during an infinitesimal displacement $\vec{dl}$ is:

$$dw = \vec{F}.\vec{dl}$$

The total work between two points A and B is given by:

$$W(\vec{F})_A^B = \int_A^B \overbrace{\vec{F}}^{q.\vec{E}} .\vec{dl} \Rightarrow \quad W(\vec{F})_A^B = q\int_A^B \overbrace{\vec{E}.\vec{dl}}^{-dV}$$

$$\Rightarrow \quad W(\vec{F})_A^B = -q\int_A^B dV$$

$$\Rightarrow \quad W(\vec{F})_A^B = -q(V_B - V_A)$$

$$\boxed{W(\vec{F})_A^B = q(V_A - V_B)} \quad (*)$$

We know that the potential energy: $\boldsymbol{E_P = q\,V}$

$$(*) \quad \Rightarrow \boldsymbol{W(\vec{F})_A^B = qV_A - qV_B}$$

$$\boldsymbol{W(\vec{F})_A^B = E_P(A) - E_P(B)}$$

The work done by the electric force $\vec{\boldsymbol{F}}$ (Coulomb force) between two positions A and B depends on its potential energy at these two positions.

$$\boxed{\boldsymbol{W(\vec{F})_A^B = -\Delta E_p\big]_A^B}}$$

3. Internal Energy of an Electric Charge Distribution

The internal energy of a system of **n** charges is defined as the work done by an operator to assemble the two charges, initially without interaction.

a- System composed of two point charges

The internal energy of a system of two charges is defined as the work done by an operator to assemble the two charges, initially without interaction. It is the potential energy of the second charge in the field of the first (or the potential energy of the first charge in the field of the second), given by the relation:"

$$U = K\frac{q_1\, q_2}{r_{12}}$$

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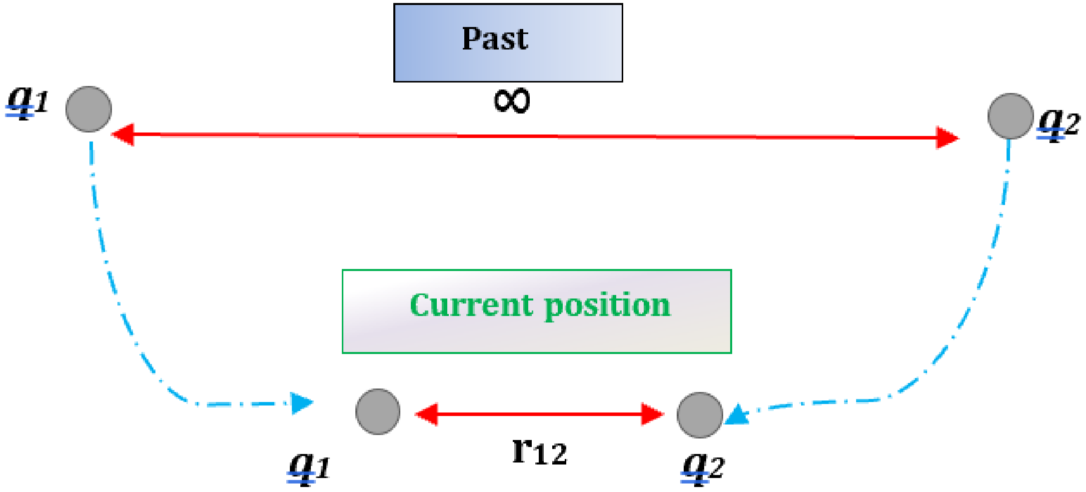
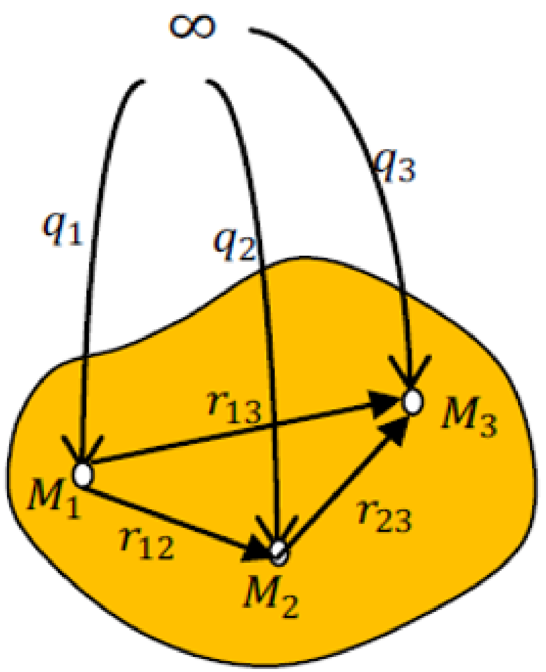


b- System composed of three-point charges

In this case, the internal energy of a system is the sum of all potential energies between charges taken two by two (without repeating the same potential energy twice), thus:

$$U = K\frac{q_1\, q_2}{r_{12}} + K\frac{q_1\, q_3}{r_{13}} + K\frac{q_2\, q_3}{r_{23}}$$



C - System composed of n point charges

The internal energy of a system composed of n charges is given by the relation:

$$\boxed{U_{syst} = \sum_i \sum_j U_{ij} = \frac{k}{2} \sum_i \sum_j \frac{q_i q_j}{r_{ij}}} \qquad (i \neq j)$$

We can express the potential energy as a function of the potential

$$V(i) = \sum_j k \frac{q_j}{r_{ij}} \qquad (i \neq j)$$

$V(i)$ is the potential created at the point where charge q_i is located by the rest of the charges.

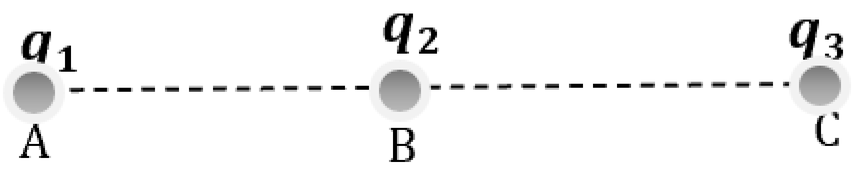
Then we have:

$$\boxed{U_{syst} = \frac{1}{2} \sum_i q_i . V(i)}$$

Example:

Let's consider three electric charges $\boldsymbol{q_1, q_2}$ ***and*** $\boldsymbol{q_3}$ placed at points A, B, and successively (see the figure below).

As given $\boldsymbol{q_1 = -3\mu C}$, $\boldsymbol{q_2 = 2\mu C}$ ***et*** $\boldsymbol{q_3 = 8\mu C}$ **and** $\boldsymbol{BC = AB = 3cm}$



- Calculate the internal energy of this system.

Solution:

The internal energy of this system is:

$$U = K\frac{q_1\, q_2}{AB} + K\frac{q_1\, q_3}{AC} + K\frac{q_2\, q_3}{BC}$$

NA :

$$U = 9\,10^9 * 10^{-12}\left(\frac{-3 *\ 2}{3\,10^{-2}} + \frac{-3 *\ 8}{6\,10^{-2}} + \frac{2 * 8}{3\,10^{-2}}\right)$$

$$U = \ -0.6\,J$$