Chapter I
Electrostatics

Course 4

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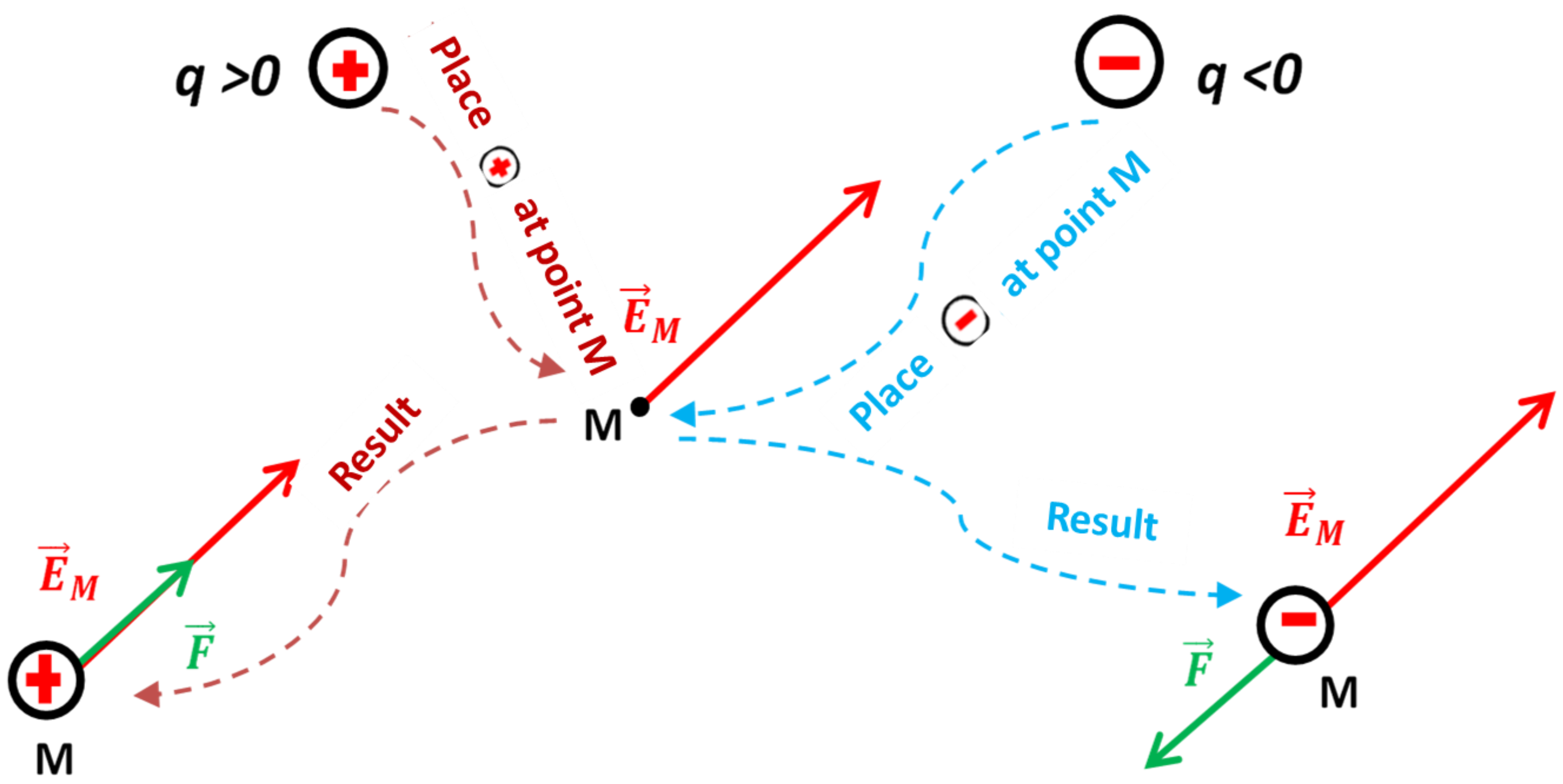
3- Relation between the electric field $\vec{E}$ and the electric force $\vec{F}$

If we place an electric charge **q** ***at*** **point M** where the electric field is $\vec{\boldsymbol{E}}(\boldsymbol{M})$, there will appear on this charge an electric force $\vec{\boldsymbol{F}}$ (see Figure below), given by the following relation:

$$\overrightarrow{\boldsymbol{F}} = \boldsymbol{q}.\overrightarrow{\boldsymbol{E}}(\boldsymbol{M})$$

$$\boldsymbol{If} \begin{cases} q > 0 \implies \overrightarrow{\boldsymbol{F}} \; and \; \overrightarrow{\boldsymbol{E}} \; in\ the\ same\ direction \\ q < 0 \implies \overrightarrow{\boldsymbol{F}} \; et \; \overrightarrow{\boldsymbol{E}} \; in\ the\ opposit\ direction \end{cases}$$



A charge q (+) or (-) is placed at point M where the field is $\vec{E}(M)$

$\Rightarrow$ An electric force $\vec{F}$ applied on q

the direction of $\vec{F}$ depends on the nature of the charge q.

Note:

In the case of two point charges $\mathbf{q_1}$ and $\boldsymbol{q_2}$ the electrical force exerted by $\mathbf{q_1}$ on $\boldsymbol{q_2}$ is given by Coulomb's law :

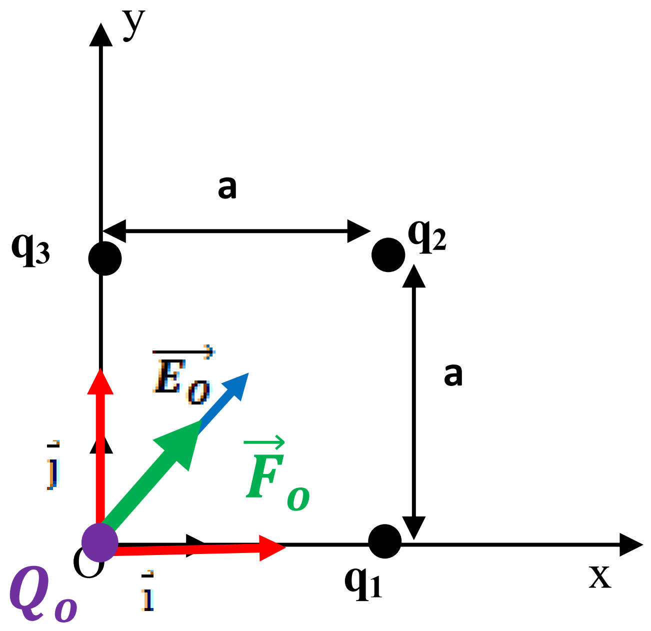
$$\vec{F}_{1/2} = K\frac{q_1 q_2}{r^2}\vec{u}$$

$\vec{E}_1(r)$: **The electric field created by the charge q_1 at position (2)**

$$\Rightarrow \vec{F}_{1/2} = q_2 . \vec{E}_1$$

Example 2 : Returning to the previous example (example 1). A charge $\mathbf{Q_0=8.10^{-6}}$ **C** is placed at point O.

Deduce the electric force applied to $\mathbf{Q_0}$.



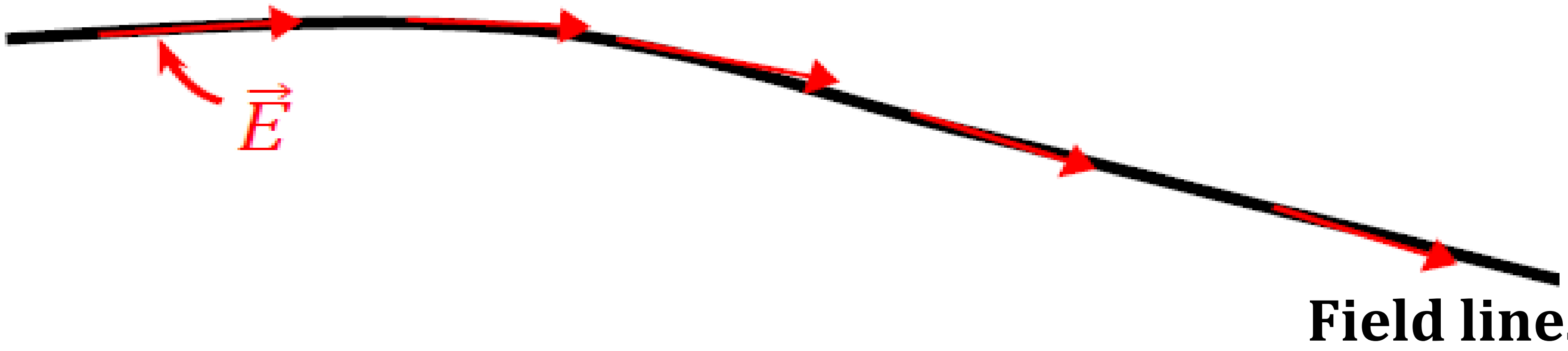
Solution :

Previously, we determined the electric field at the point where $\mathbf{Q_0}$ is located (point O); therefore, we can directly deduce the force applied to the charge $\mathbf{Q_0}$

$$\vec{F_o} = Q_o \vec{E_o} \rightarrow \left\|\vec{F_o}\right\| = |Q_o| \left\|\vec{E_o}\right\| = 0.21N$$

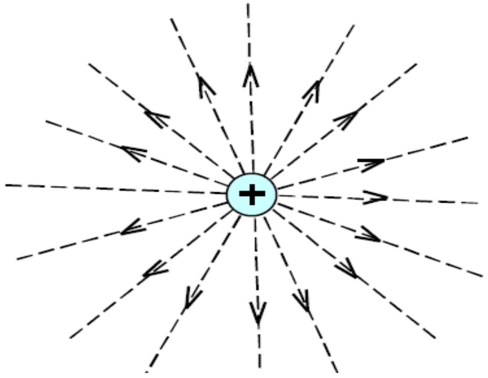
The field lines and equipotential surfaces

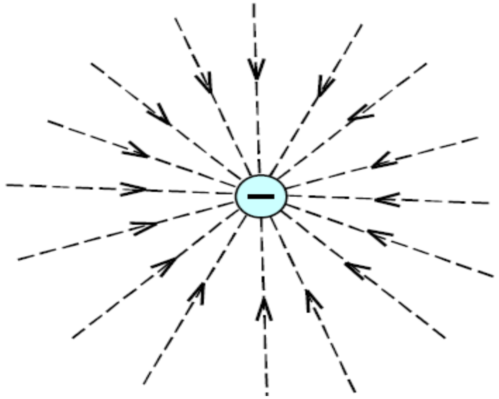
The field lines are lines tangent to the electric field vector at each of its points.

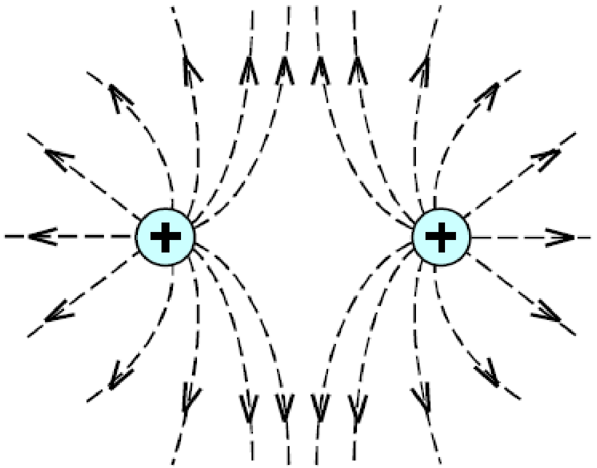


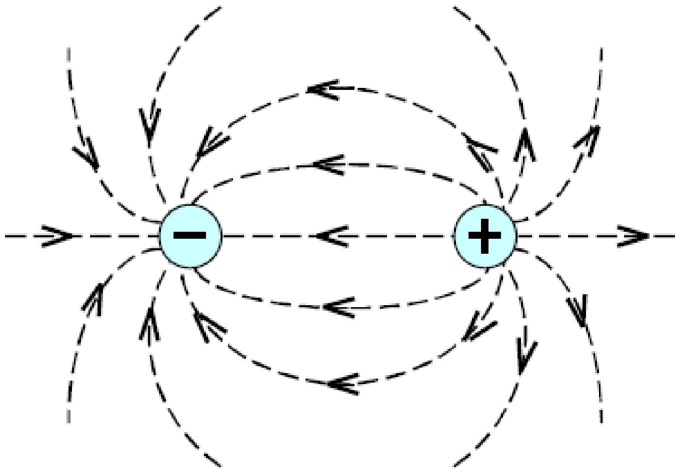
Example

The presence of two charges of equal magnitude leads to a deformation of the field lines, resulting in a new topography. At each point, the field line is tangent to the resultant field.

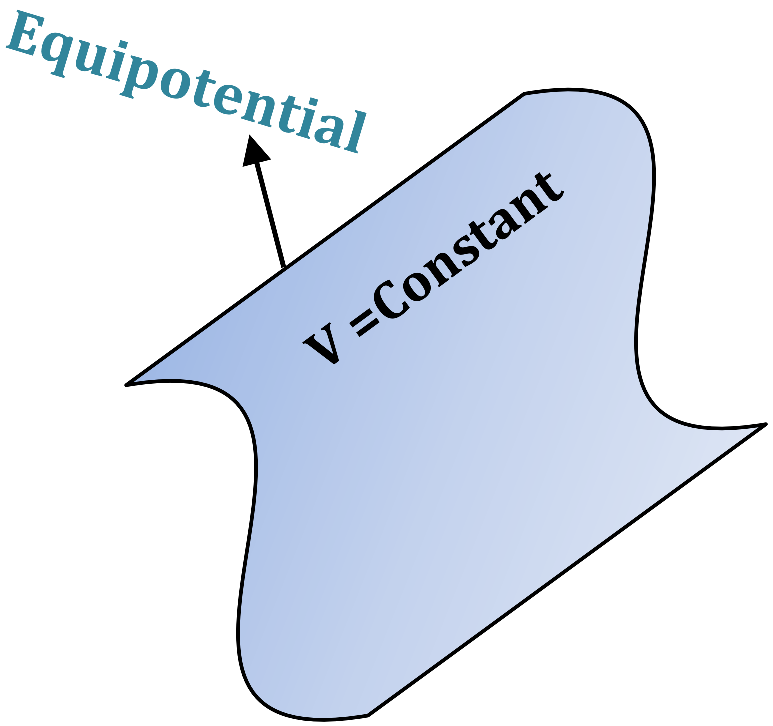




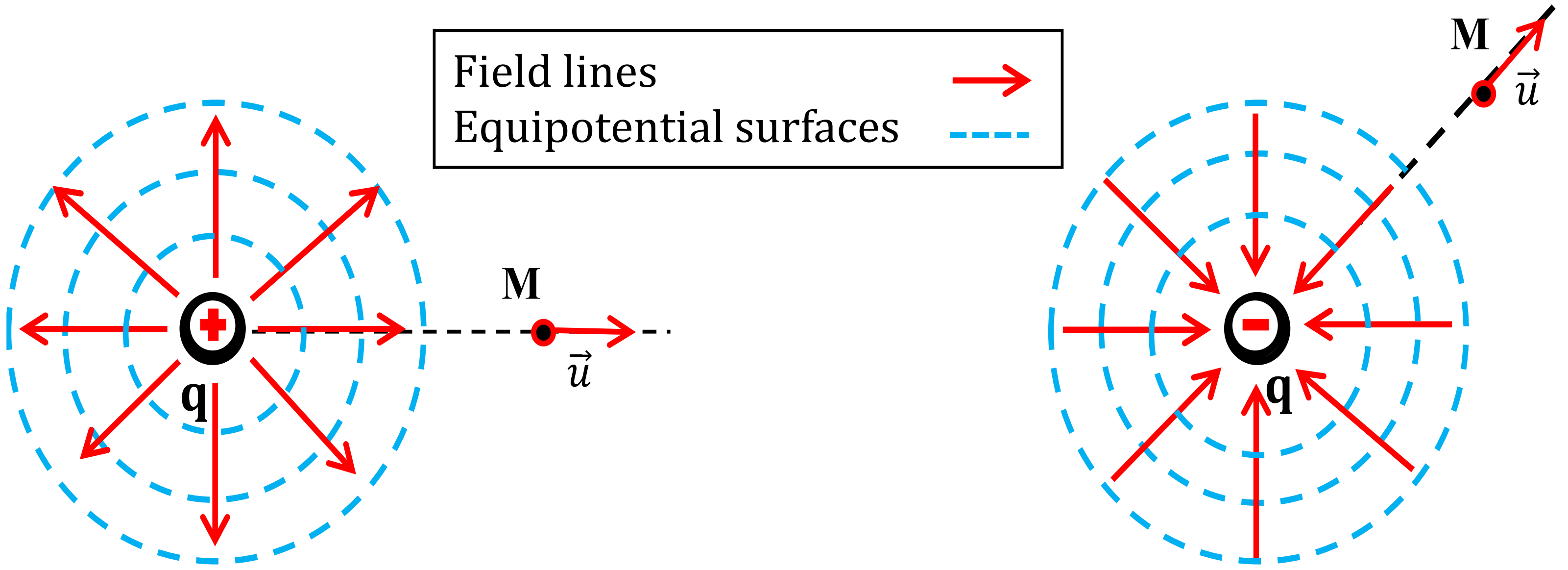


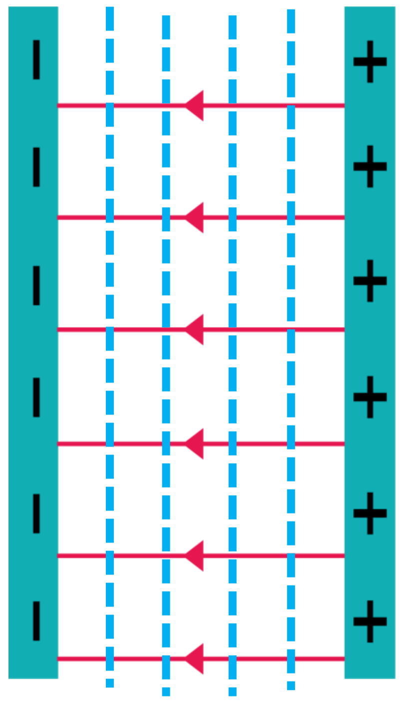


The equipotential surfaces are surfaces where the potential is the same at every point; V on this surface is constant (dV=0), as shown in Figure below.

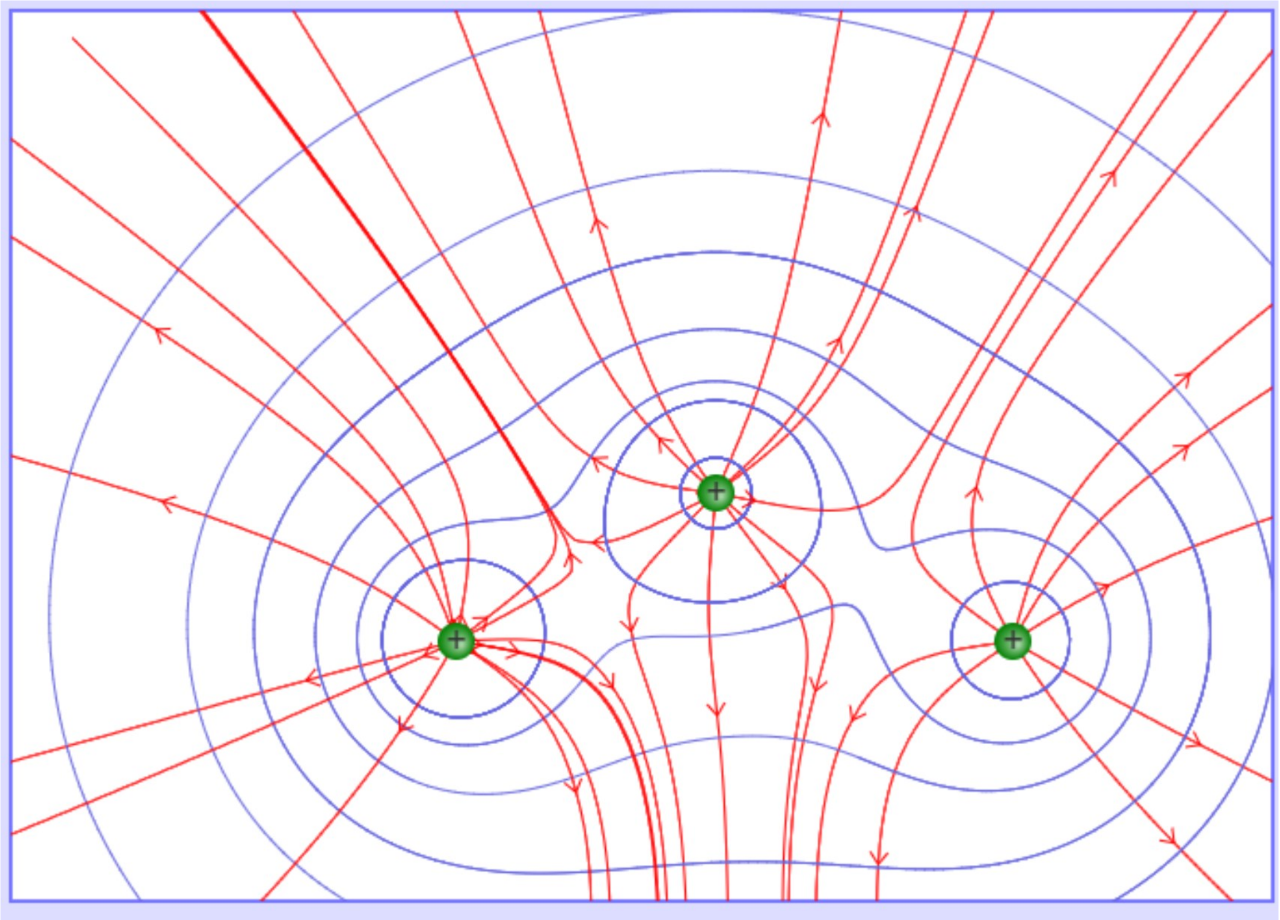


In the case of a point charge, the equipotential surfaces are concentric spheres centered at O, and the field lines are radial, as depicted in Figure below



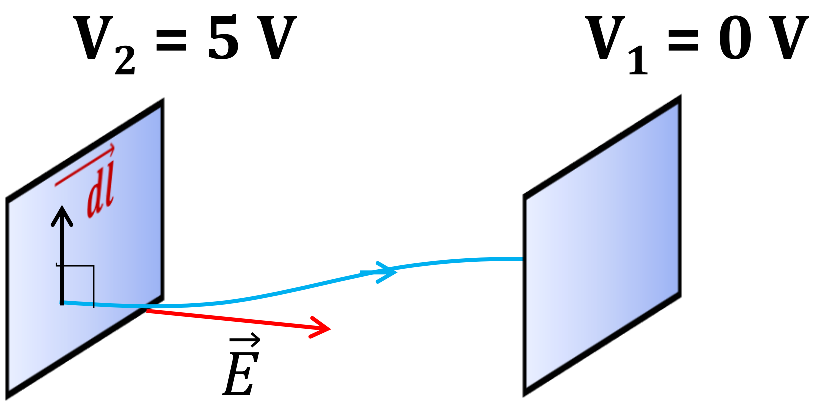


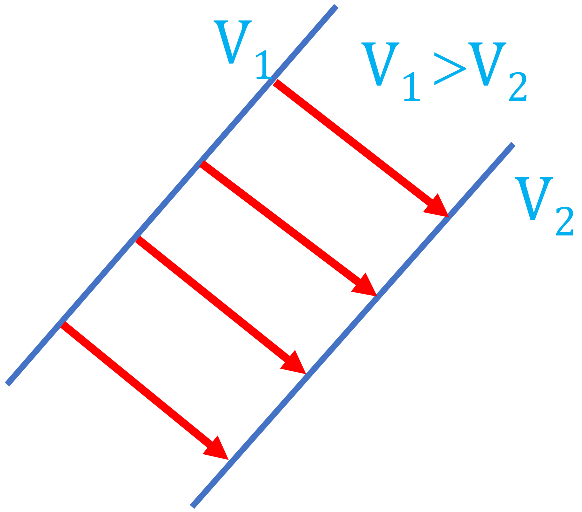
Between two parallel planes.

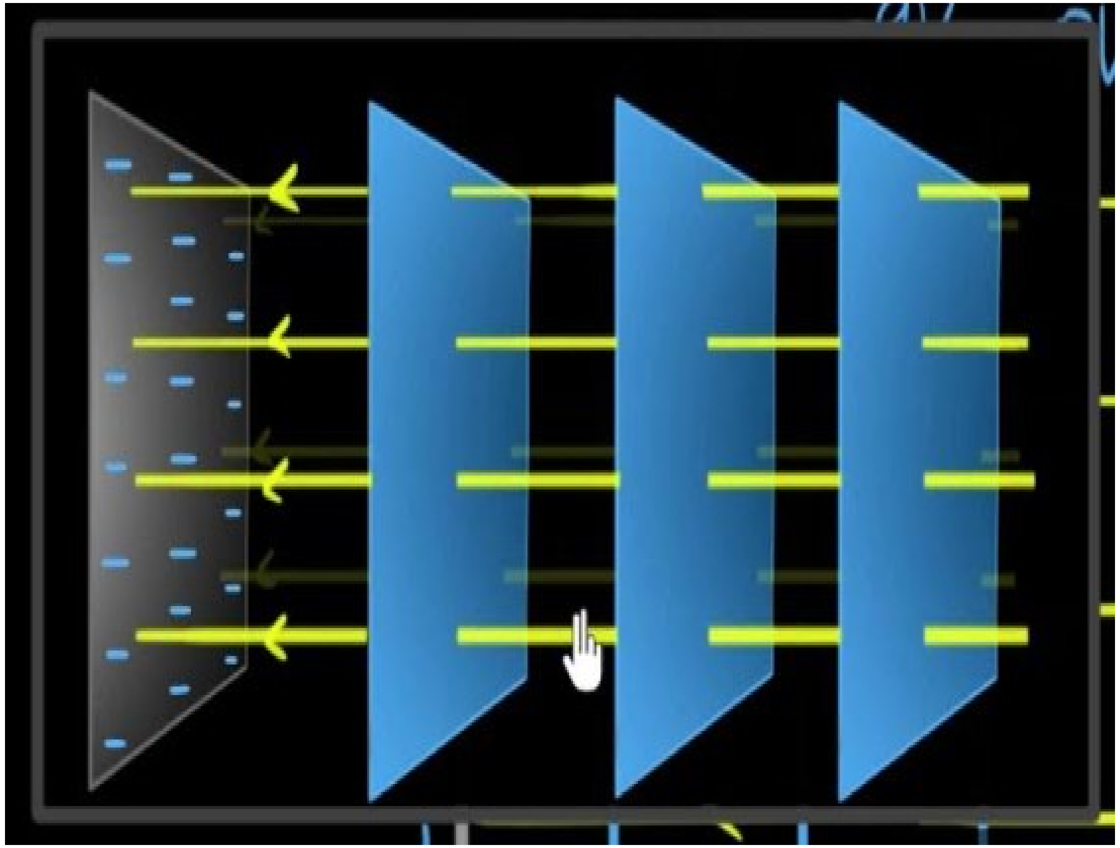


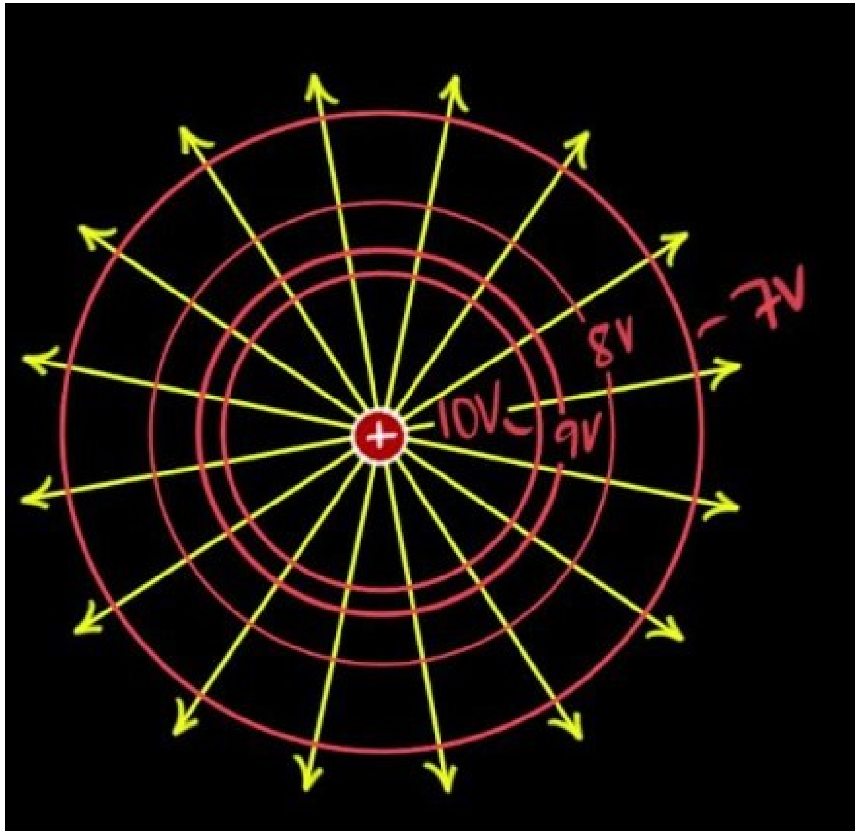
Note :

- The field line is oriented from the highest potential to the lowest potential, meaning the potential decreases along a field line.
- The electric field (or field line) is perpendicular to the equipotential V.
- The electric field is more intense where equipotential lines are closer together.








10V
9V
8V
7V

1) Relation between the electric field and the electric potential

The mathematical relationship between the electric field $\vec{E}$ and the electric potential **V** is defined by:

$$\vec{E}(\vec{r}) = -\overrightarrow{grad}(V(r))$$

In Cartesian coordinates:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

In cylindrical coordinates:

$$\vec{E} = -\left(\frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{\partial V}{\partial z}\vec{k}\right)$$

In spherical coordinates:

$$\vec{E} = -\left(\frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{1}{r\,sin\theta}\frac{\partial V}{\partial \varphi}\vec{u}_\varphi\right)$$

From this relationship:

- If we know the expression of the potential **V** at every point in space, we can determine the electric field $\vec{E}$ by taking the derivative of the expression for **V.**
- Conversely, if the expression for the electric field $\vec{E}$ is known, then we can calculate the electric potential **V** by integrating the expression for $\vec{E}$.