Chapter I
Electrostatics

Course 2

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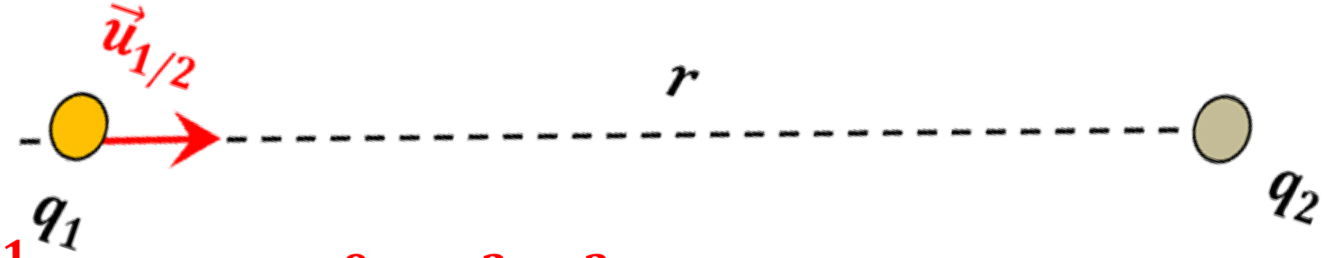
I.2 Electrostatic Forces : Coulomb's Law

The electrical force exerted by the point charge (شحنه نقطيه) q_1 on the point charge q_2, both placed in free space (الفراغ), is given by the law

$$\vec{F}_{1/2} = K\frac{q_1 q_2}{r^2}\vec{u}_{1/2}$$



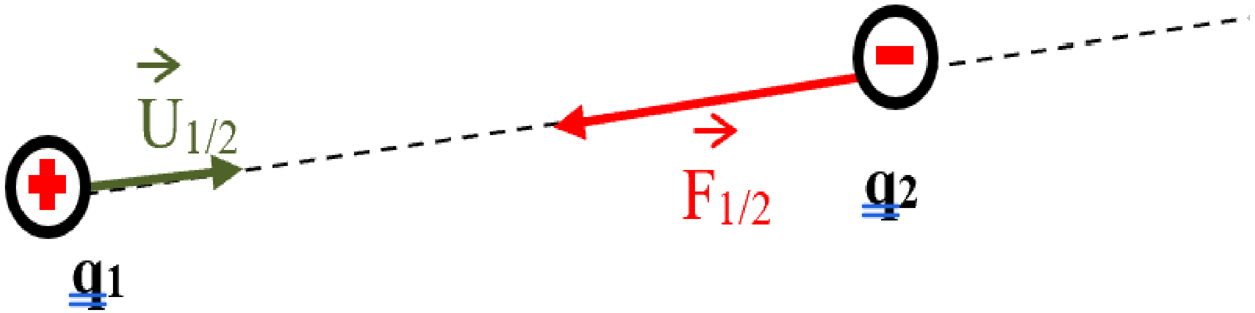
K is the Coulomb's constant, $K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2/C^2$

r: is the distance between the two charges.

$\vec{u}_{1/2}$ is a unit vector pointing outwards from the charge causing the force q_1 towards the affected q_2. (cause towards effect)

$\varepsilon_0 = \frac{10^{-9}}{36\pi}$ is the electric permittivity of free space (in matter, $\varepsilon = \varepsilon_0\varepsilon_r$ is used where ε_r is the relative permittivity of the material compared to free space).

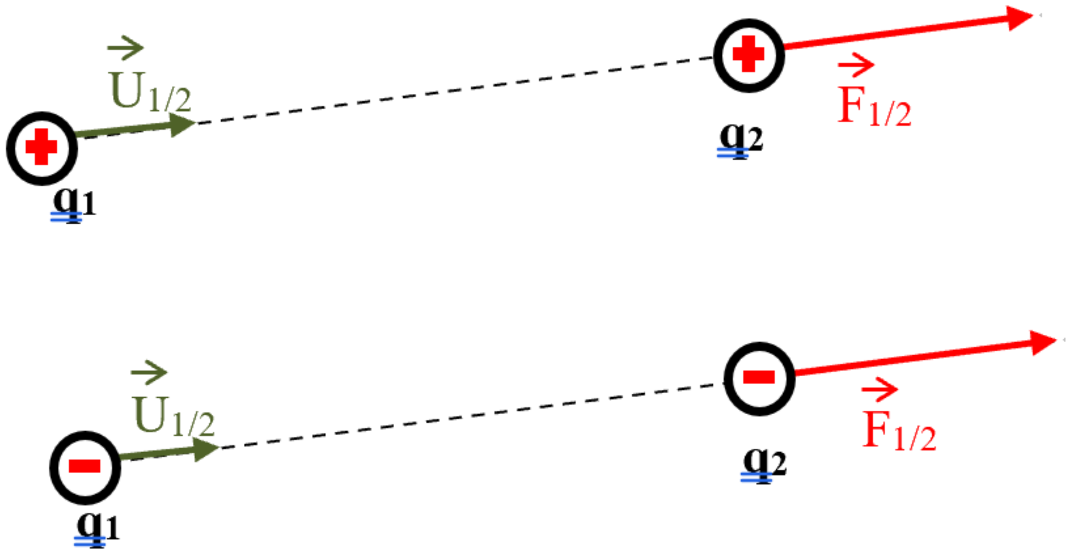
- If q_1 and q_2 are of different nature: the force exerted by q_1 on q_2 is attractive. Therefore, $\vec{F}_{1/2}$ and $\vec{u}_{1/2}$ have opposite directions



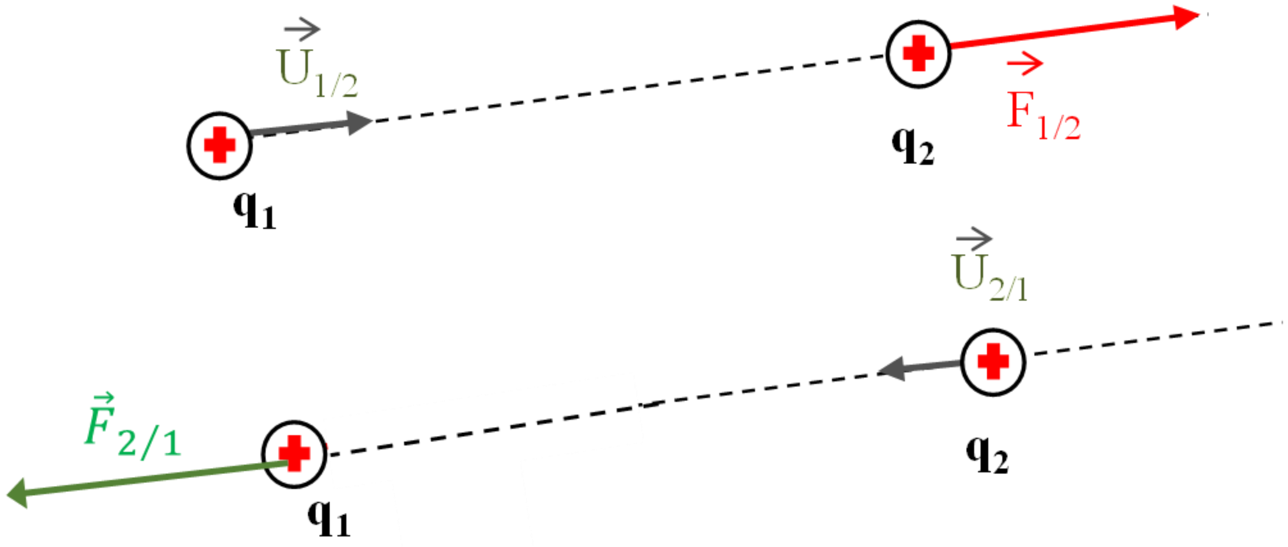
$$\vec{F}_{1/2} = K\frac{q_1 q_2}{r^2}\vec{u}_{1/2}$$

- If q_1 et q_2 are of the same nature : : the force exerted by q_1 sur q_2 is repulsive.

Therefore $\vec{F}_{1/2}$ have same direction $\vec{u}_{1/2}$.



Note: If charge q_1 exerts a force on charge q_2 , according to the principle of action and reaction, q_2 will exert an equal and opposite force on q_1.



$$\vec{F}_{1/2} = -\vec{F}_{2/1}$$

$$\left\|\vec{F}_{1/2}\right\| = \left\|\vec{F}_{2/1}\right\| = K\frac{|q_1||q_2|}{r^2}$$

Example

Calculate the electric force exerted by the charge q_1 on the charge q_2, given that:

$q_1 = 3.10^{-3}$ C, $q_2 = -5.10^4\ C$ and $r = 20\ mm$

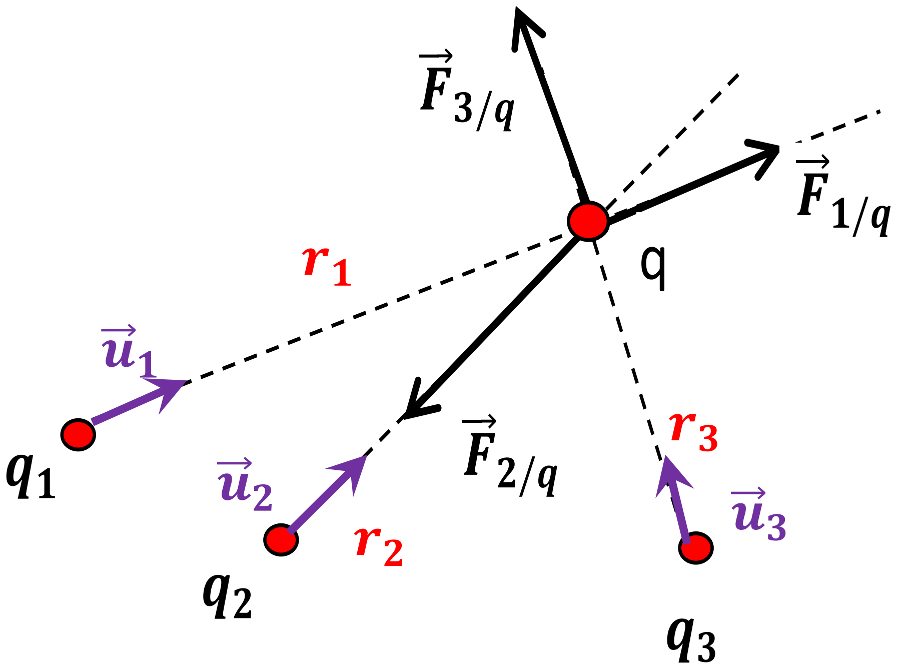
$$\left\|\vec{F}_{1/2}\right\| = K\,\frac{|q_1||q_2|}{r^2} = 9.10^9\,\frac{3.10^{-3}\ 5.10^{-4}}{(20.10^{-3})^2} = 33{,}7510^6\ C$$

q_1 and q_2 have opposite signs $\vec{F}_{1/2}$ is in opposite direction to $\vec{u}_{1/2}$, therefore the two charges attract each other (attractive force).

- Superposition principle

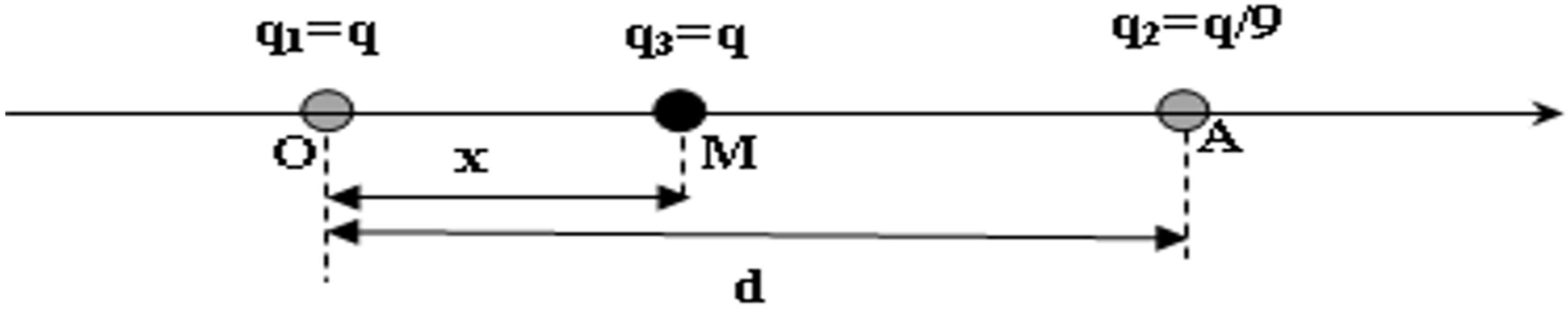
The force applied by multiple point charges q_i on another point charge q is equal to the sum of all the forces exerted by the individual charges, one by one, as if the others do not exist.

$$\vec{F}_{(q_i\ sur\ q)} = K \sum_{1}^{n} \frac{q\ q_i}{r^2} \vec{u}_i$$



Exercise

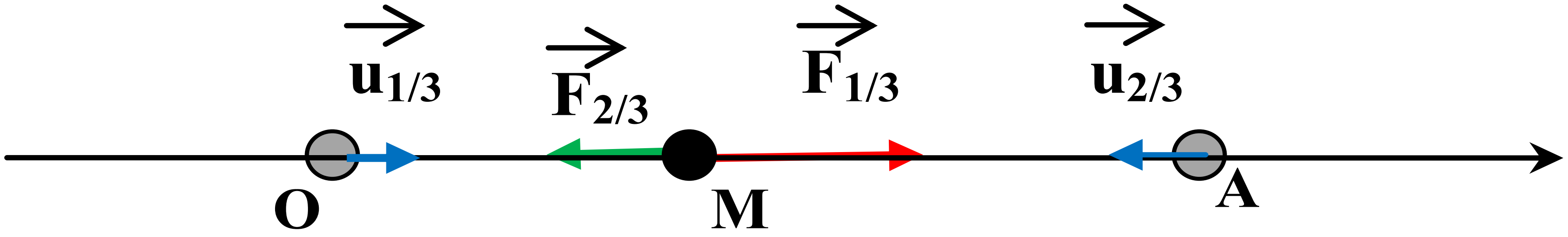
Let's consider the system of point charges as represented in the figure below. The charges $\mathbf{q_1}$ and $\mathbf{q_2}$ are fixed at positions O and A, respectively, with a separation distance of d. A third charge $\mathbf{q_3}$, is placed between them.



1. Find the expression for the force $\vec{\mathbf{F}}$ exerted on q_3 at position M.
2. At what abscissa $\mathbf{x}$ is the charge q_3 in an equilibrium position? d = 4 cm.

Solution

1-Expression of $\overrightarrow{\mathbf{F}}$



$$\overrightarrow{F_3} = \vec{F}_{1/3} + \vec{F}_{2/3}$$

$$\vec{F}_{1/3} = K\frac{q_1 q_3}{r_{13}^2}\vec{i} = K\frac{q^2}{x^2}\vec{i}$$

$$\vec{F}_{2/3} = K\frac{q_2 q_3}{r_{23}^2}(-\vec{i}) = -K\frac{q^2}{9(d-x)^2}\vec{i}$$

$$\overrightarrow{F_3} = kq^2\left[\frac{1}{x^2} - \frac{1}{9(d-x)^2}\right]\vec{i}$$

2. Calculate x: At equilibrium

$$\overrightarrow{F_3} = \vec{0} \Longrightarrow \left[\frac{1}{x^2} - \frac{1}{9(d-x)^2}\right] = 0 \Longrightarrow \frac{1}{x^2} = \frac{1}{9(d-x)^2}$$

$$x^2 = 9(d-x)^2 \Longrightarrow x = \mp 3(d-x)$$

Two solutions: $\begin{cases} x_1 = 3cm \\ \quad or \\ x_2 = 6cm \end{cases}$

The second solution $\boldsymbol{x_2}$ is rejected because it is greater than d. Then the equilibrium is at $x_1 = 3cm$